Structural properties of urban street patterns and the Multiple Centrality Assessment

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Complex Networks - Equilibrium and Vulnerability Analysis with Applications – Catania Italy

Our Group

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  - Alessio Cardillo
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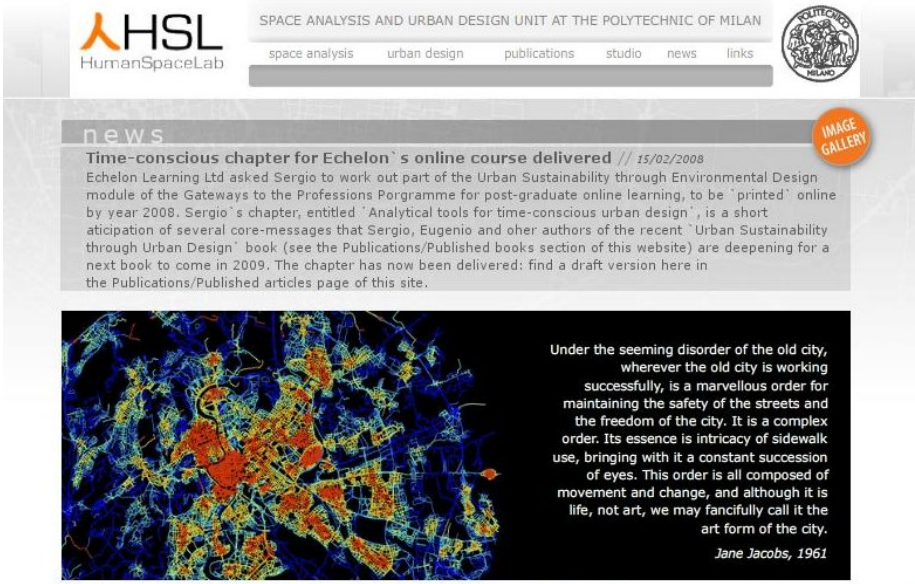
  - Sergio Porta
  - Emanuele Strano



http://www.humanspacelab.com
http://www.ct.infn.it/~latora/
http://www.ct.infn.it/~cardillo/

Outline

- Structural Properties ;

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- Multiple Centrality Assessment (MCA);

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- Relation between Centrality and Commercial Activity;

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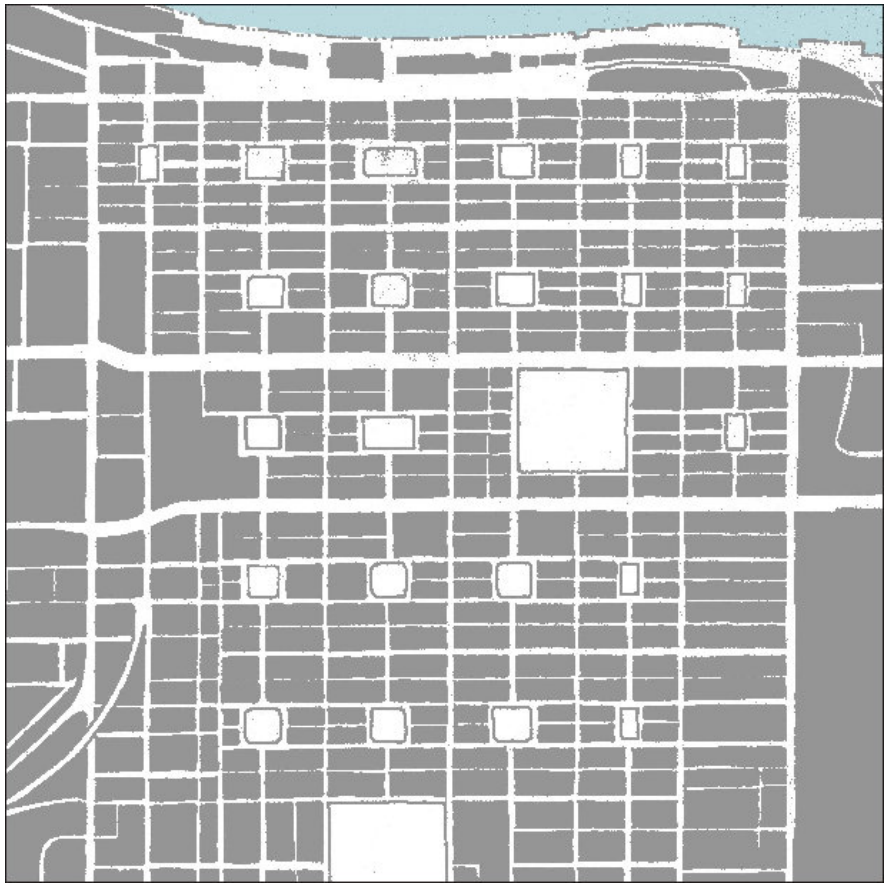
- Structural Properties ;
- Multiple Centrality Assessment (MCA);
- Relation between Centrality and Commercial Activity;
- Conclusions.

Complex Networks & Cities

Question:

How can we map a city into a graph?



Complex Networks & Cities

Answer:

We can follow two ways:

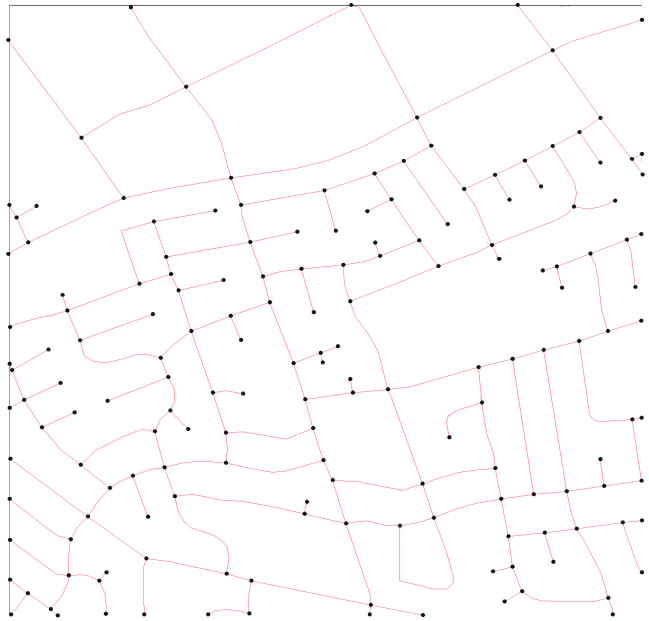
1. A Primal Approach;
2. A Dual Approach.

Complex Networks & Cities

Primal Approach

Complex Networks & Cities

Primal Approach



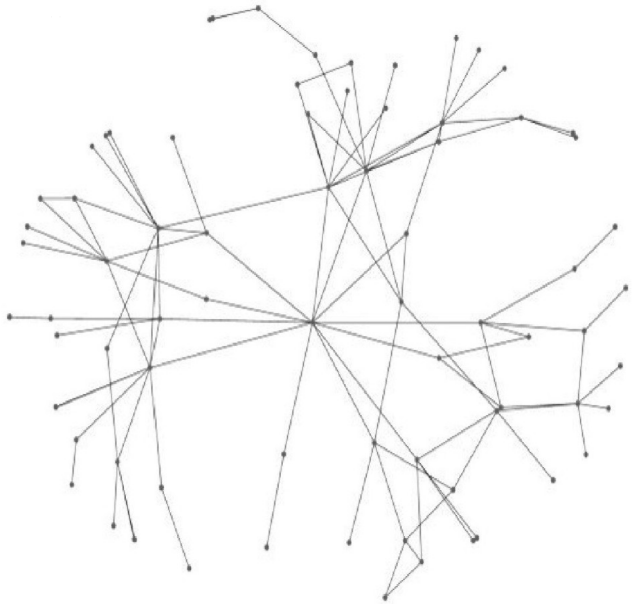
- Intersections as nodes, streets as edges;
- Focus on intersection.

Complex Networks & Cities

Dual Approach

Complex Networks & Cities

Dual Approach



- Streets as nodes, intersections as edges;
- Focus on streets;
- Not unique (name based, line of sight).

From Trees to GT

Question:

How to compare a graph?

From Trees to GT

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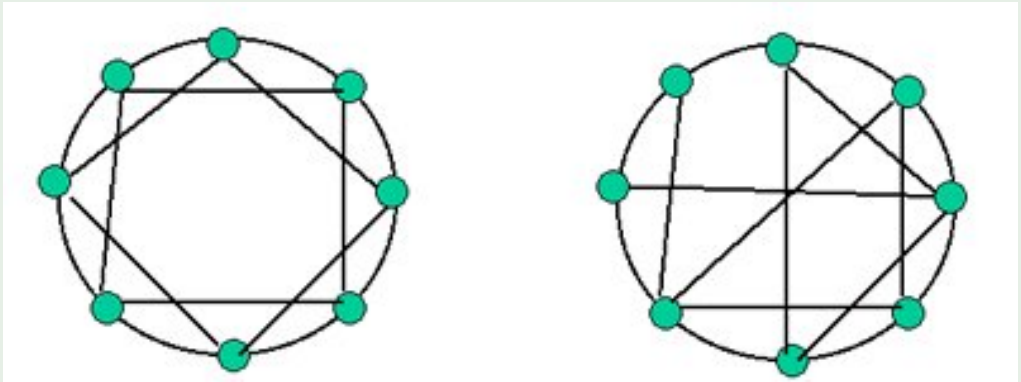
How to compare a graph?

Answer:

Usually real networks are compared with their randomized version.



From Trees to GT

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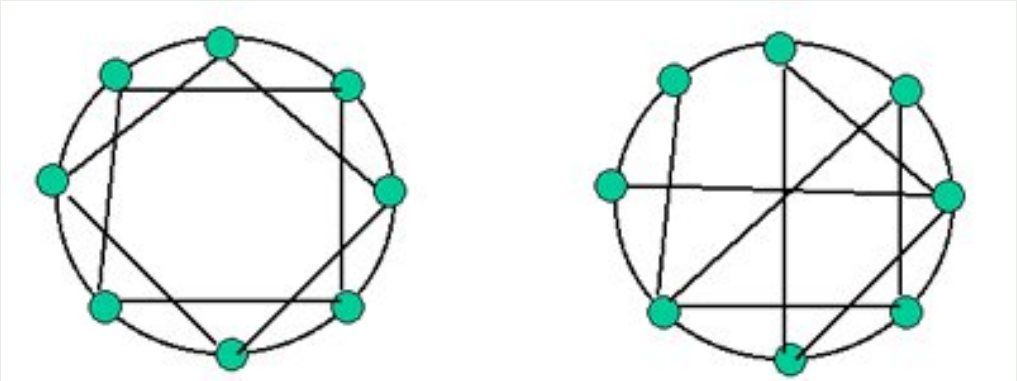
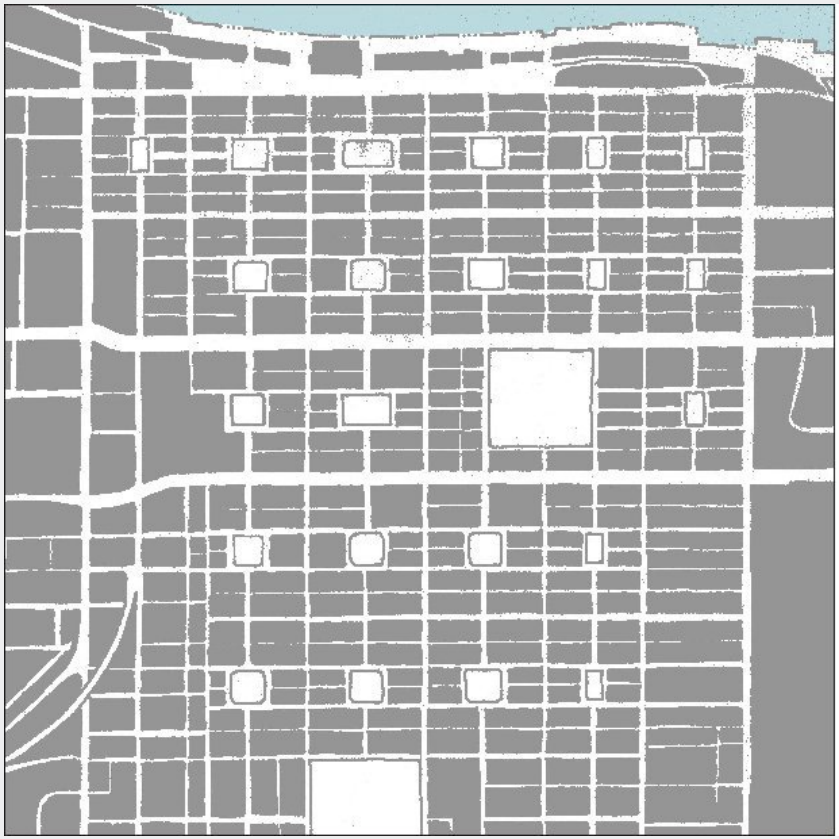


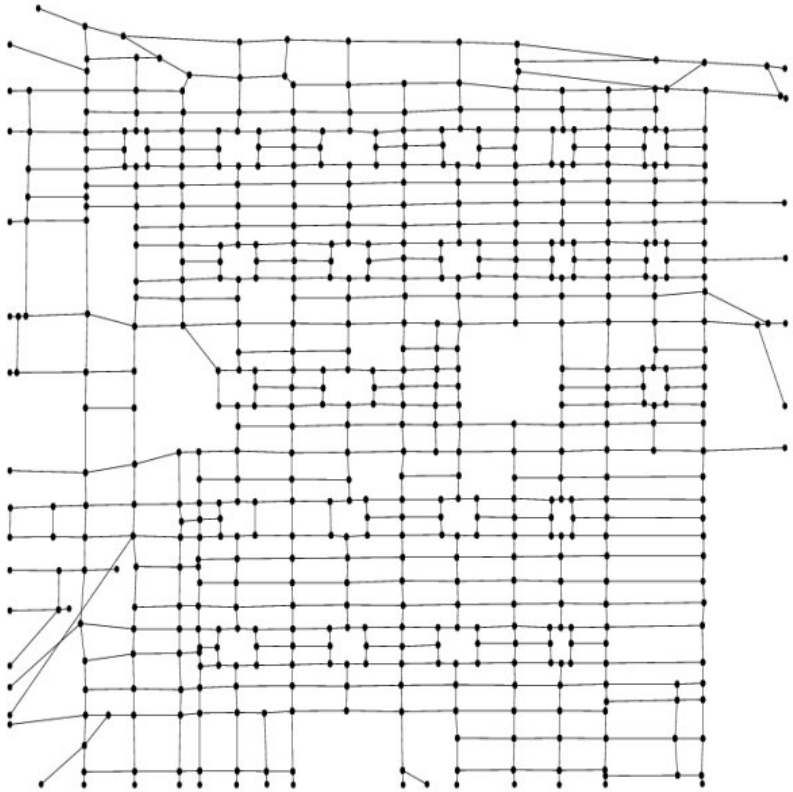
Randomization $\longrightarrow$ Loss of Planarity;
Use of extreme cases:

- Minimum Spanning Tree (MST);
- Greedy Triangulation (GT).

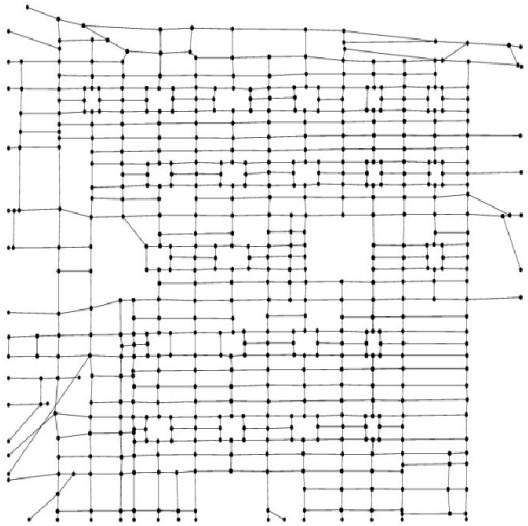
From Trees to GT

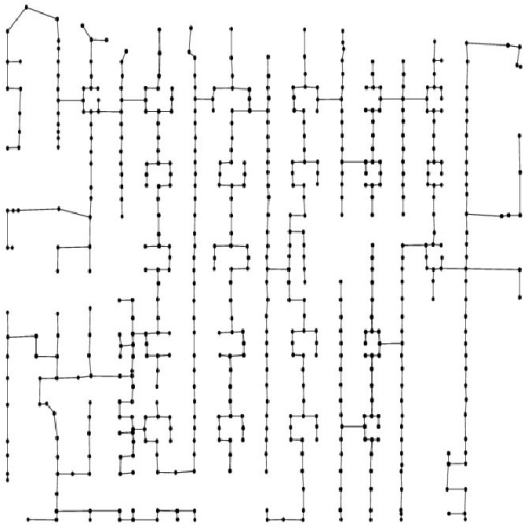
An example: the city of Savannah





From Trees to GT

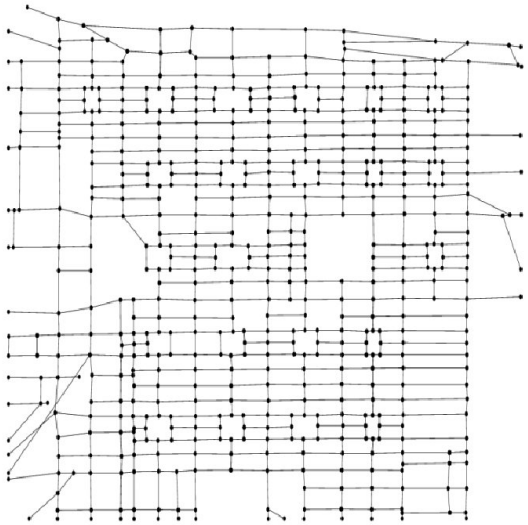


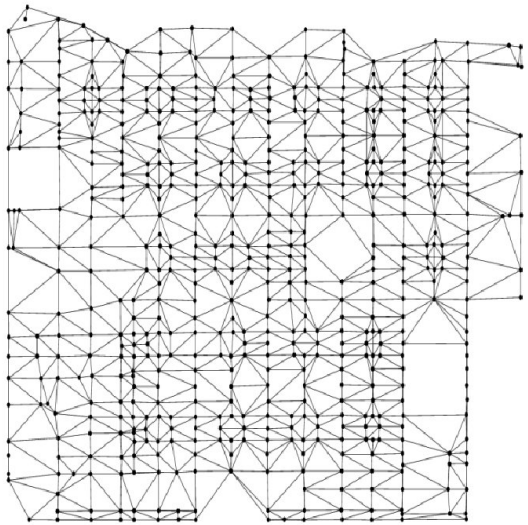


Definition

The minimum spanning tree (MST) is the shortest tree which connects every node into a single connected component.

From Trees to GT

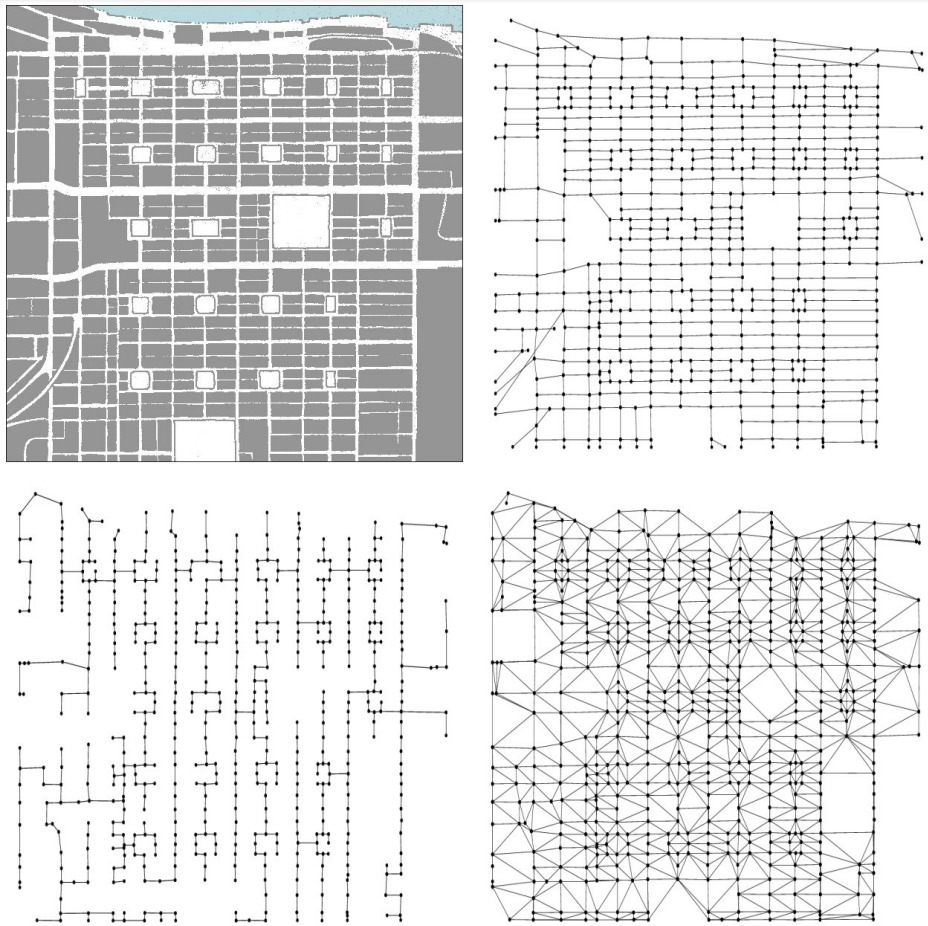




Definition

The greedy triangulation (GT) is the planar graph with the highest number of edges K_{max}, and that minimize the total length.

From Trees to GT



A. Cardillo, S. Scellato, V. Latora, S. Porta, Phys. Rev. E **73** (2006) 066107.

S. Scellato, A. Cardillo, V. Latora, S. Porta, Eur. Phys. Journ. B **50** (2006) 221.

Topological Properties

- Fractal Dimension d_{BOX}

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Topological Properties

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- Global Properties
  - Cost W;
  - Global Efficiency $E(G)$.

Local Properties

Meshedness Coefficient

The Meshedness Coefficient M is more general measures of the structure of cycles than the clustering coeficient.

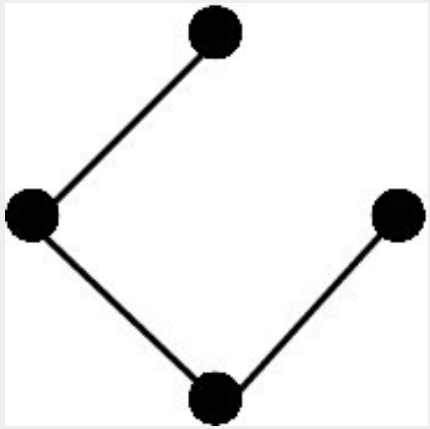
$$M = \frac{f}{f_{max}} = \begin{cases} 0 & \text{if } G \text{ is a tree,} \\ 1 & \text{if } G \text{ is a complete planar graph.} \end{cases}$$

where
$$f = m - n + 1\,,$$
$$f_{max} = 2n - 5\,.$$

J. Buhl et.al. Eur. Phys. J. B **42**, 123 – 129 (2004)

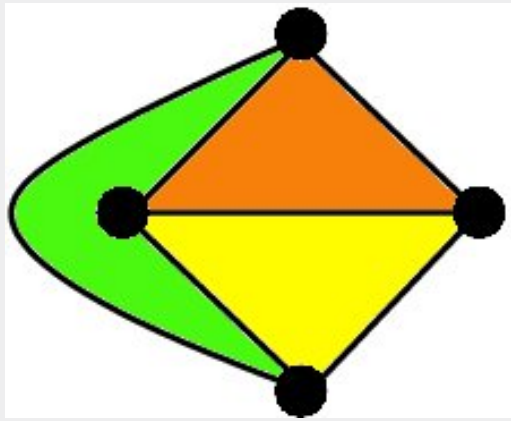
Local Properties

An example of Meshedness:



$$f = m - n + 1 = 3 - 4 + 1 = 0$$

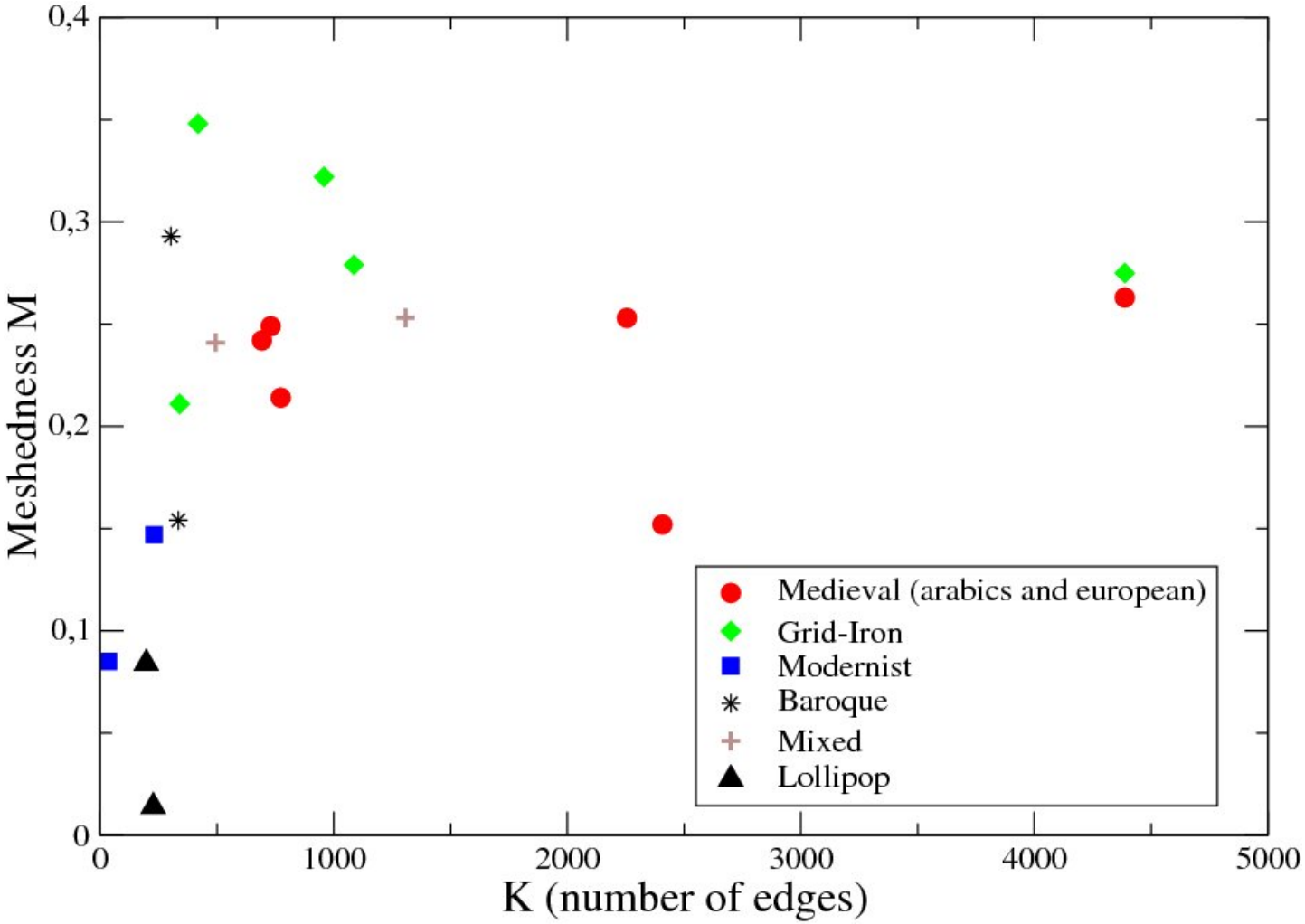
$$M = \frac{0}{2 \cdot 4 - 5} = \frac{0}{3} = 0$$



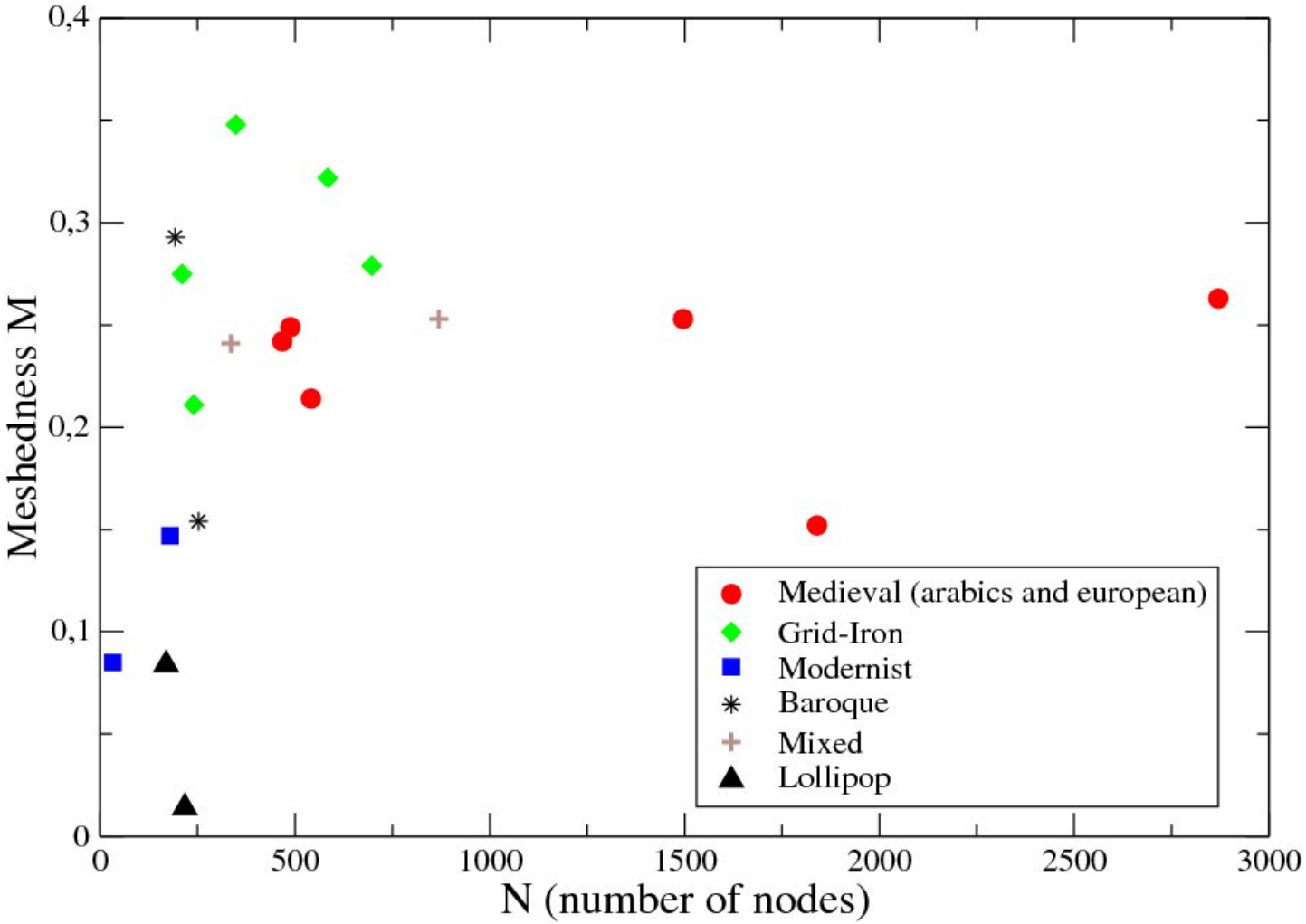
$$f = m - n + 1 = 6 - 4 + 1 = 3$$

$$M = \frac{3}{2 \cdot 4 - 5} = \frac{3}{3} = 1$$

Local Properties



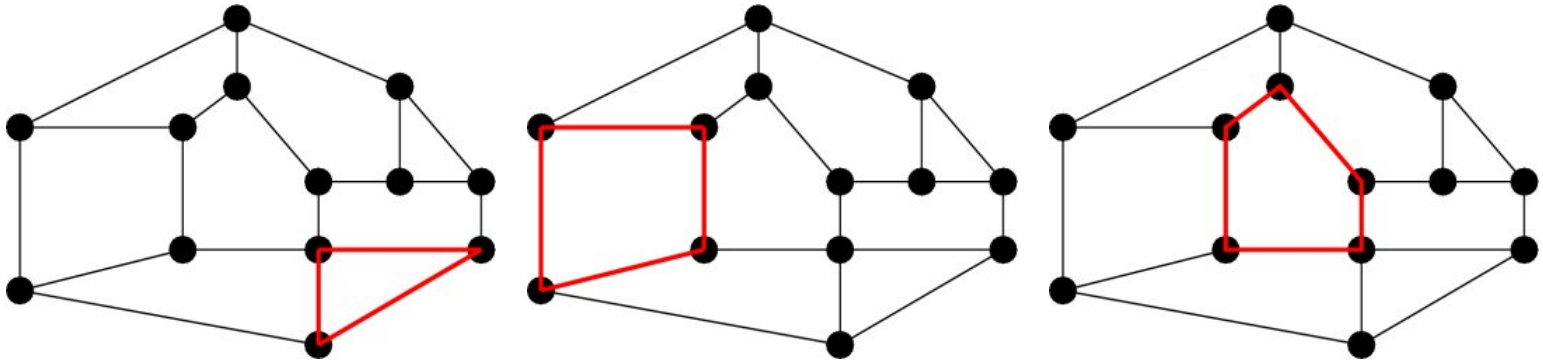
Local Properties



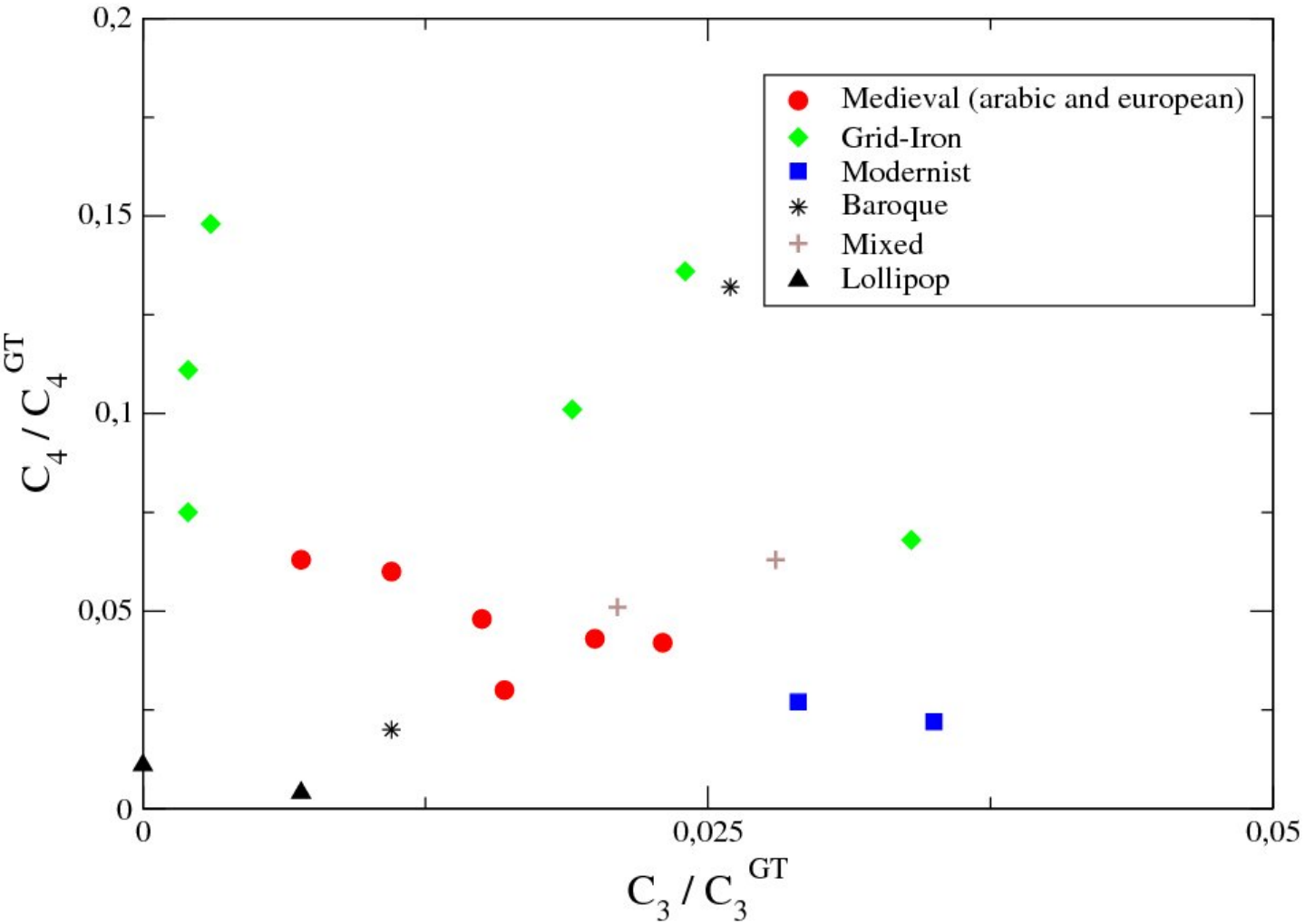
Local Properties

Motifs

In order to evaluate the complexity of graph inner structure one could count the number of cycles of length three (triangles), four (squares) and five (pentagons). A motif is significant if its number is higher than the number found in an equivalent random graph.



Local Properties



Global Properties

Cost

To take into account the ammount of resources needed to “build” a street one could consider several functional forms. The simplest one is the Euclidean distance between start and arrival node. This is the so called Cost.

$$W_{ij} = \sqrt{(x_j - x_i)^2 - (y_j - y_i)^2} \; ;$$

Global Properties

Global Efficiency

A measure of the typical separation between two nodes in the graph is given by the average shortest path length, also known as characteristic path length L, defined as:

$$L = \frac{1}{N(N-1)} \sum_{i,j \in G \, i \neq j} d_{ij} \; ;$$

An alternative approach is to consider the harmonic mean of geodesic lengths, and to define the so-called efficiency of G as:

$$E(G) = \frac{1}{N(N-1)} \sum_{i,j \in G \, i \neq j} \frac{1}{d_{ij}} \; ;$$

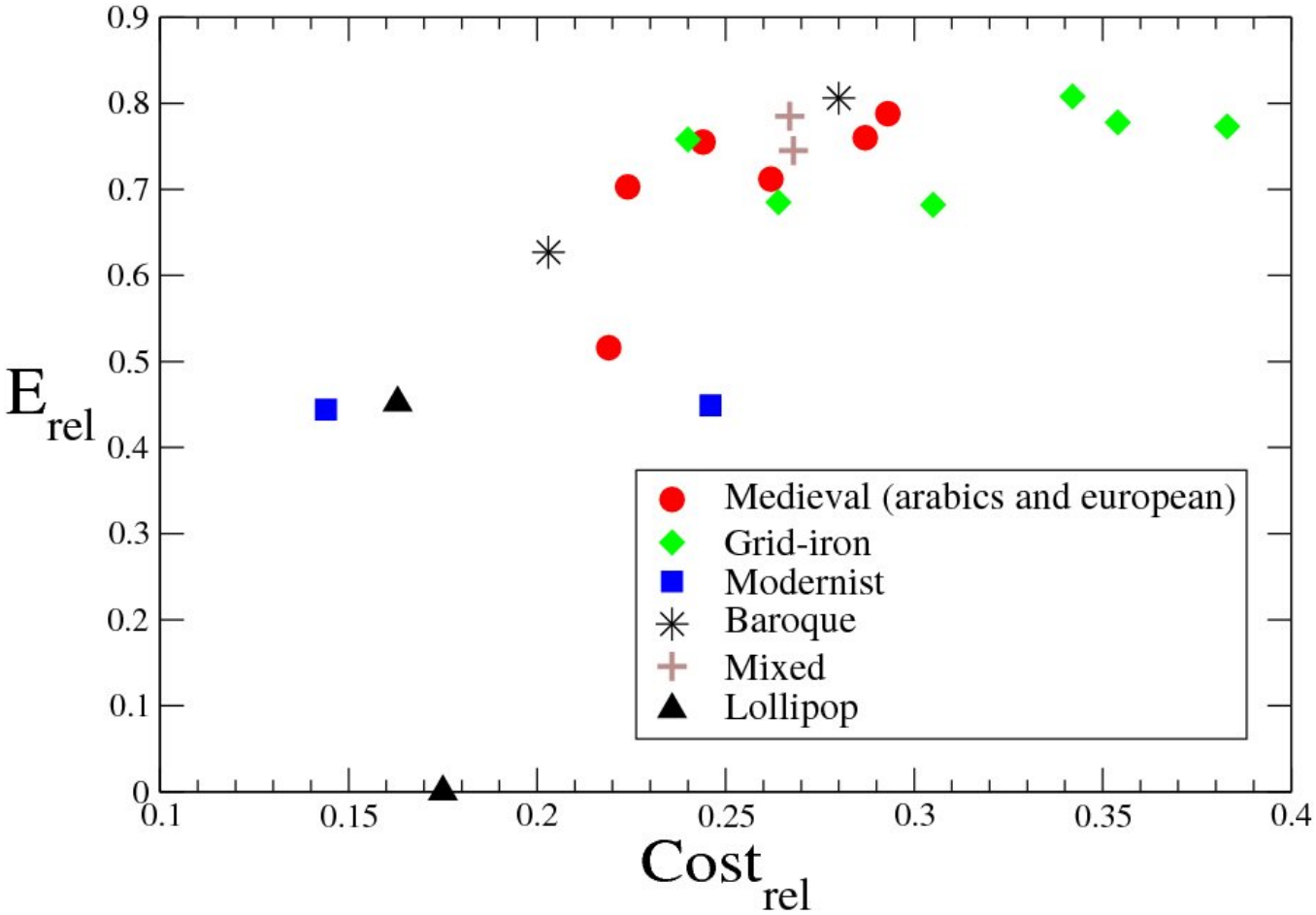
V. Latora, M. Marchiori, PRL **87**, 198701 (2001)

Data Analisys

In order to compare different city we used two quantities: Relative Efficiency E_{rel} and Relative Cost W_{rel} defined as:

$$E_{rel} = \frac{E - E^{MST}}{E^{GT} - E^{MST}}; \qquad W_{rel} = \frac{W - W^{MST}}{W^{GT} - W^{MST}};$$

Data Analisys



A. Cardillo, S. Scellato, V. Latora, S. Porta, Phys. Rev. E**73** (2006) 066107.

Multiple Centrality Assessment (MCA)

Main Scope

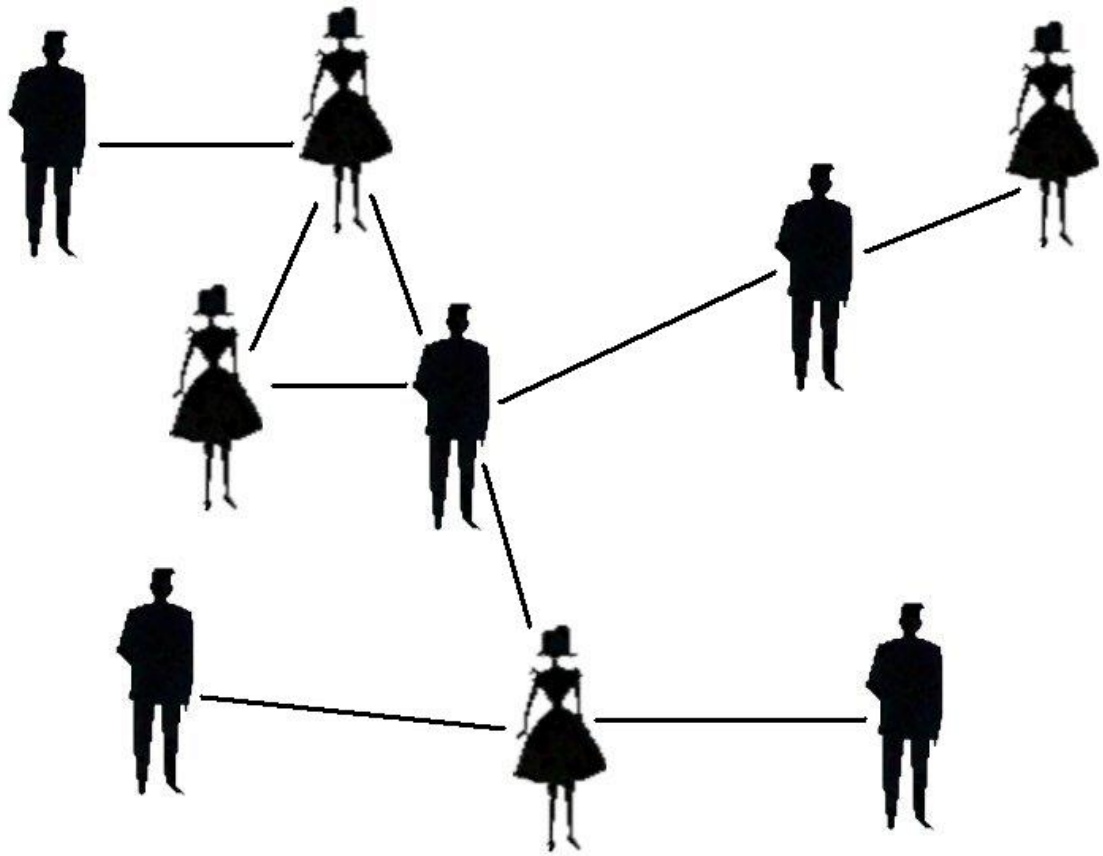
Set up a methodology for investigation of spatial systems as complex networks to spot the hot places, the critical components and the central routes of a city.

P. Crucitti, V. Latora, S. Porta, Chaos **16** (2006) 015113

Centrality Indices

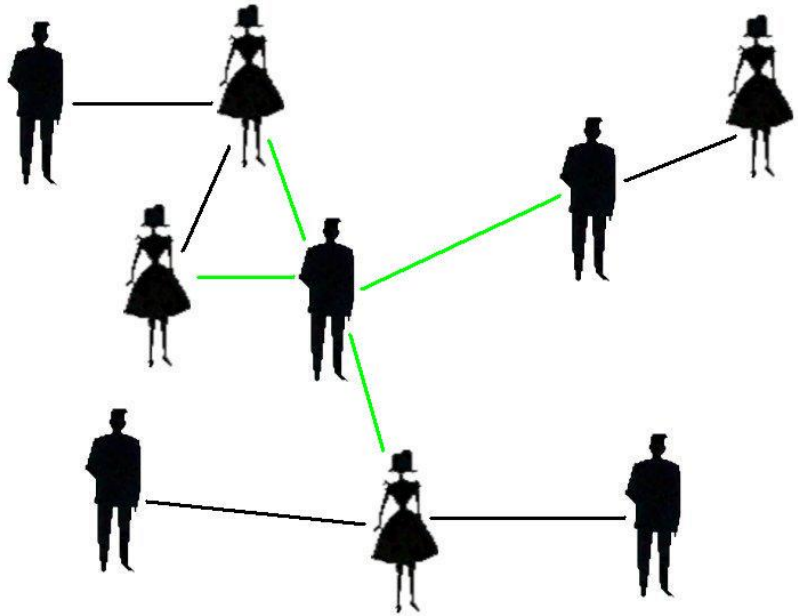
Question:

What does it mean being "Central"? (Bavelas 1948)



Centrality Indices

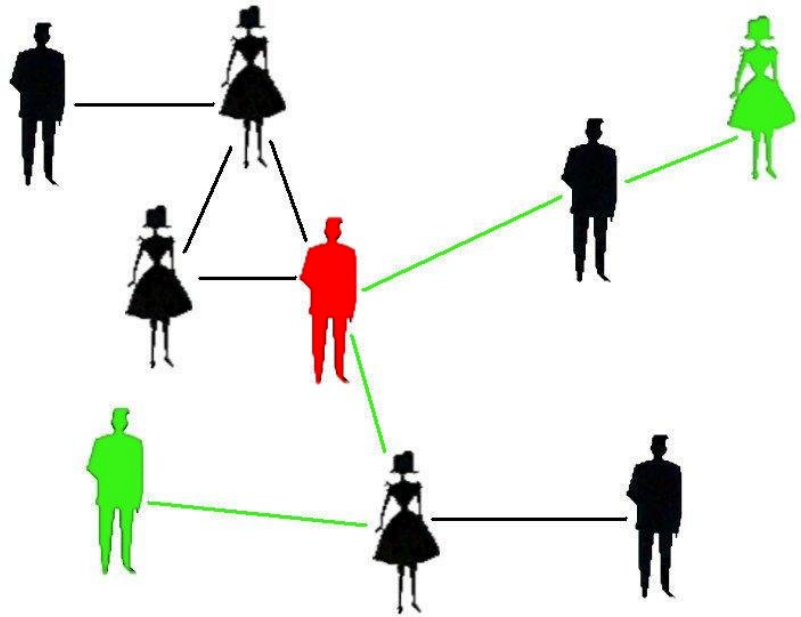
It means know “many” people? Degree Centrality; (Nieminem 1974)



$$k_i = \sum_{j=1}^{N} a_{ij} ;$$

Centrality Indices

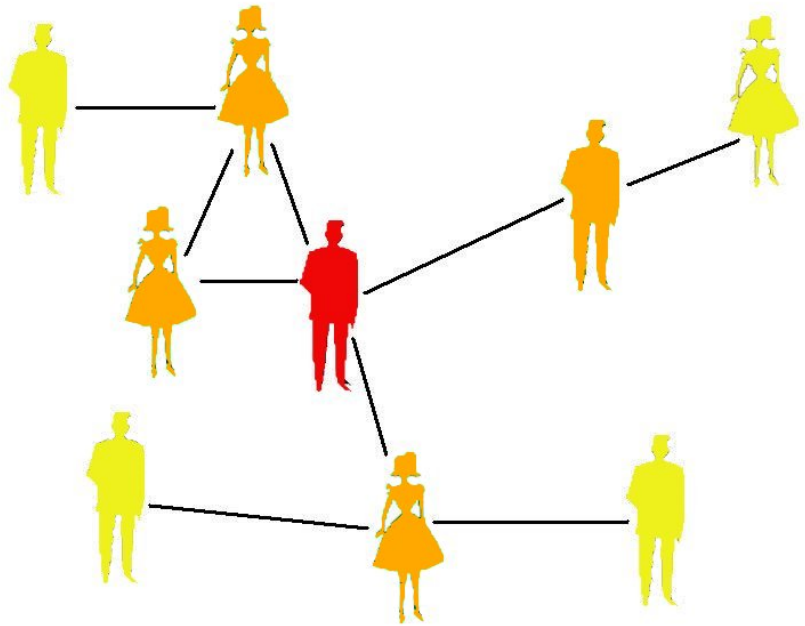
Or act like a "bridge" between two nodes?
Betweenness Centrality; (Freeman 1977)



$$C_i^B = \frac{1}{(N-1)(N-2)} \sum_{\substack{j,k \in G \\ j \neq k \neq i}} \frac{n_{jk}(i)}{n_{jk}};$$

Centrality Indices

Or being more “close” to other nodes?
Closeness Centrality; (Sabidussi 1966)

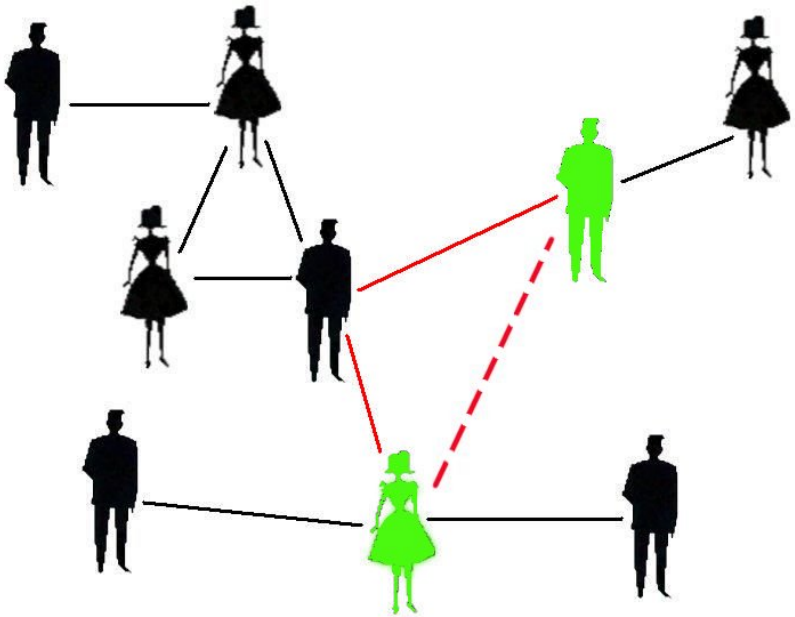


$$C_i^C = \frac{N-1}{\sum_{j \in G} \frac{1}{d_{ij}}};$$

P. Crucitti, V. Latora and S. Porta Phys. Rev. E **73** (2006) 036125

Centrality Indices

Or how much you are in a straight line with others? Straightness Centrality; (Crucitti 2006)

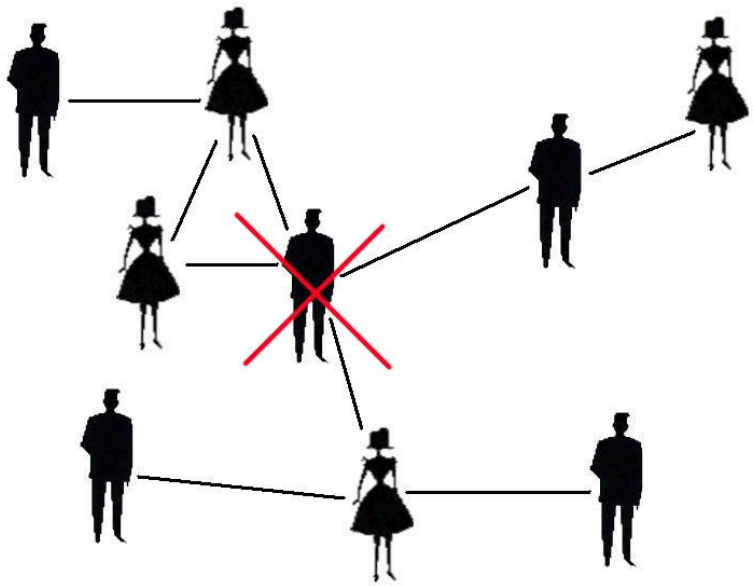


$$C_i^S = \frac{1}{N-1} \sum_{\substack{j \in G \\ j \neq i}} \frac{d_{ij}^{Eucl.}}{d_{ij}} ;$$

P. Crucitti, V. Latora and S. Porta Phys. Rev. E **73** (2006) 036125

Centrality Indices

Or how much your removal affect network efficiency Latora (2005)? Information Centrality; (Latora 2007)



$$C_i^I = \frac{\Delta E}{E} = \frac{E(G) - E(G')}{E(G)},$$

$$E(G) = \frac{\sum_{i,j \in G} \frac{1}{d_{ij}}}{N(N-1)};$$

V. Latora, M. Marchiori, Phys. Rev. E **71** (2005) 015103R.

V. Latora, M. Marchiori, New. J. Phys **9** (2007) 188

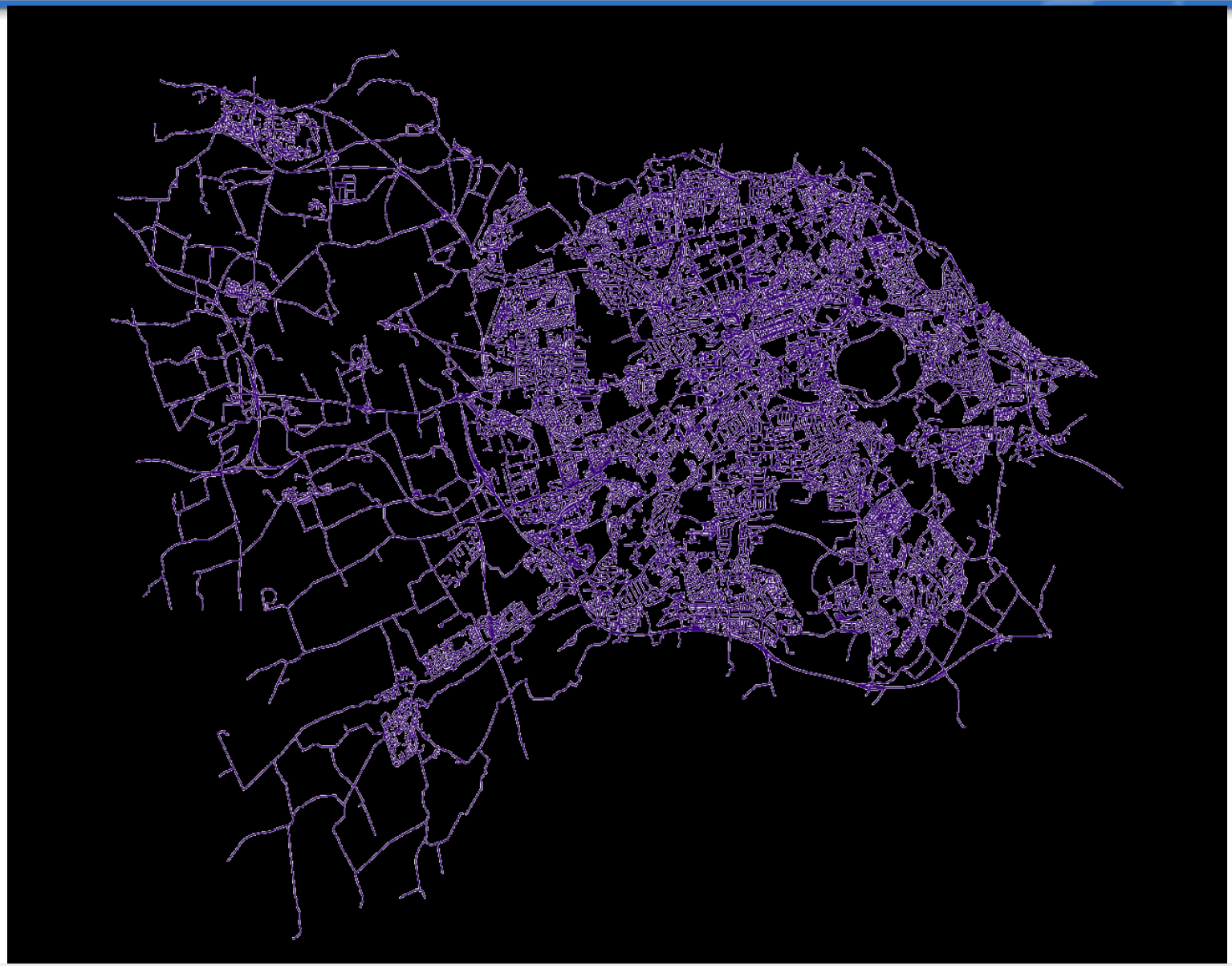
Centrality Indices

Answer:

All of these features make you "Central" !!!

S. Porta, P. Crucitti, V. Latora, Environmental and Planning B**33** (2006) 705.
P. Crucitti, V. Latora, S. Porta, Chaos **16** (2006) 015113.

Some Example of MCA



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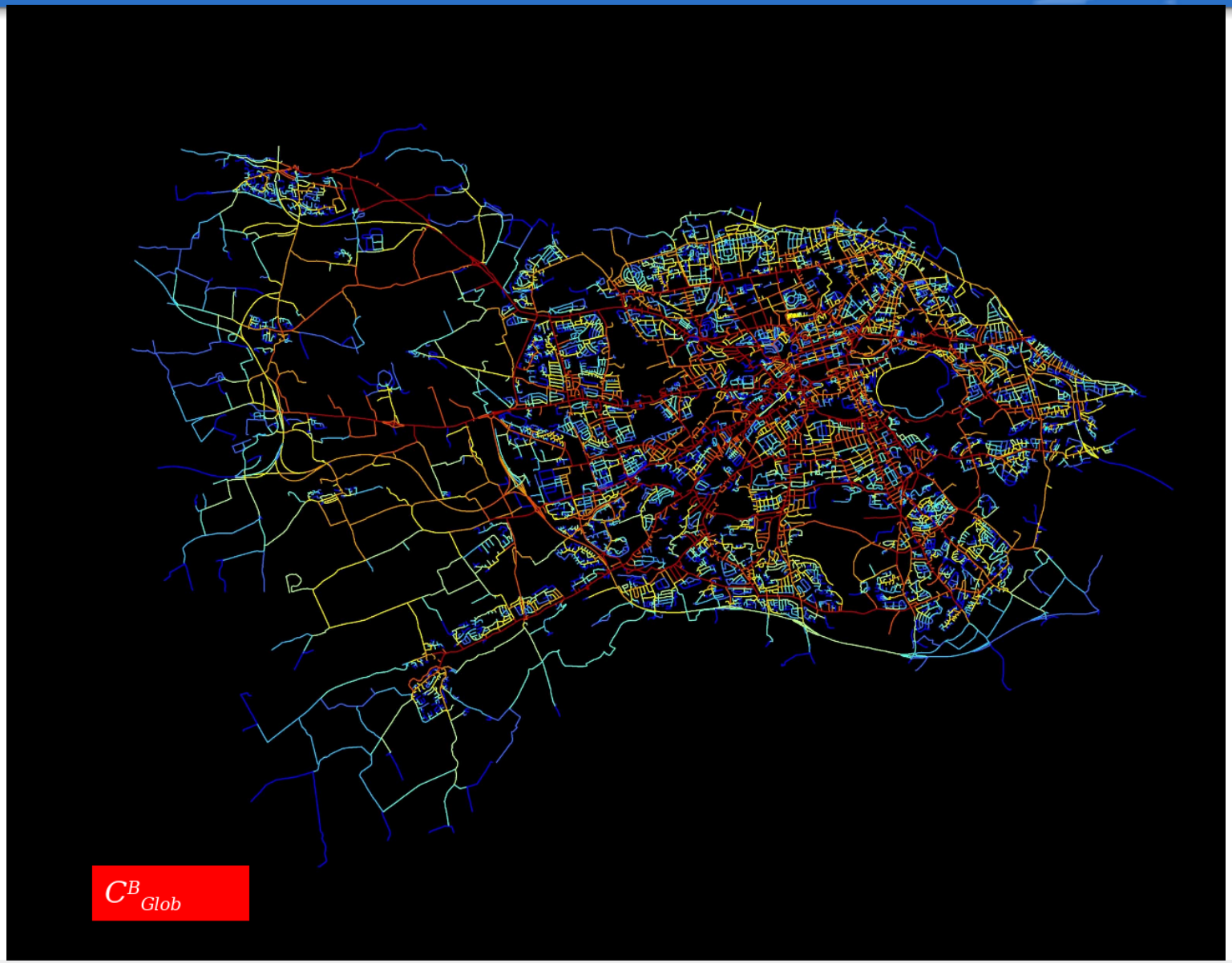

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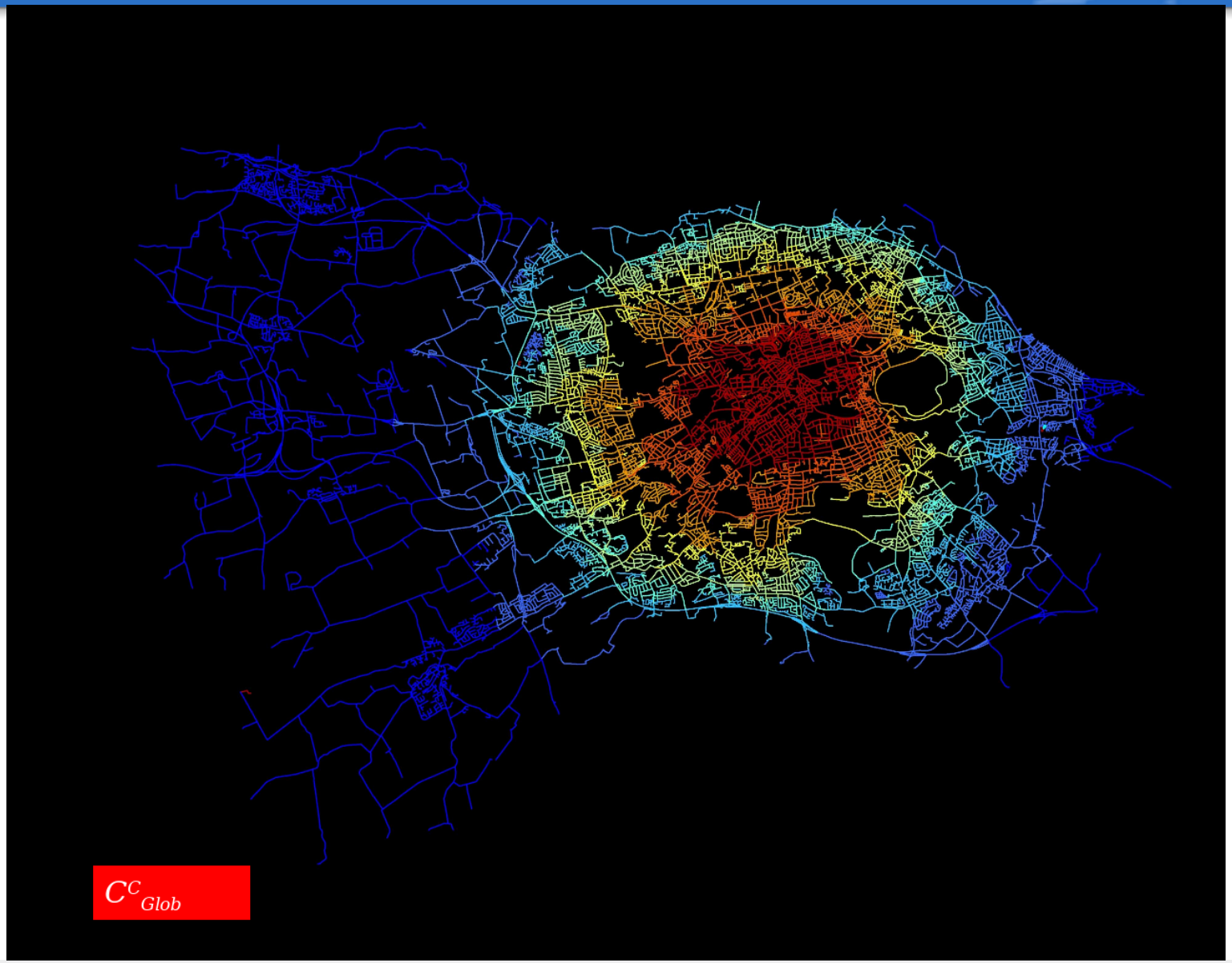

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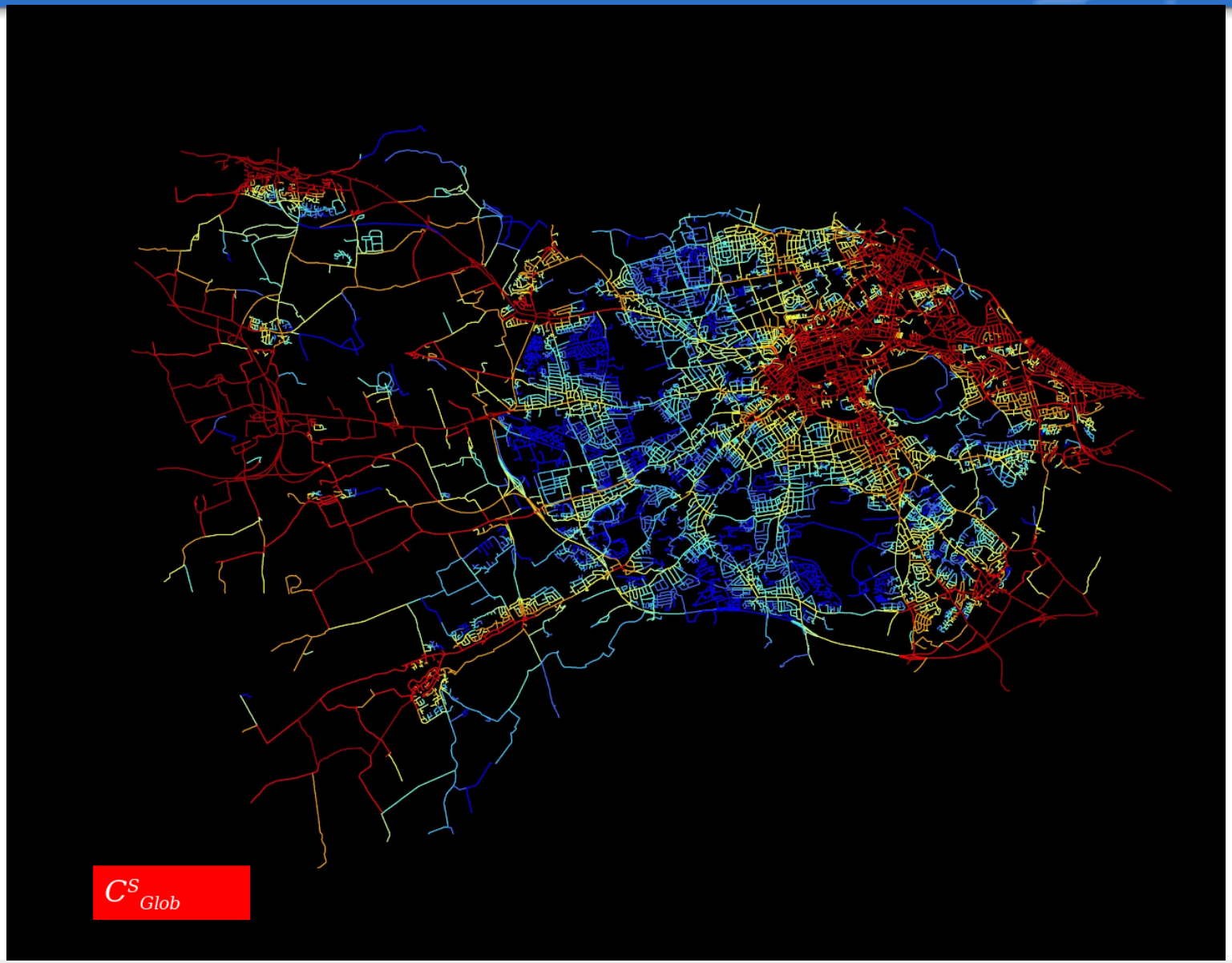
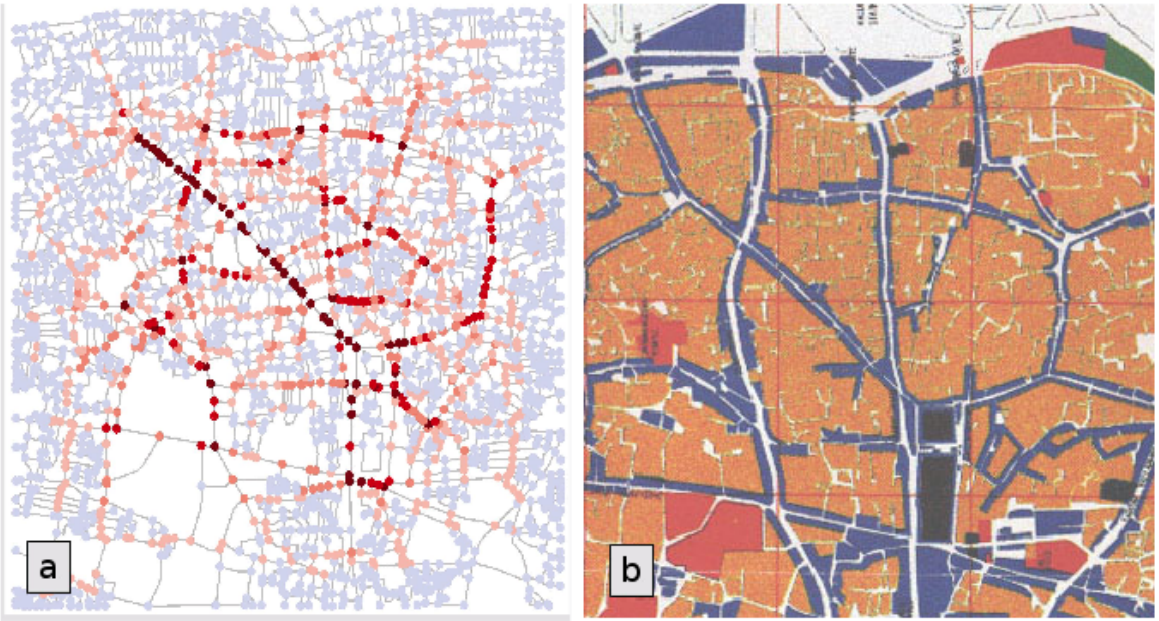


Relationship Between Centrality and Commerce Retail

The Grocer's Mantra

- Mario runs his business at the street corner just in front of my door.
- He told me that he spent one week in the street before making the choiche for the location of his grocery
- "The first thing – he said – is where do the people walk"
- Mario is quite good in his work, he says "You have to be central, people are where centrality is".

Relationship Between Centrality and Commerce Retail

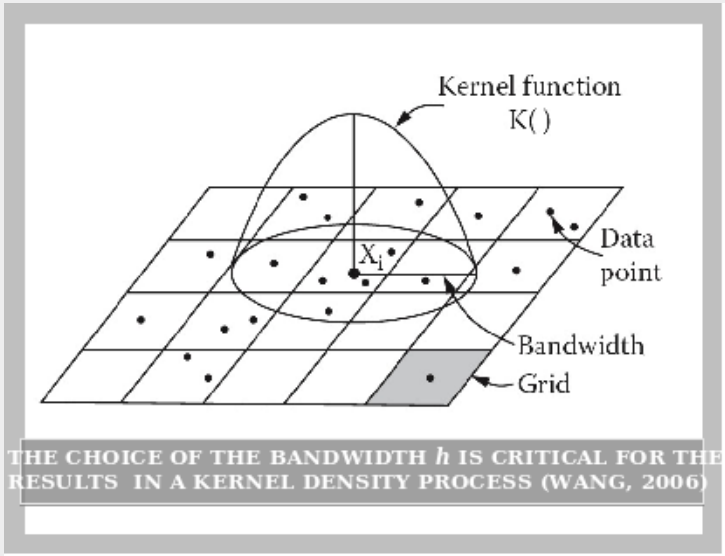


Question:

Is there empirical evidence of the correlation between street centrality and economic activities?

Relationship Between Centrality and Commerce Retail

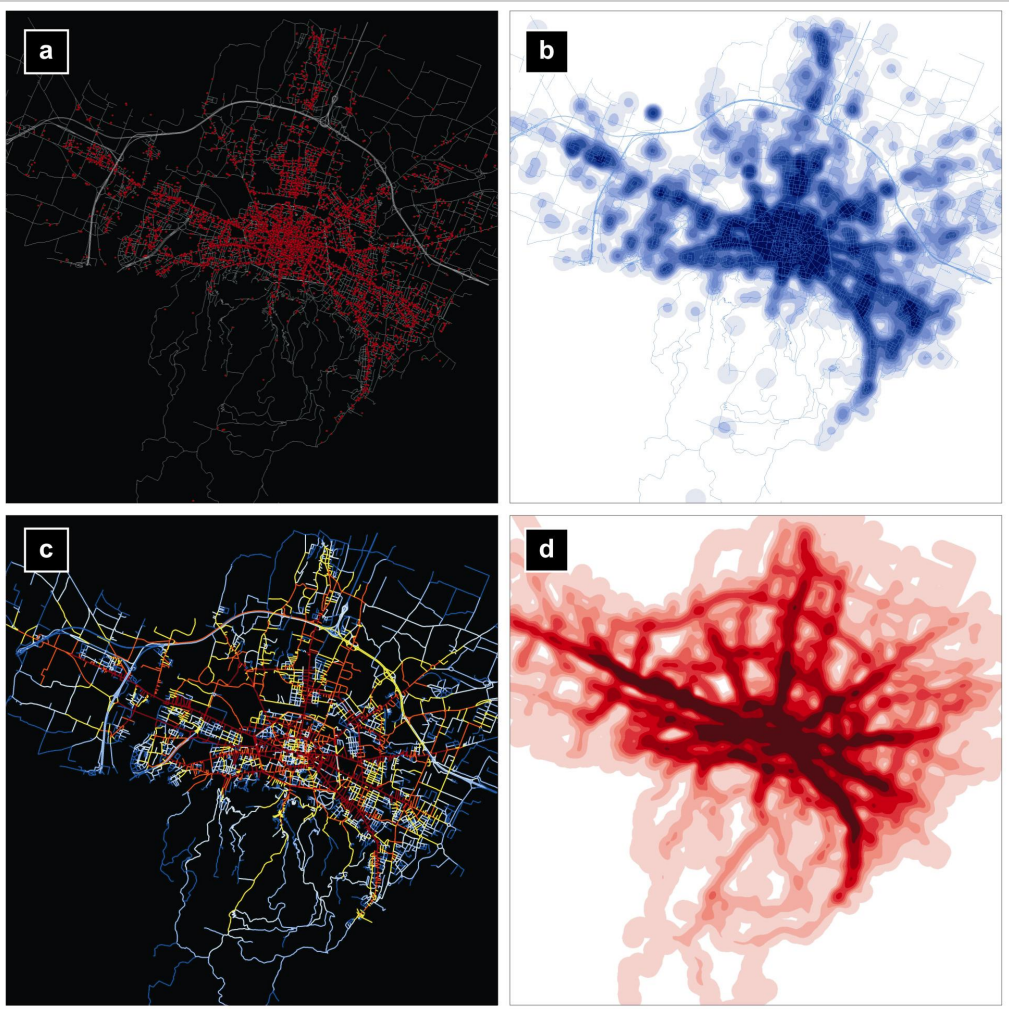
- In order to study the relation between centrality and commerce the space has been divided into square cells. Per each cell a Kernel Density Estimator (KDE) function has been calculated. The same operation has been done with centrality.



THE CHOICE OF THE BANDWIDTH h IS CRITICAL FOR THE RESULTS IN A KERNEL DENSITY PROCESS (WANG, 2006)

Some Results

Case study: the city of Bologna



Porta S et. al. – Street Centrality and Densities of Retails and Services in Bologna, Italy – **physics/0701111**. In press in Env. Plann. B

What are we doing

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What are we doing

- Application of MCA to large scale graphs (entire cities);
- Comparison between city structure and biological systems like leafs ;
- Elaboration of a city growth model based not only on geometrical costraint but also on centrality and retail.

Summary

- Analysis of structural properties of planar graphs based on urban street patterns;

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- Elaboration of a method to compare different spatial graphs;

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- Analysis of city structure based on multiple centrality indices (MCA);

Summary

- Analysis of structural properties of planar graphs based on urban street patterns;
- Elaboration of a method to compare different spatial graphs;
- Analysis of city structure based on multiple centrality indices (MCA);
- Relation between centrality and commercial activities $\longrightarrow$ Geomarketing.

References

- A. Cardillo, S. Scellato, V. Latora, S. Porta,Phys. Rev. E **73** (2006) 066107
- S. Scellato, A. Cardillo, V. Latora, S. Porta, Eur. Phys. Journ. B **50** (2006) 221
- P. Crucitti, V. Latora and S. Porta Phys. Rev. E **73** (2006) 036125
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