Comparing spatial networks: A 'one size fits all' efficiency-driven approach

Alessio Cardillo

Dept. Engineering Mathematics – University of Bristol – United Kingdom

Spatially Embedded Networks Workshop – University of Bristol – United Kingdom
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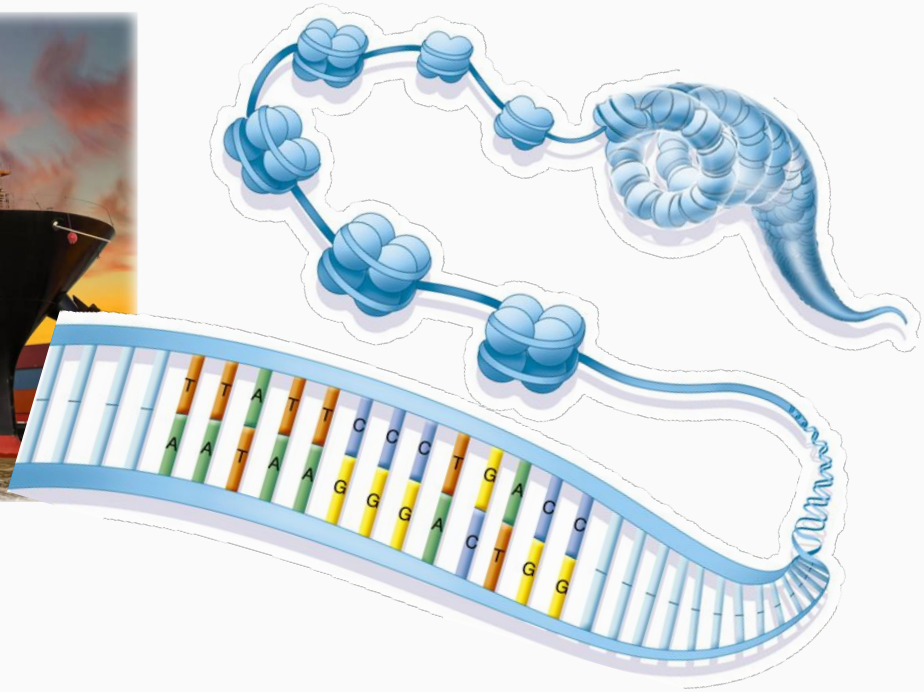
Systems embedded into space

Systems embedded into space



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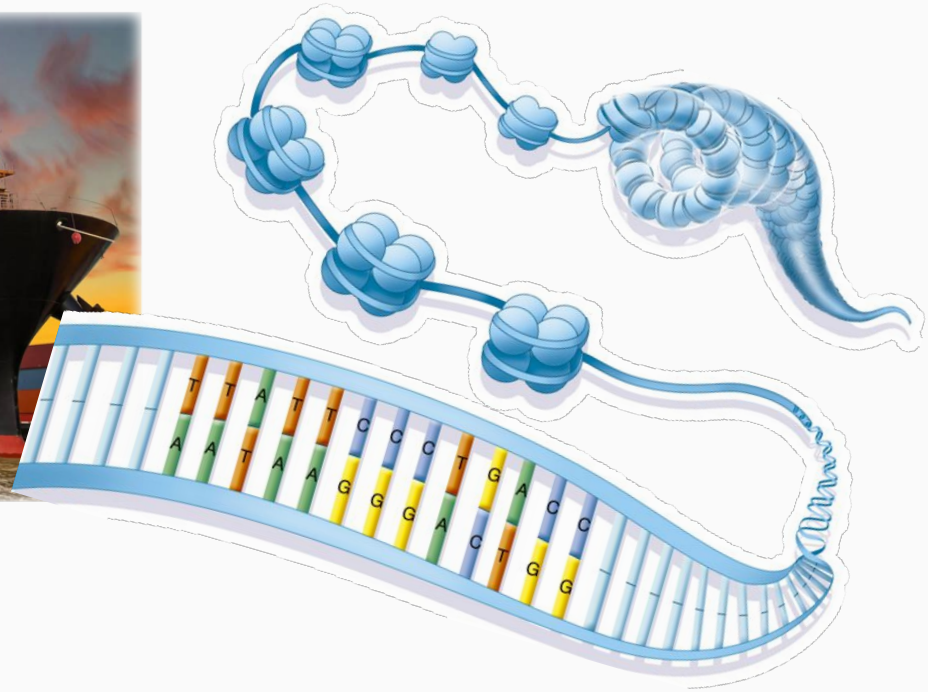

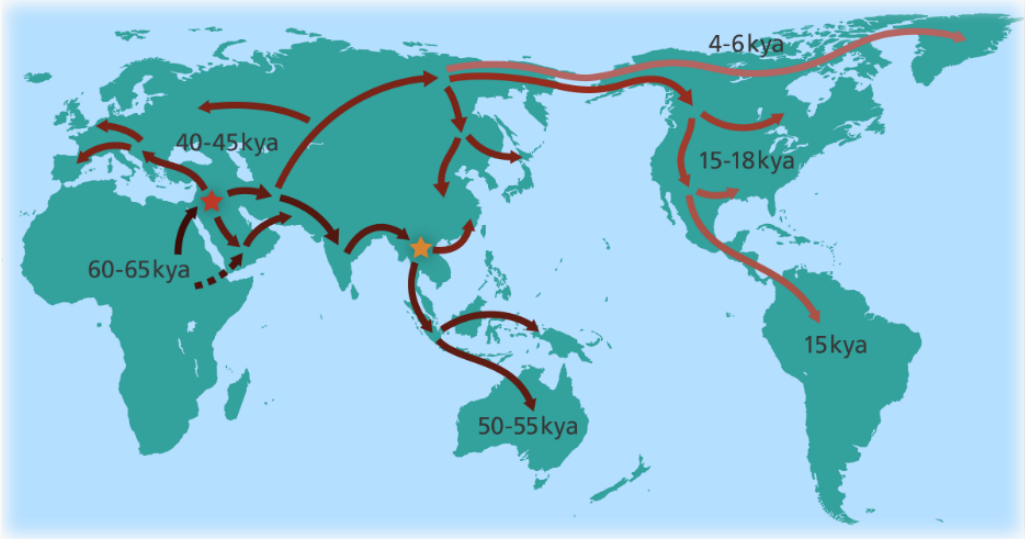

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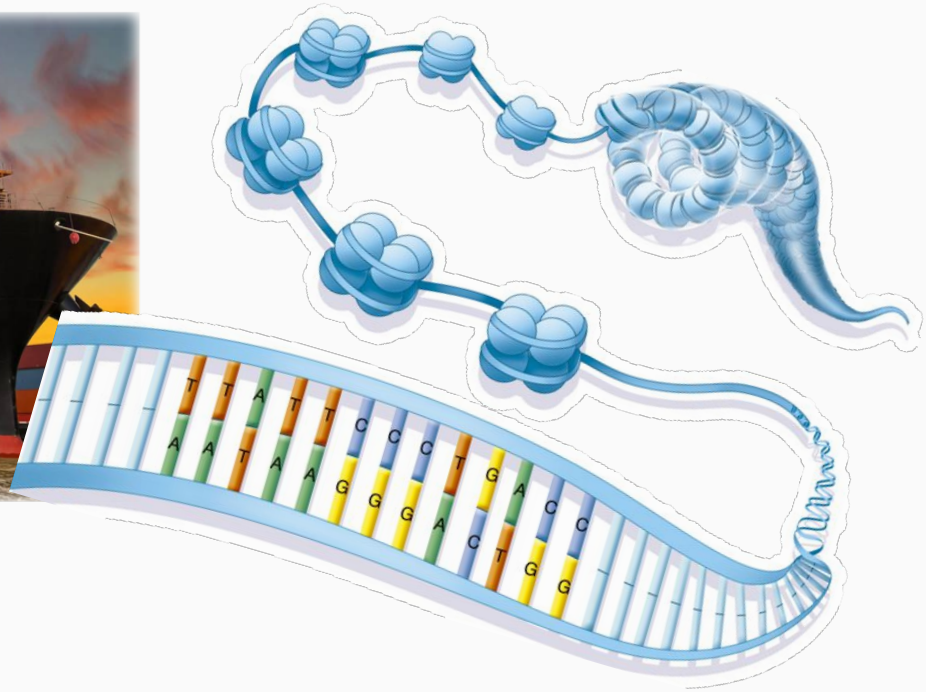

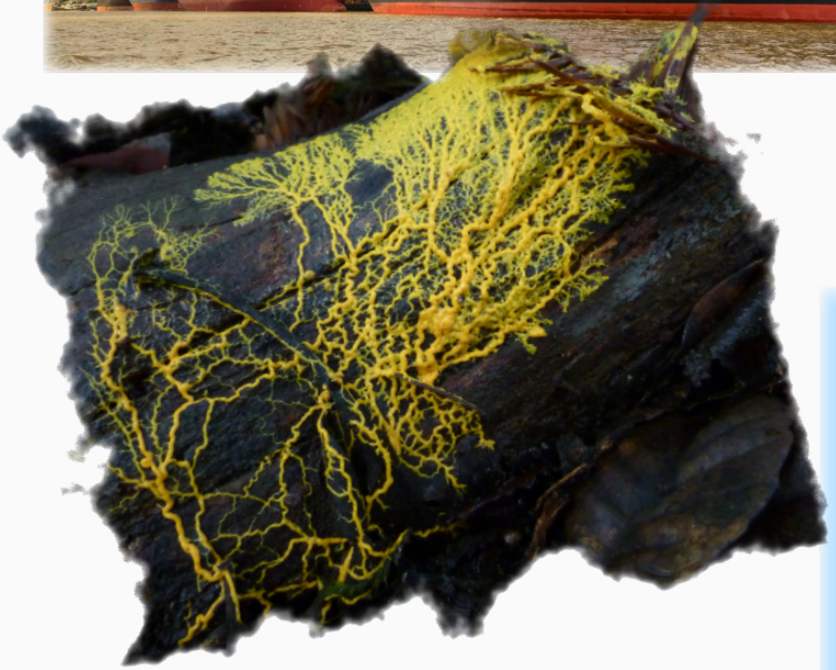

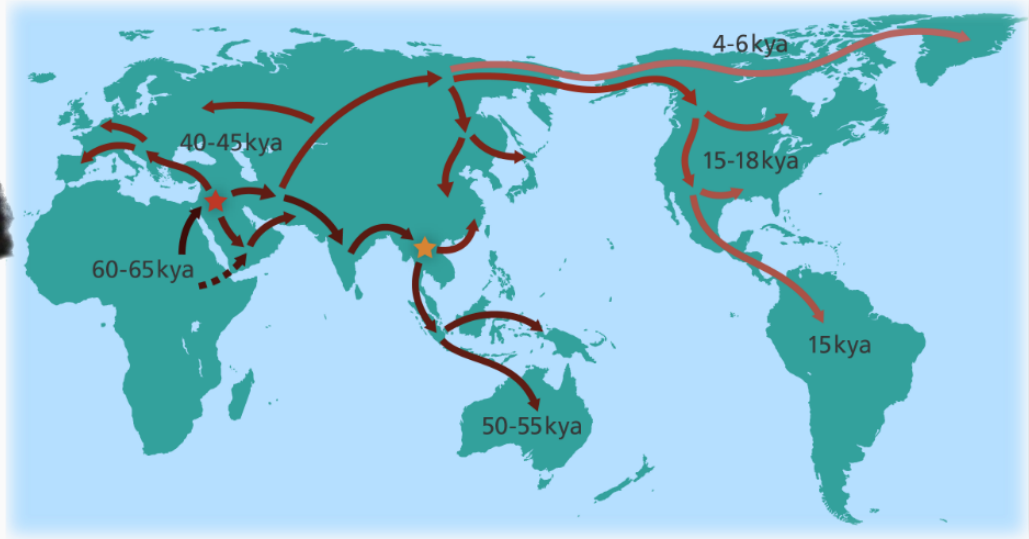

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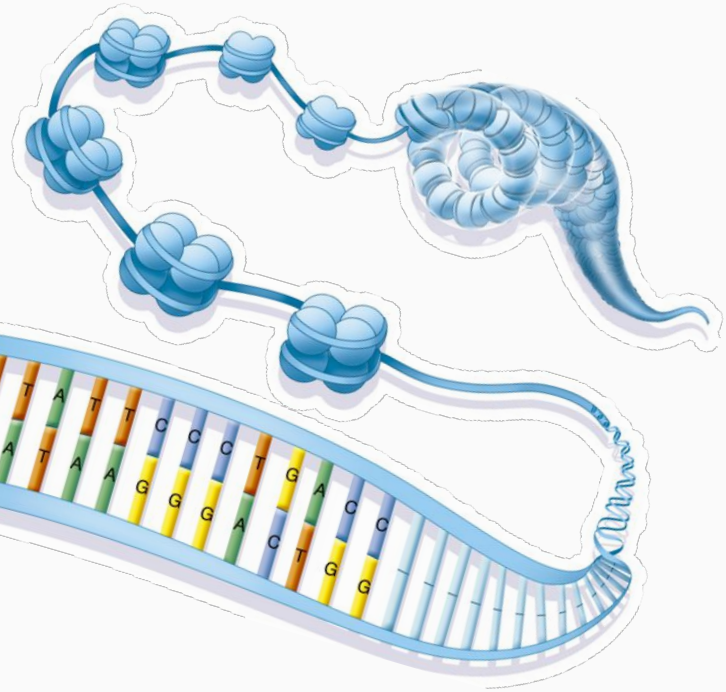
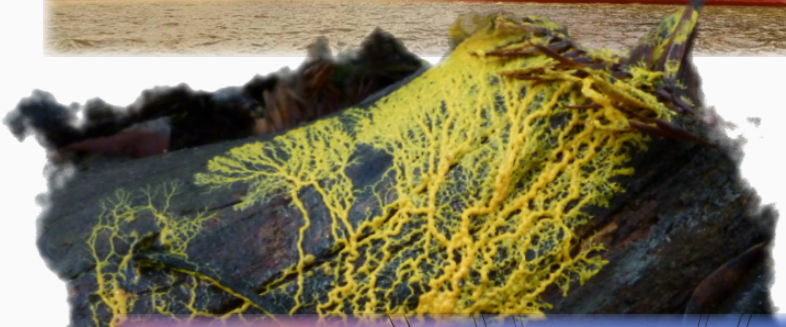
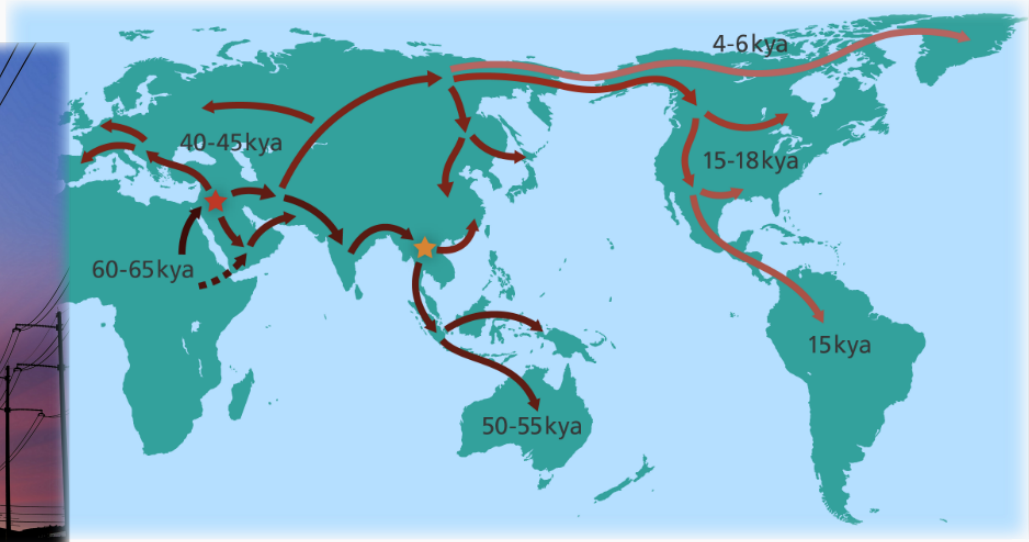






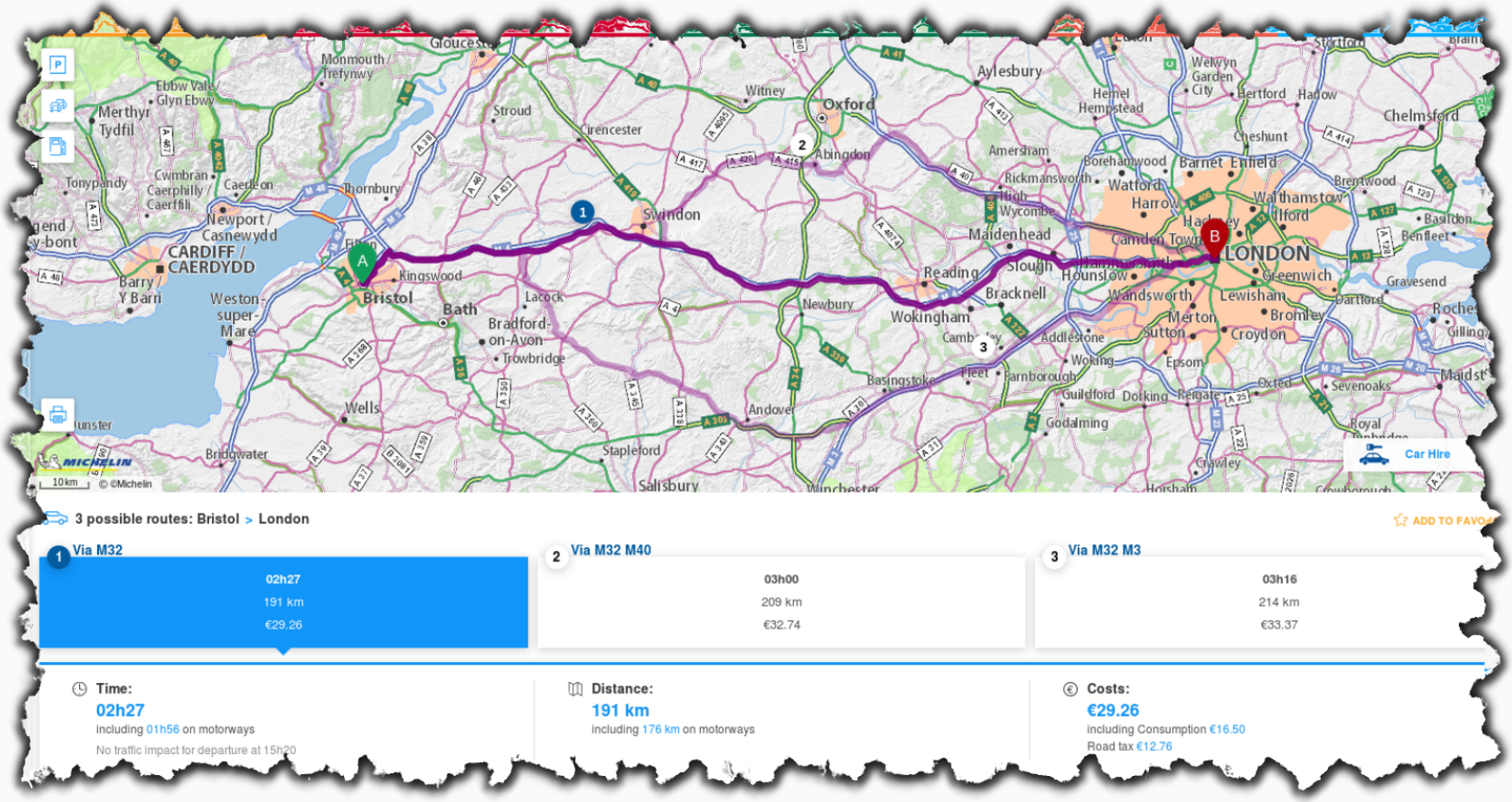
Systems embedded into space


How can we measure (and compare)
the performances of a spatial network?


SAMSUNG
SAMSUNG
SAMSUNG

Cost

One simple example: route search



Cost

The limitation of the total amount of resources available reflects on the structure of the network

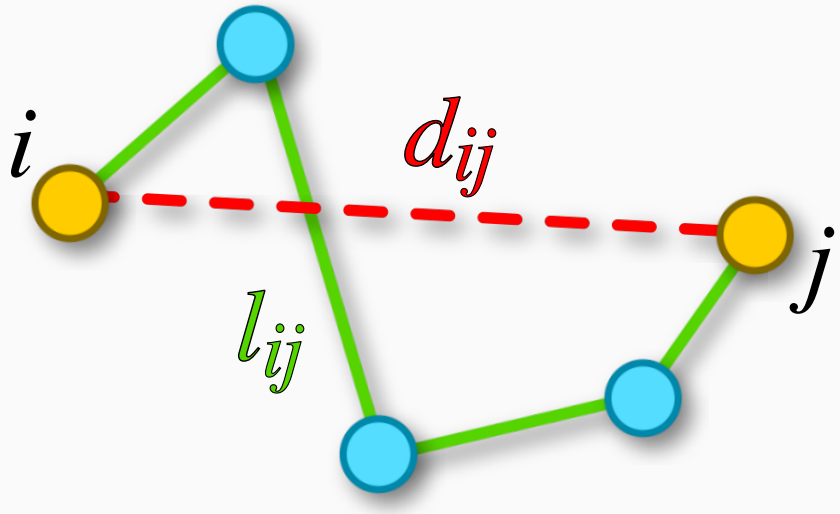
Cost

Total cost

$$L_{\text{tot}} = \sum_{ij} a_{ij}\, d_{ij}$$

- Barthélemy M 2011 *Phys. Rep.* **499** 1–101

Efficiency



Detour index/distance ratio

$$e_{ij} = \frac{d_{ij}}{l_{ij}} \in [0, 1]$$

Efficiency

Global Efficiency

$$E_{\text{glob}} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{d_{ij}}{l_{ij}} \qquad E_{\text{glob}} \in [0, 1] \,.$$

- Latora V and Marchiori M *Phys. Rev. Lett.* **87** 198701 (2001).

Efficiency

Global Efficiency

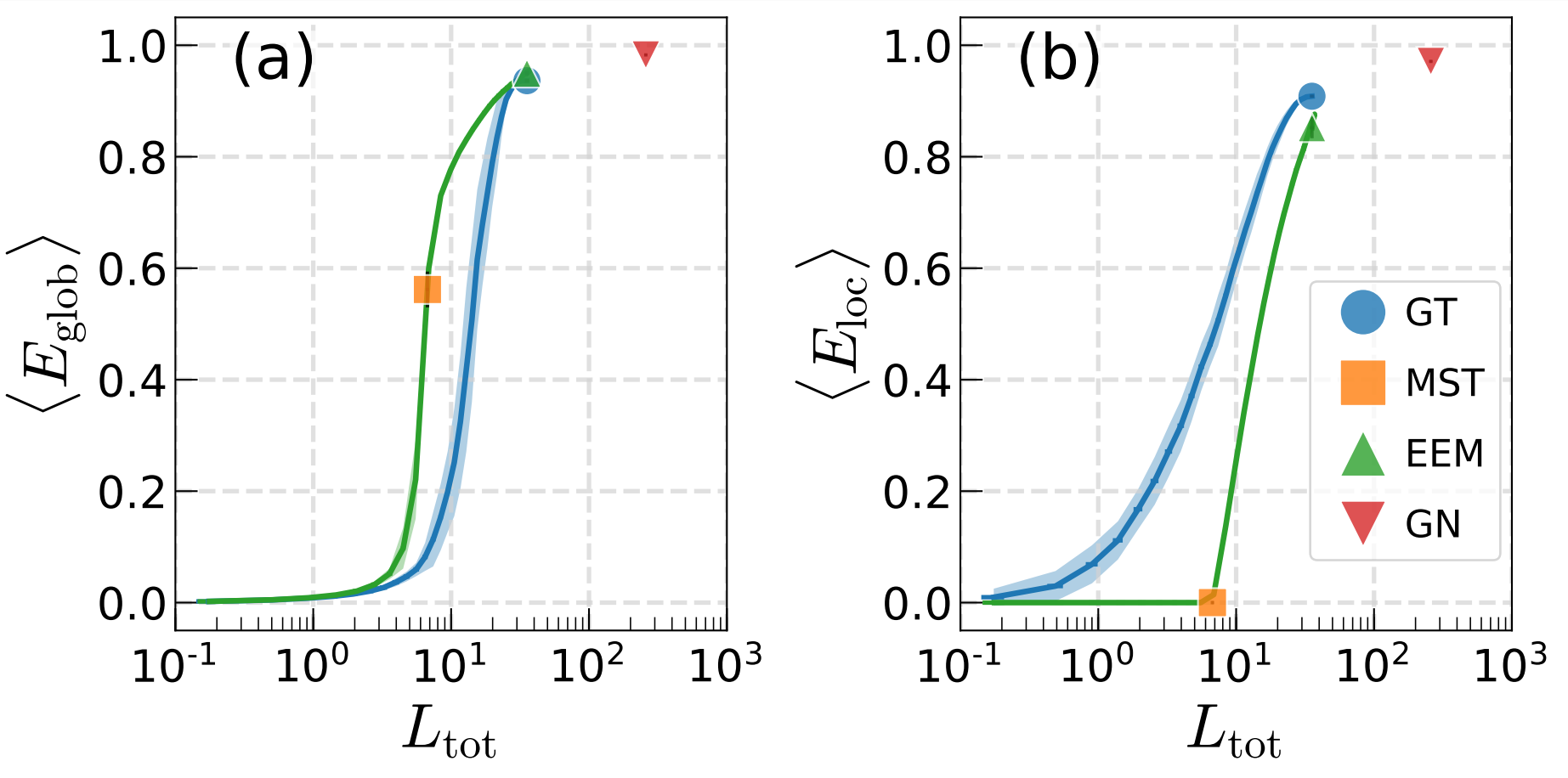
$$E_{\text{glob}} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{d_{ij}}{l_{ij}} \qquad E_{\text{glob}} \in [0,1] \,.$$

Local Efficiency

$$E_{\text{loc}} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{k_i (k_i - 1)} \sum_{j \neq m \in \Gamma_i} \frac{d_{jm}}{l_{jm/i}} \qquad E_{\text{loc}} \in [0,1] \,.$$

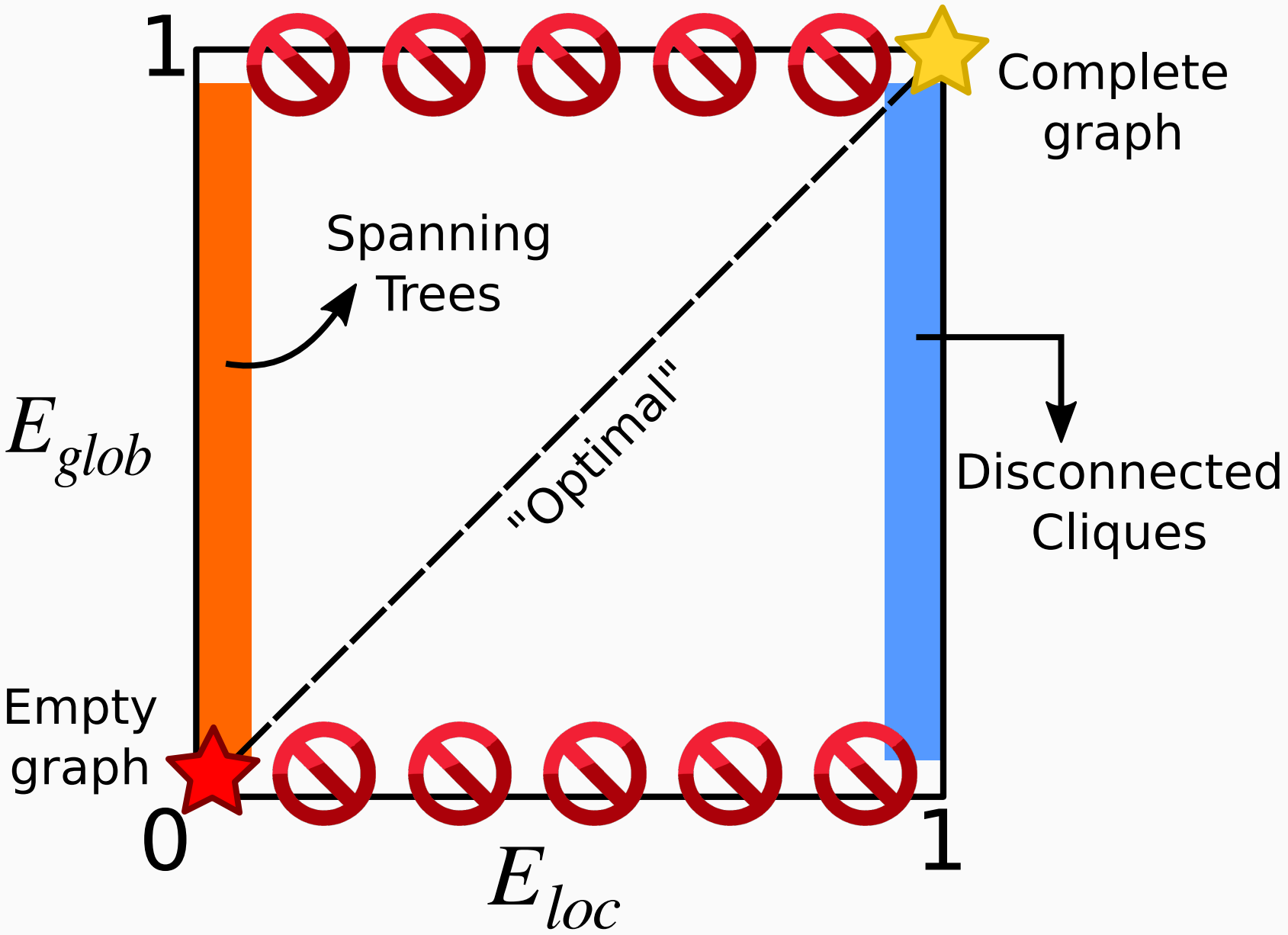
- Latora V and Marchiori M *Phys. Rev. Lett.* **87** 198701 (2001).
- Vragović I, L E and Díaz-Guilera A *Phys. Rev. E* **71** 036122 (2005).

Efficiency

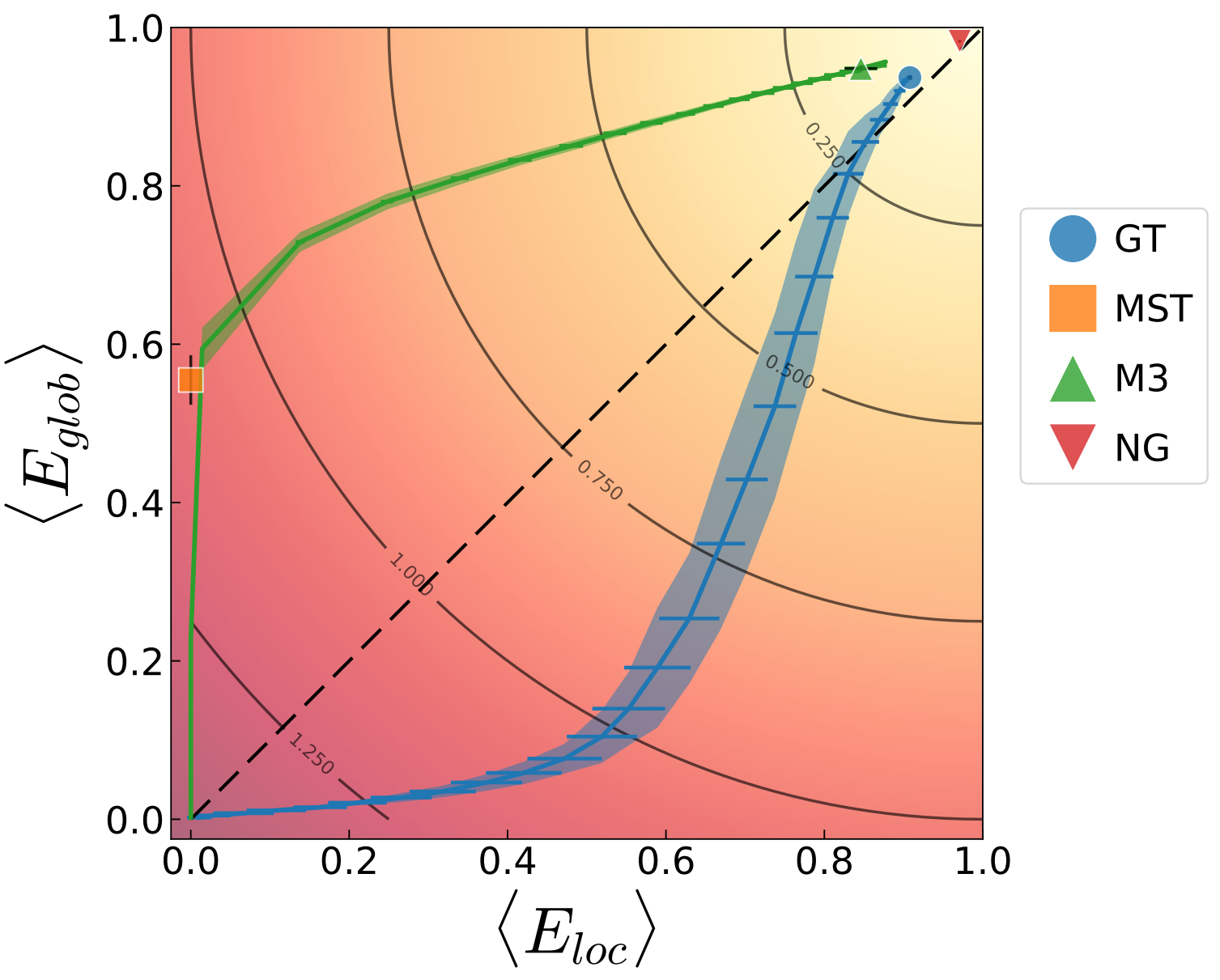


- Cardillo A, Scellato S, Latora V and Porta S *Phys. Rev. E* **73** 66107 (2006).
- Prignano L, Morer I, Fulminante F and Lozano S arXiv:1612.09321 (2016).
- Gastner M T and Newman M E J *Eur. Phys. Jour. B* **49** 247–252 (2006).

Efficiency


1
Complete graph
Spanning Trees
E_{glob}
"Optimal"
Disconnected Cliques
Empty graph
0
1
E_{loc}

Efficiency


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$\langle E_{glob} \rangle$
$\langle E_{loc} \rangle$
0.250
0.500
0.750
1.000
1.250
GT
MST
M3
NG


YES WE CAN

Another measure of Efficiency

Integrated Efficiency

$$E_{\text{int}} = 1 - \sqrt{\frac{(1 - E_{\text{glob}})^2 + (1 - E_{\text{loc}})^2}{2}}\,.$$

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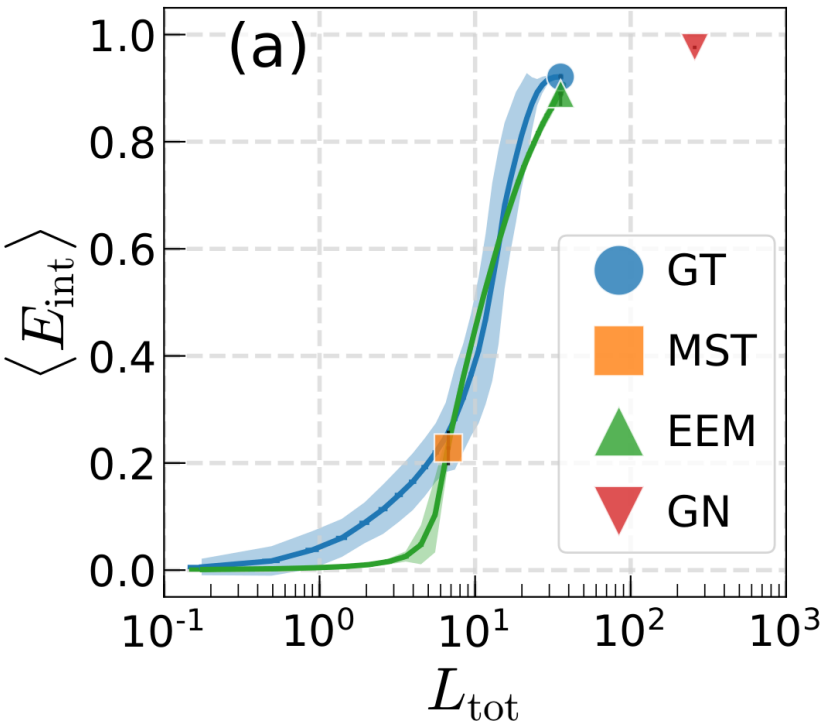

Another measure of Efficiency

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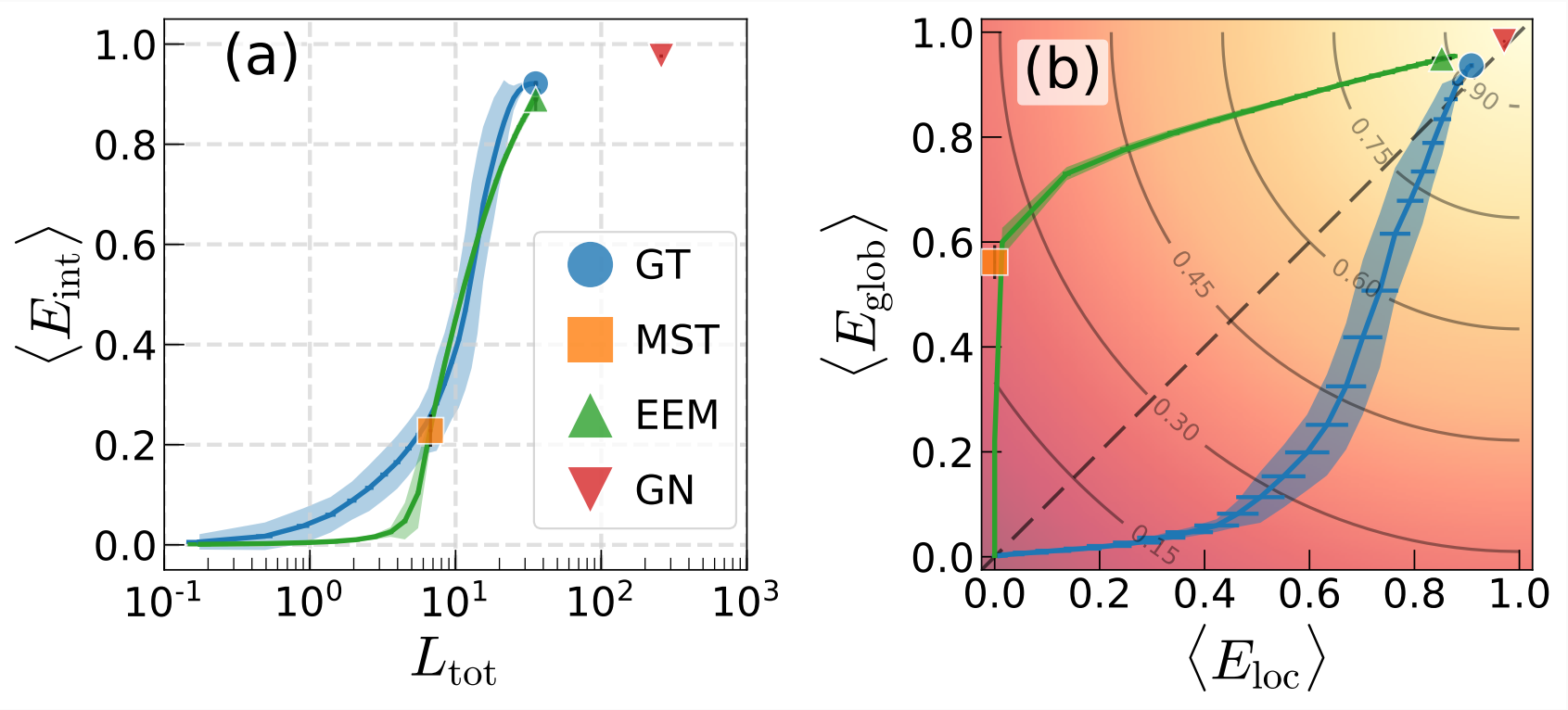


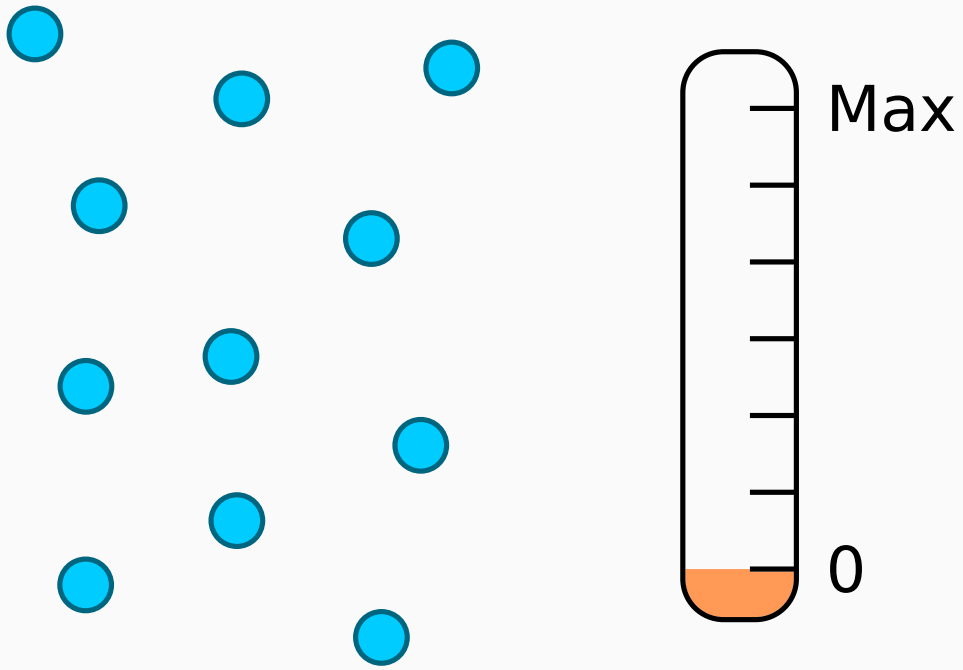
Efficiency optimal networks

Given a certain **layout** of nodes, $\tilde{\mathcal{L}}$, and a fixed **amount of resources**, $\tilde{L_{\text{tot}}}$, can we find the set of connections that **maximizes** E_{int}?

Efficiency optimal networks

Network growth algorithm

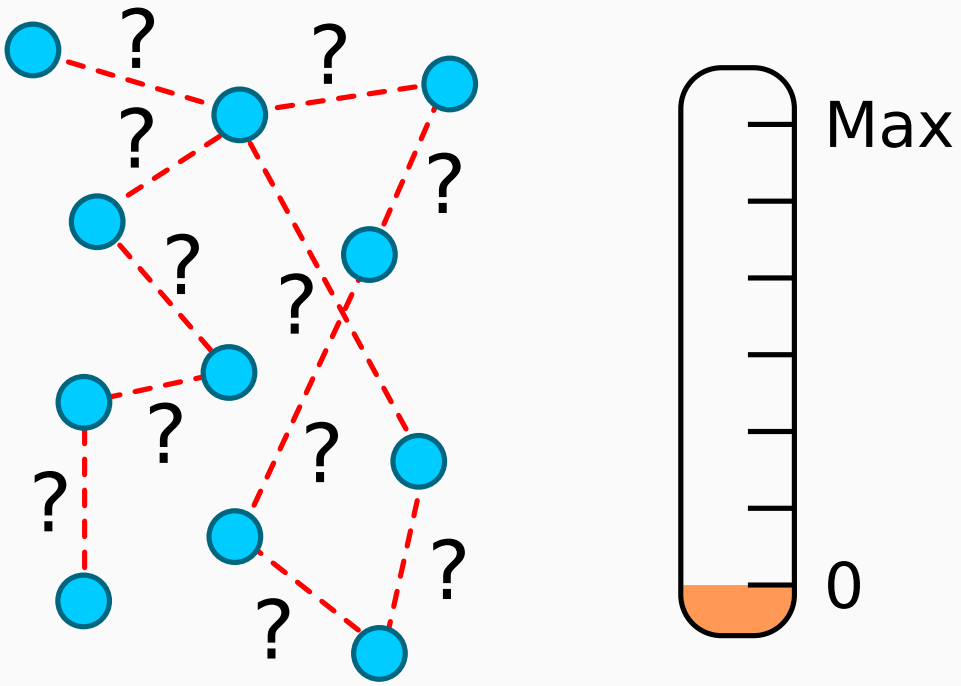
1. Start at $t = 0$ from an empty graph, G, with N nodes placed according to layout $\tilde{\mathcal{L}}$.



Efficiency optimal networks

Network growth algorithm

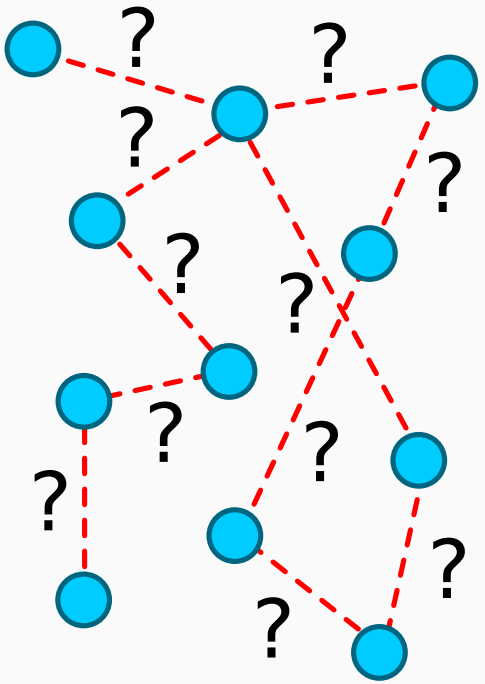
1. Start at $t = 0$ from an empty graph, G, with N nodes placed according to layout $\tilde{\mathcal{L}}$.
2. For $t > 0$, for each link $(i, j) \notin G$:

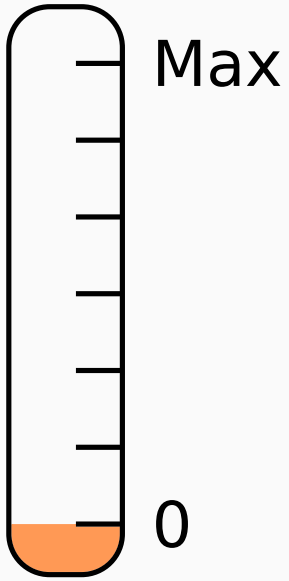


Efficiency optimal networks

Network growth algorithm

1. Start at $t = 0$ from an empty graph, G, with N nodes placed according to layout $\tilde{\mathcal{L}}$.
2. For $t > 0$, for each link $(i, j) \notin G$:
   2.1 Compute
   $$\alpha_{ij} = \left\{ \frac{\Delta E_{\text{int}}(i,j)}{d_{ij}} \right\}$$





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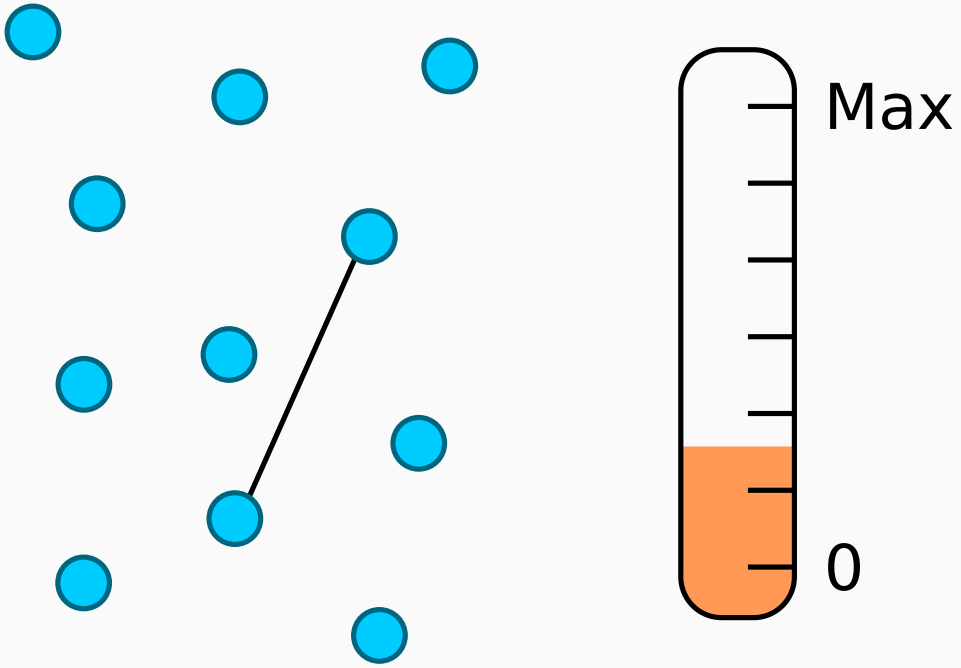
   - 2.1 Compute
     $$\alpha_{ij} = \left\{ \frac{\Delta E_{\text{int}}(i,j)}{d_{ij}} \right\}$$
   - 2.2 Add edge
     $$(i^\star, j^\star) : \alpha_{i^\star j} = \max_{i,j} (\alpha_{ij})$$



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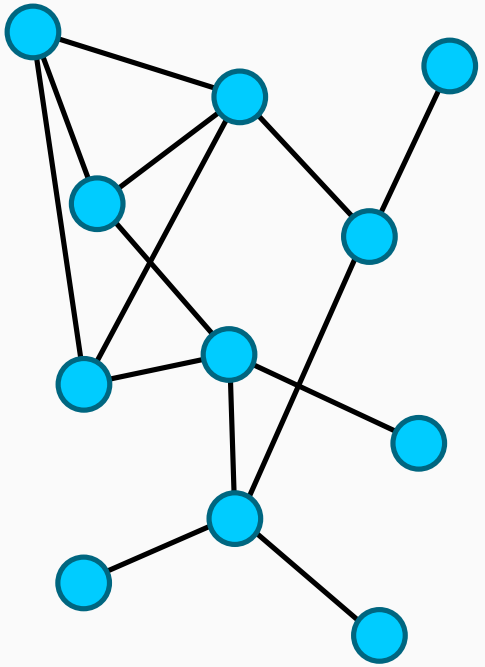
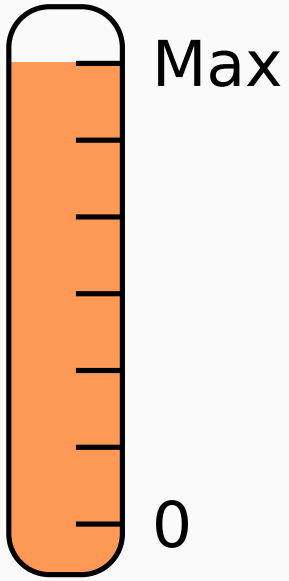
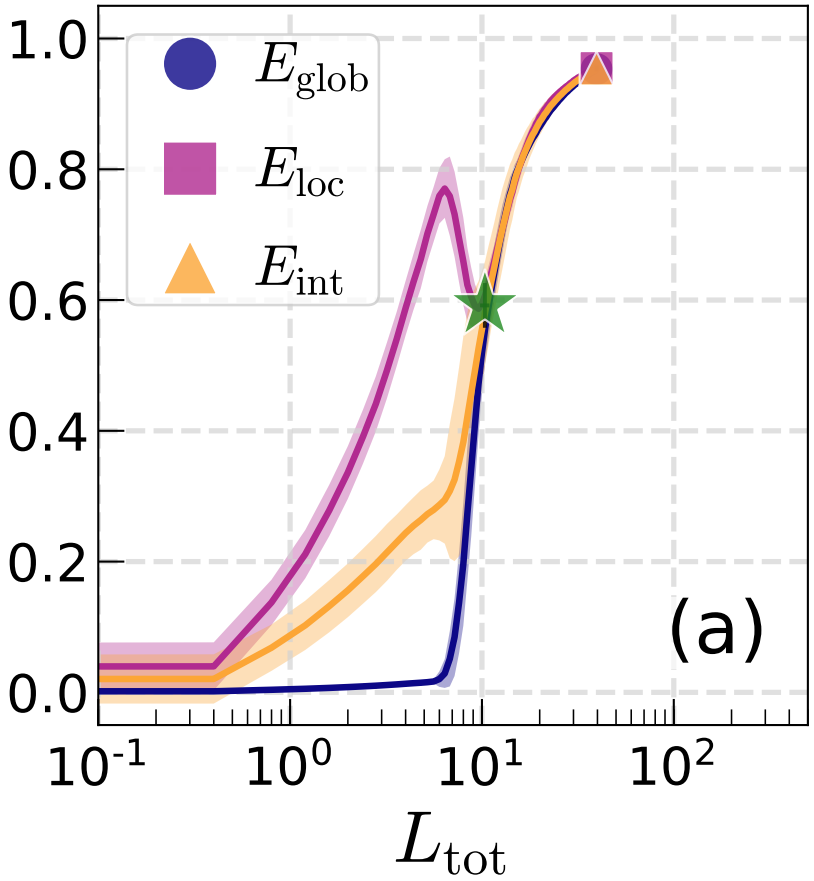
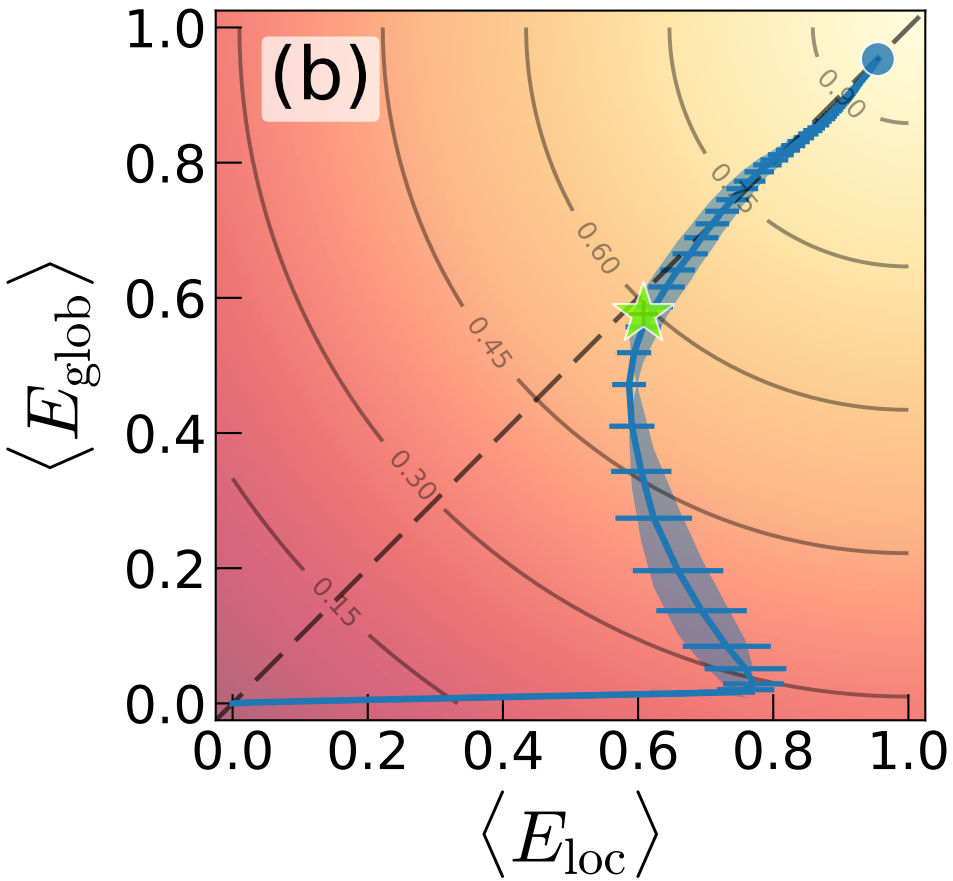
3. Repeat step 2 until $L_{\text{tot}}(G) \simeq \tilde{L_{\text{tot}}}$.



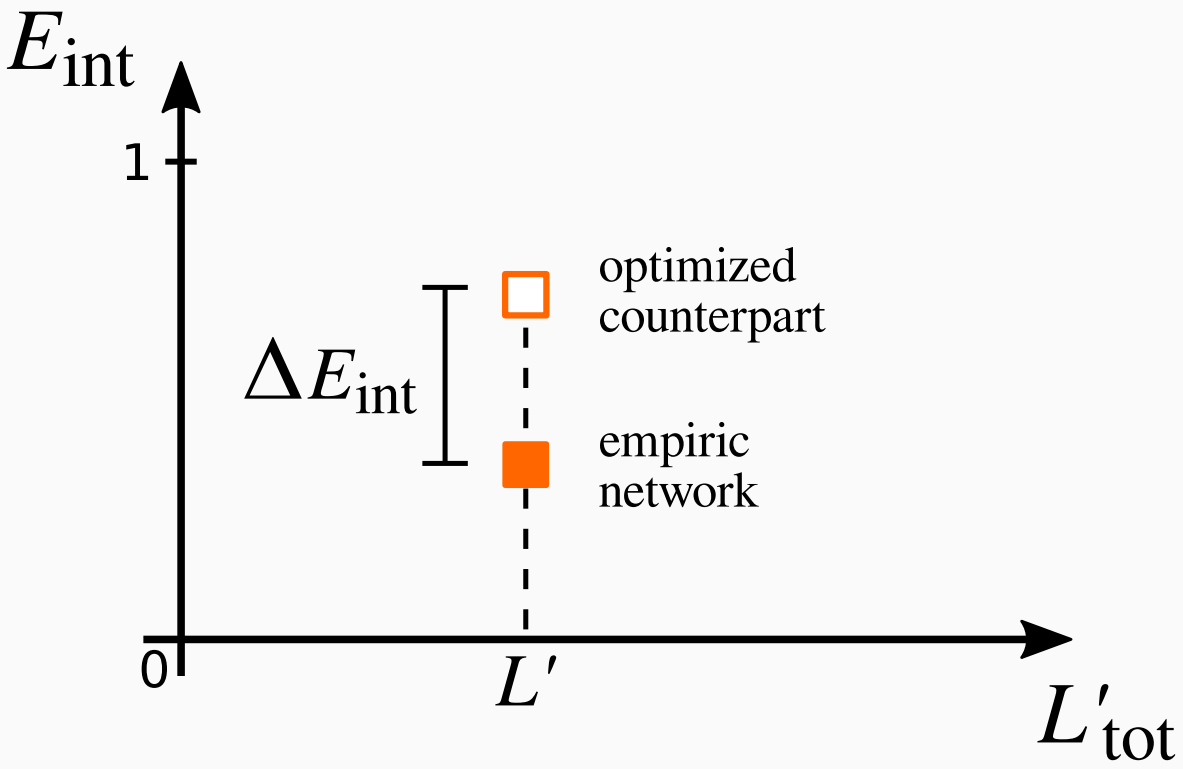


Efficiency optimal networks





Comparison between networks



Note

To ensure consistency between units, we must rescale the lengths!

$$L' = f(L, \langle d \rangle, N)$$

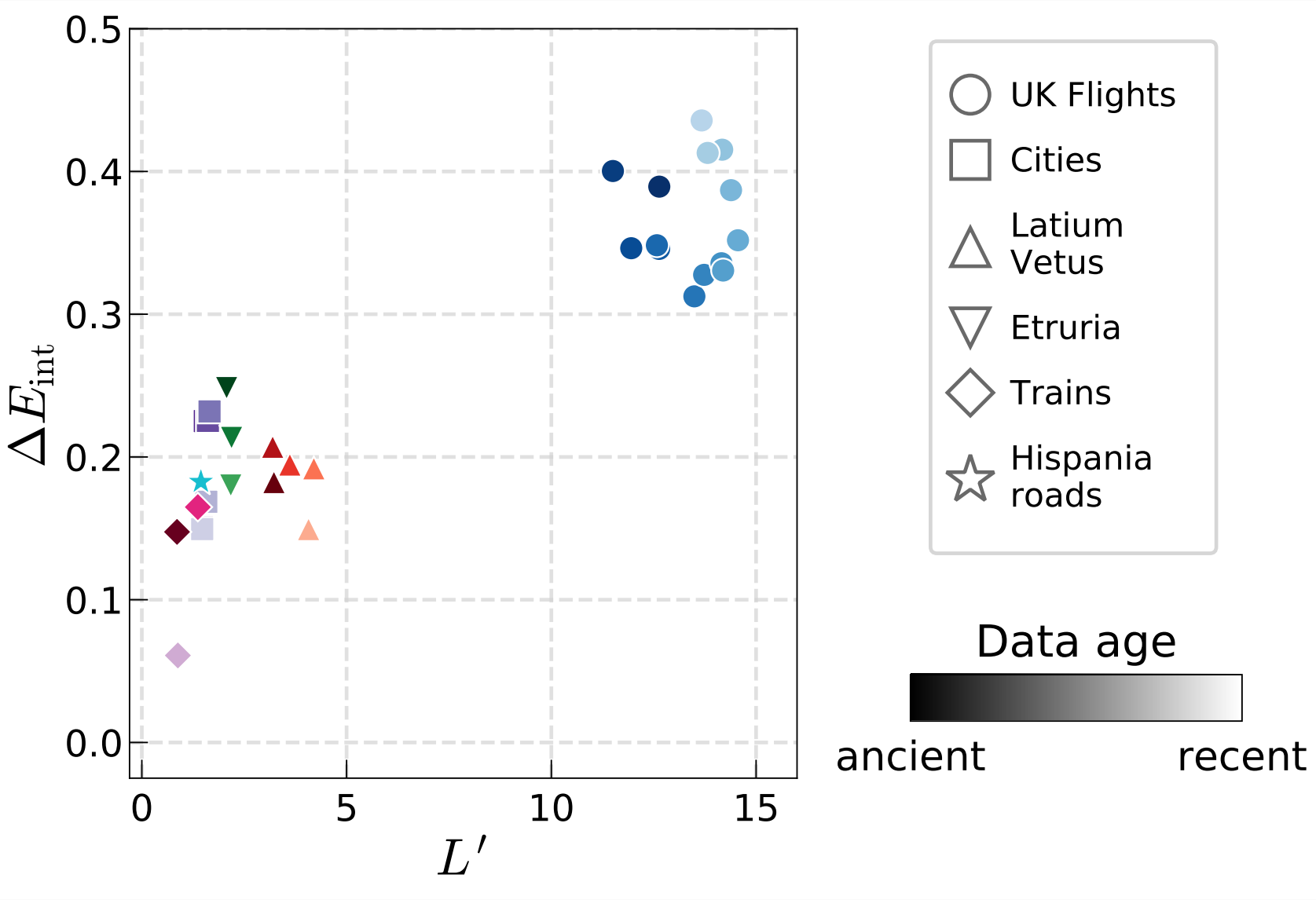
Comparison between networks

Data

UK Flights	Domestic flights operated in UK (1990 – 2003).
Cities	Road patterns in Northen Italy (1833 – 2007).
Trains	Regional (Aragon and Catalonia) railway systems in Spain.
Latium Vetus & Etruria	Trails among villages (950 – 509 BC).
Hispania	Main road network of the whole Iberian peninsula during the Roman Empire.

- R. Kujala, *et al.* Scientific Data, **5**, 180089 (2018).
- Prignano L, Morer I, Fulminante F and Lozano S arXiv:1612.09321 (2016).
- Fulminante F, Prignano L, Morer I and Lozano S *Front. Dig. Hum.* **4** 1–12 (2017).
- Strano E, Nicosia V, Latora V, Porta S and Barthélemy M *Sci. Rep.* **2** 1–8 (2012).

Comparison between networks



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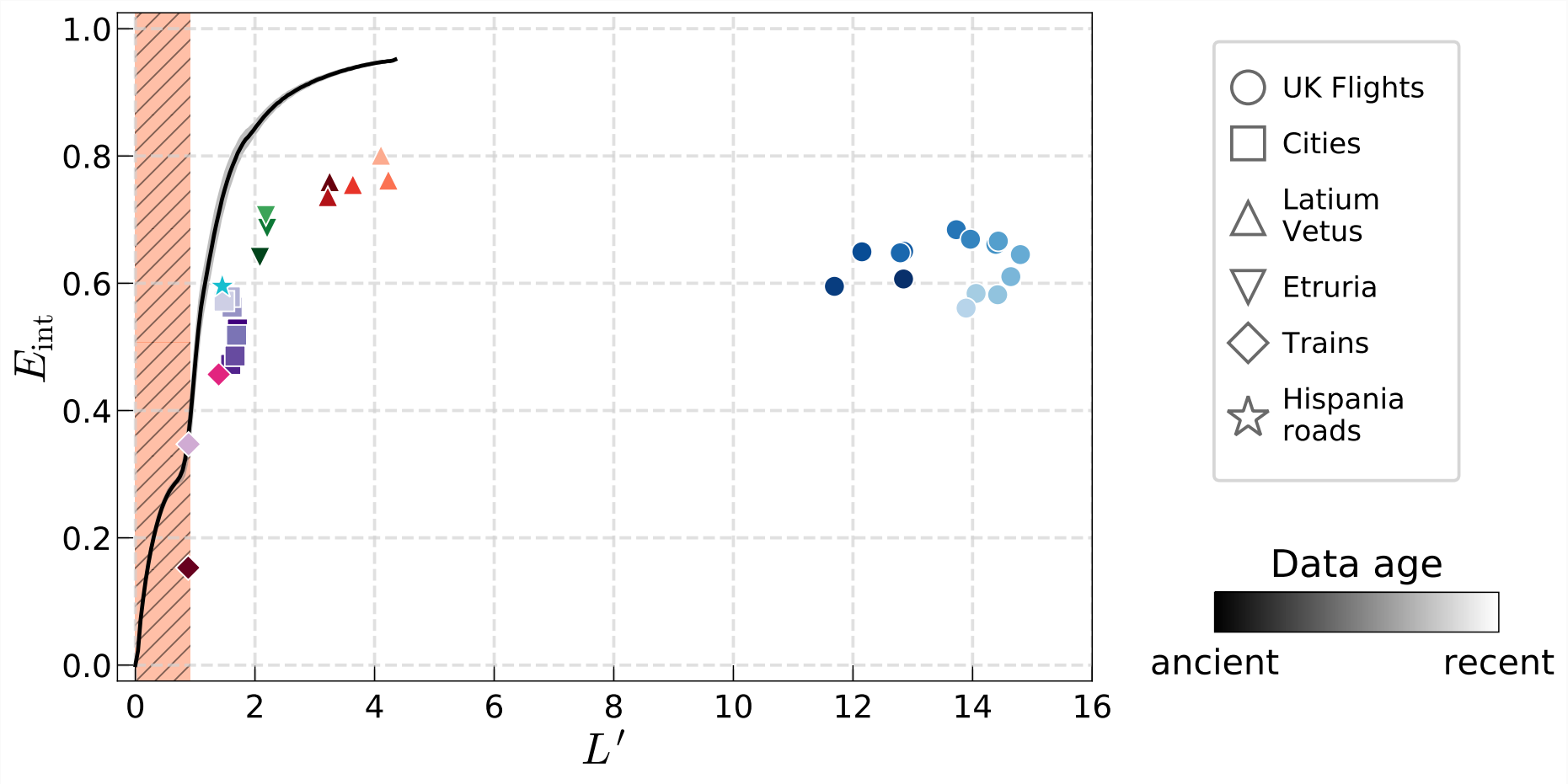
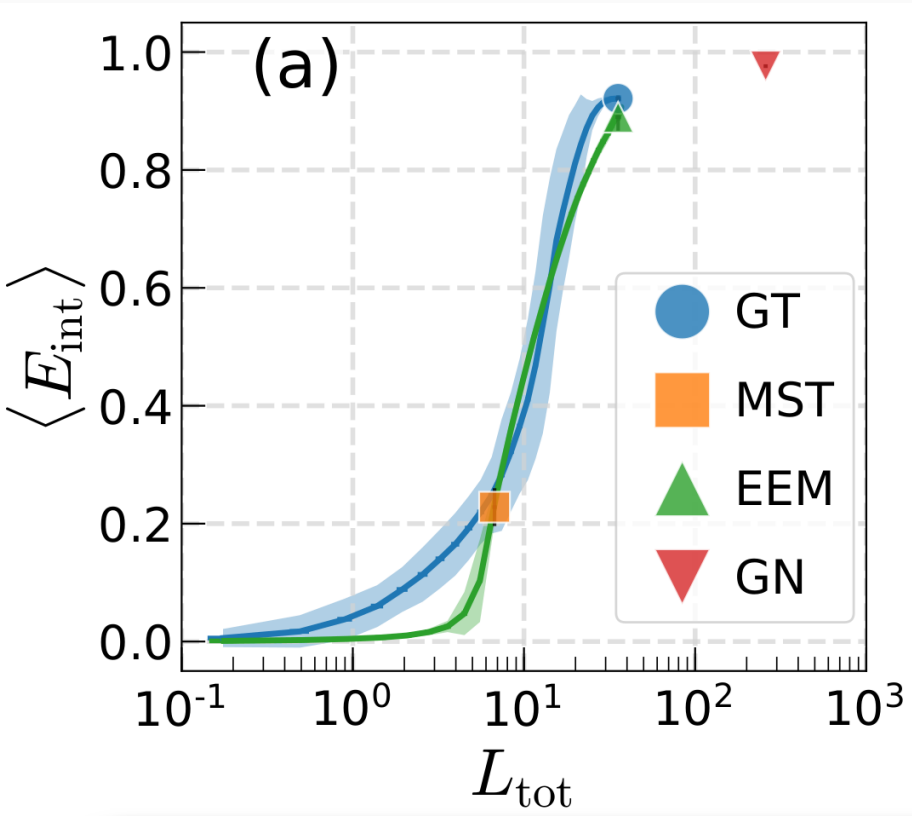
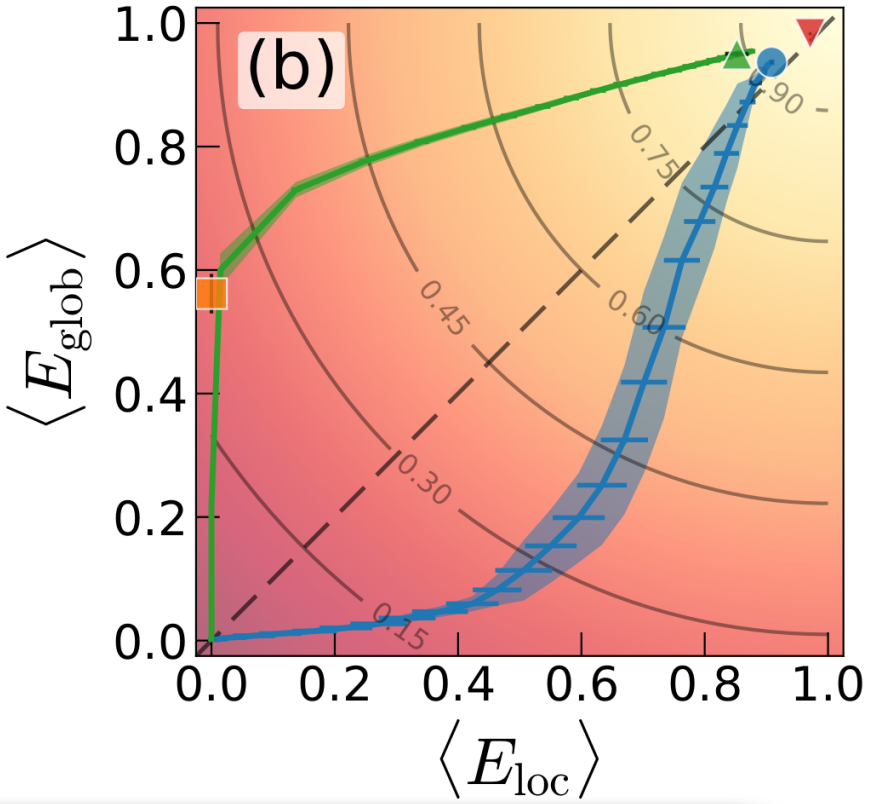
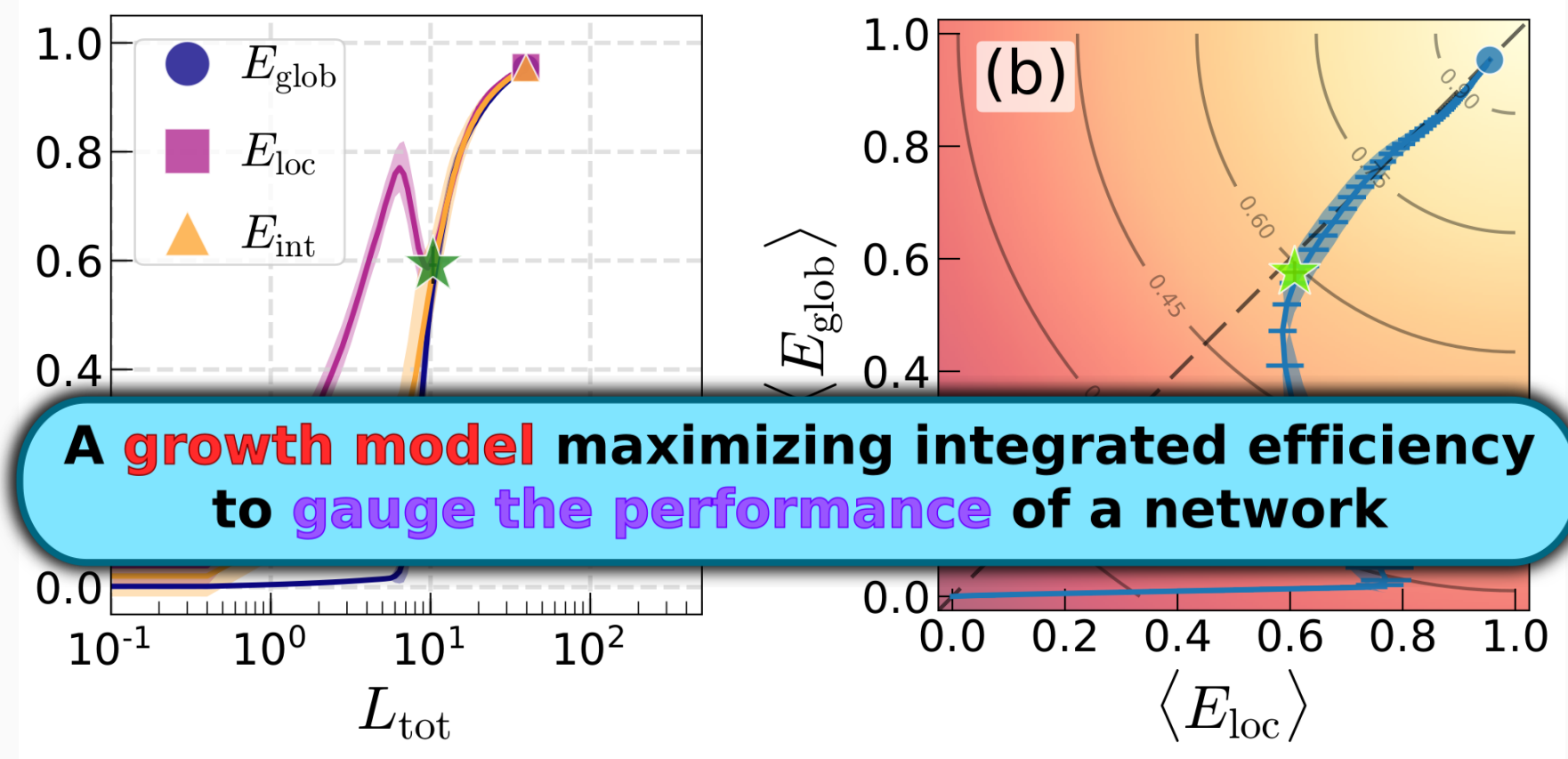


Conclusions

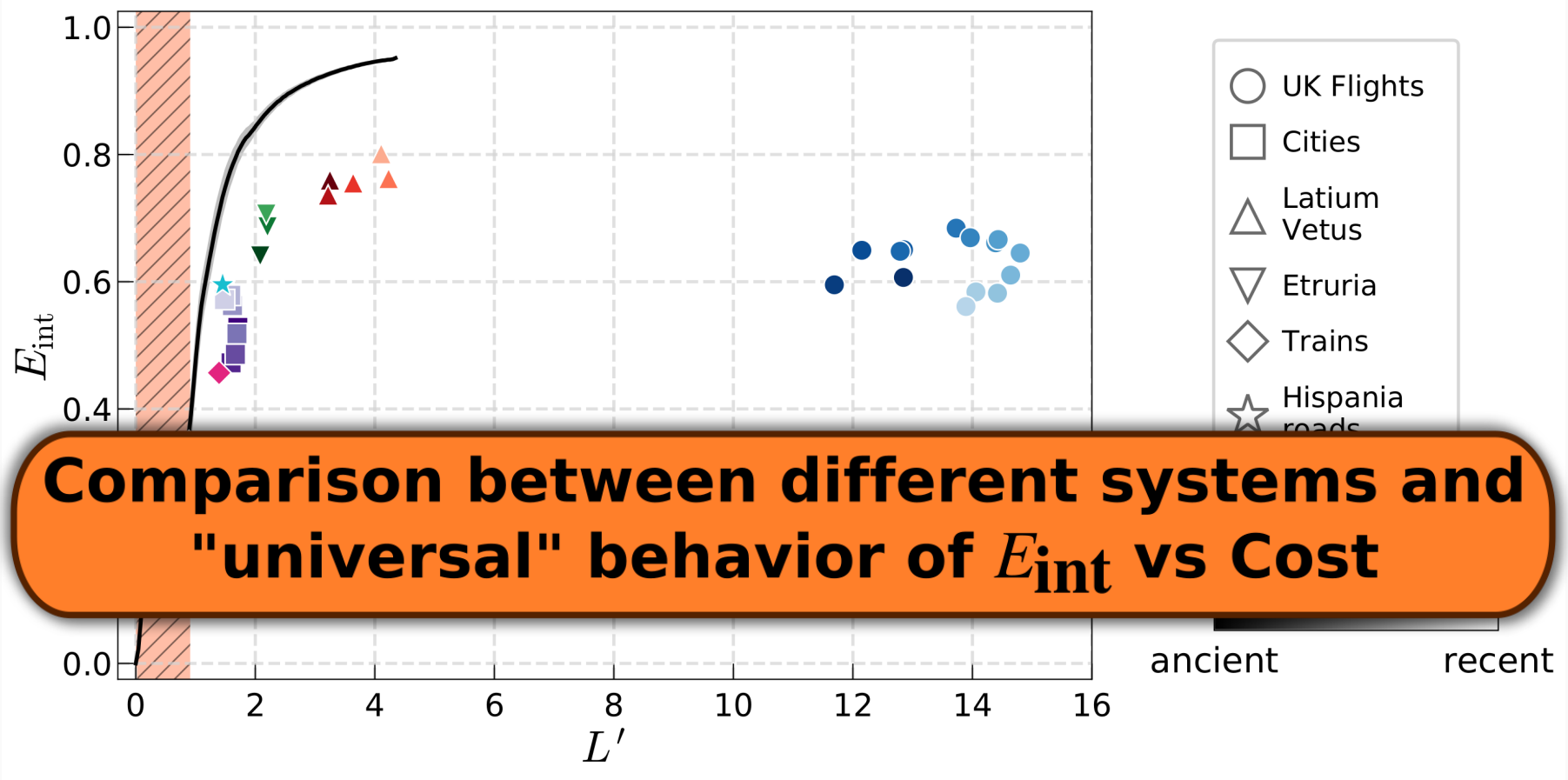




Integrated efficiency as a metric for performances at a global and local scale



A growth model maximizing integrated efficiency to gauge the performance of a network



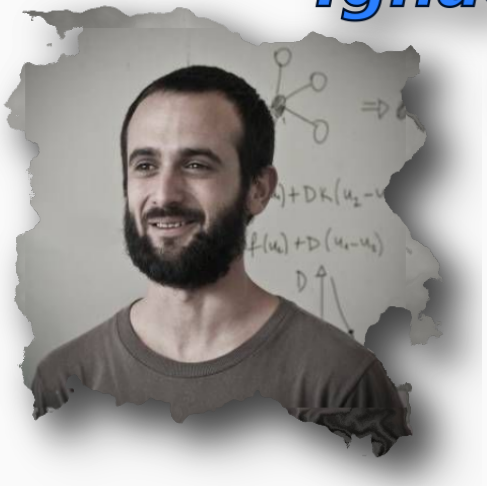
Comparison between different systems and "universal" behavior of E_{int} vs Cost

Future work

1. Extend & find a more general form of the "theoretical" curve (if it exists).
2. Use more (diverse) datasets.
3. Investigate the role of point distribution.

Acknowledgements

Ignacio Morer (UB)



Albert Diaz-Guilera (UB)

Luce Prignano (UB)

Sergi Lozano (IPHES)

Acknowledgements

Ignacio Morer et al.

Comparing spatial networks:
A 'one size fits all' efficiency-driven approach

https://arxiv.org/abs/1807.00565

UK Air Data Available!

@ alessio.cardillo@bristol.ac.uk

http://www.bifi.es/~cardillo/

@a_cardillo

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- Gastner M T and Newman M E J 2006 *Eur. Phys. Jour. B* **49** 247–252 https://doi.org/10.1140/epjb/e2006-00046-8
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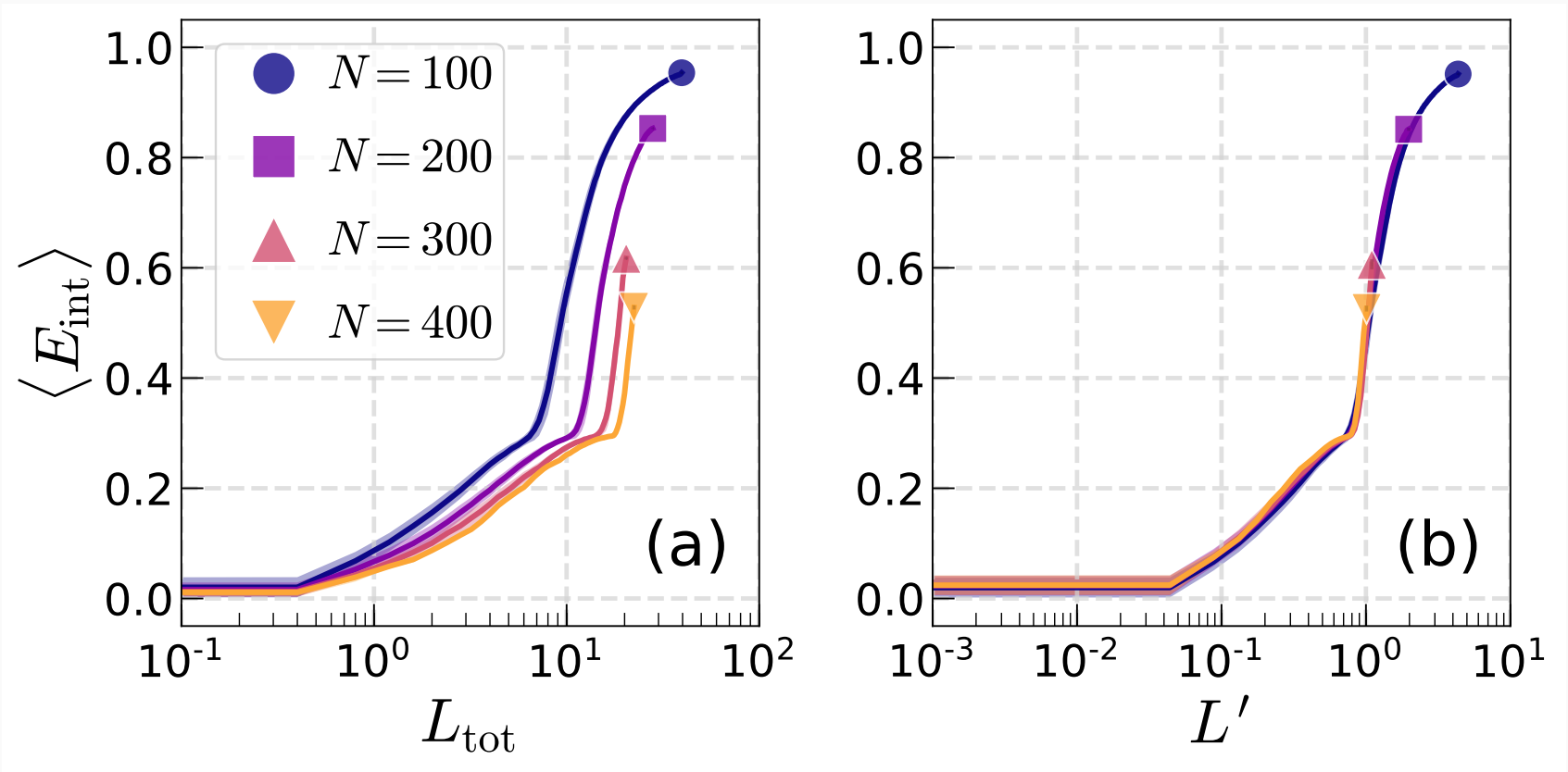
Prignano L, Morer I, Fulminante F and Lozano S 2016 Modelling terrestrial route networks to understand inter-polity interactions. A case-study from Southern Etruria arXiv:1612.09321 [physics.soc-ph]

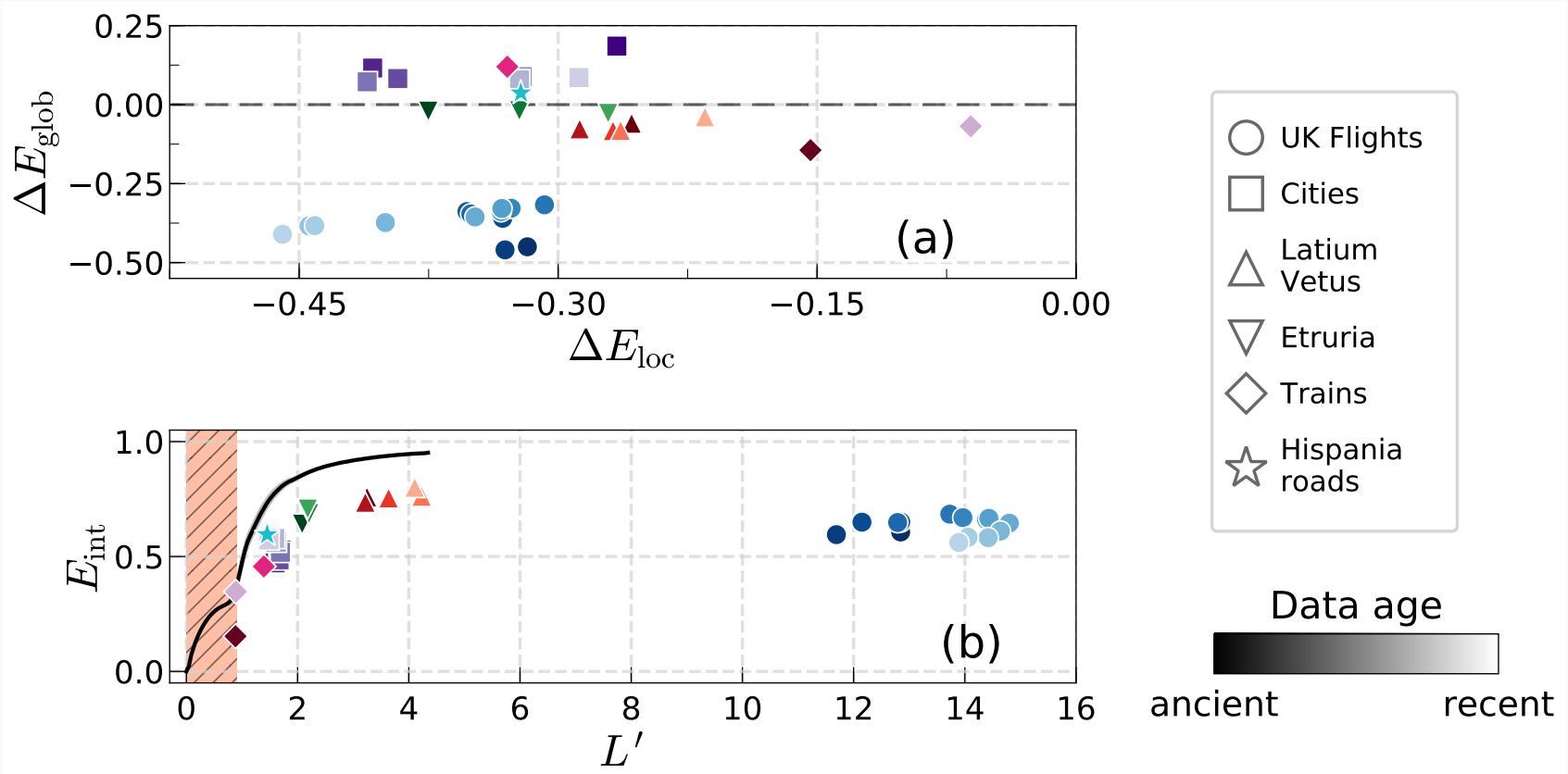

R. Kujala, C. Weckström, R. K. Darst, M. N. Mladenović, and J. Saramäki, A collection of public transport network data sets for 25 cities. Scientific Data, **5**, 180089 (2018). https://doi.org/10.1038/sdata.2018.89

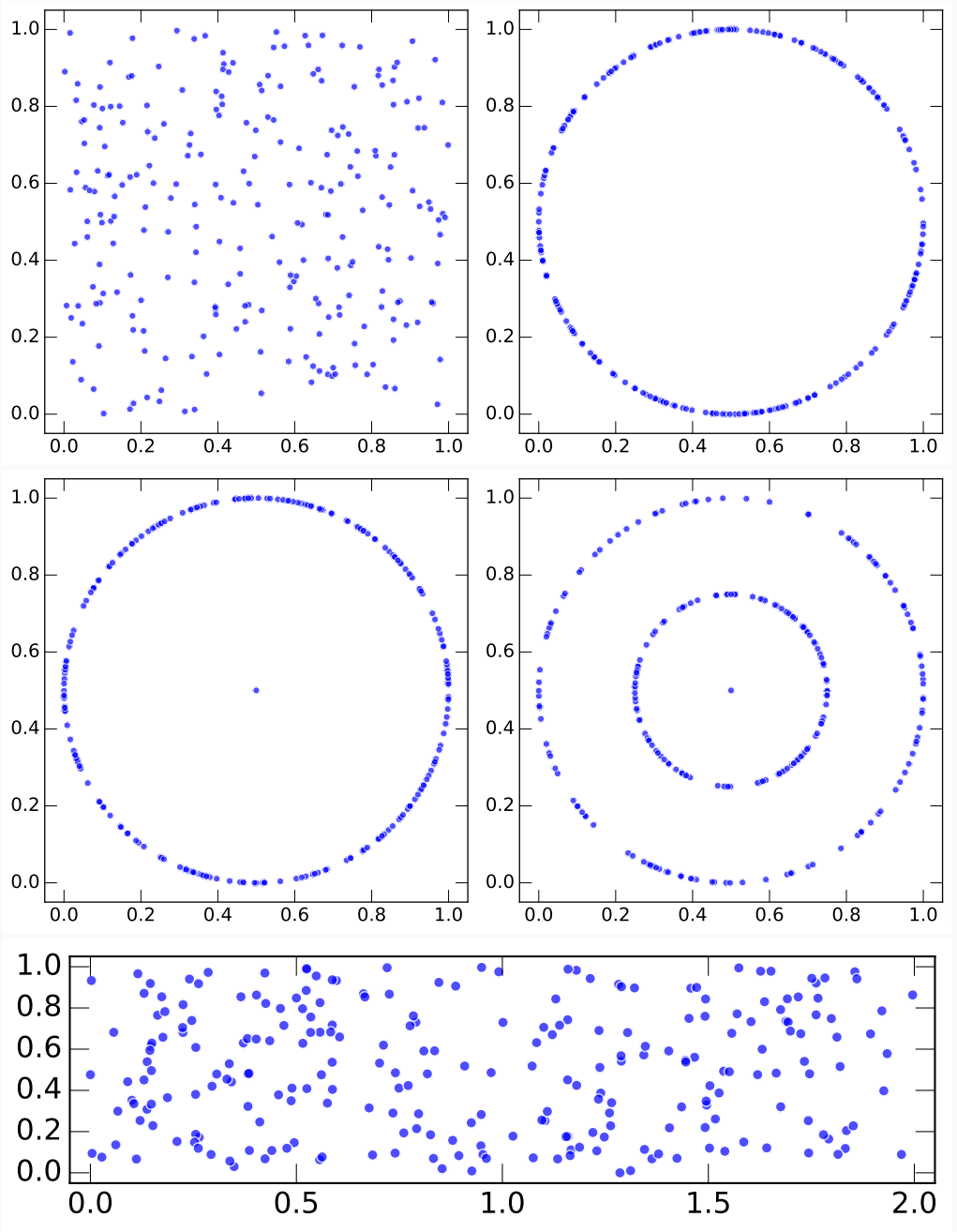

Fulminante F, Prignano L, Morer I and Lozano S 2017 *Front. Dig. Hum.* **4** 1–12 https://doi.org/10.3389/fdigh.20

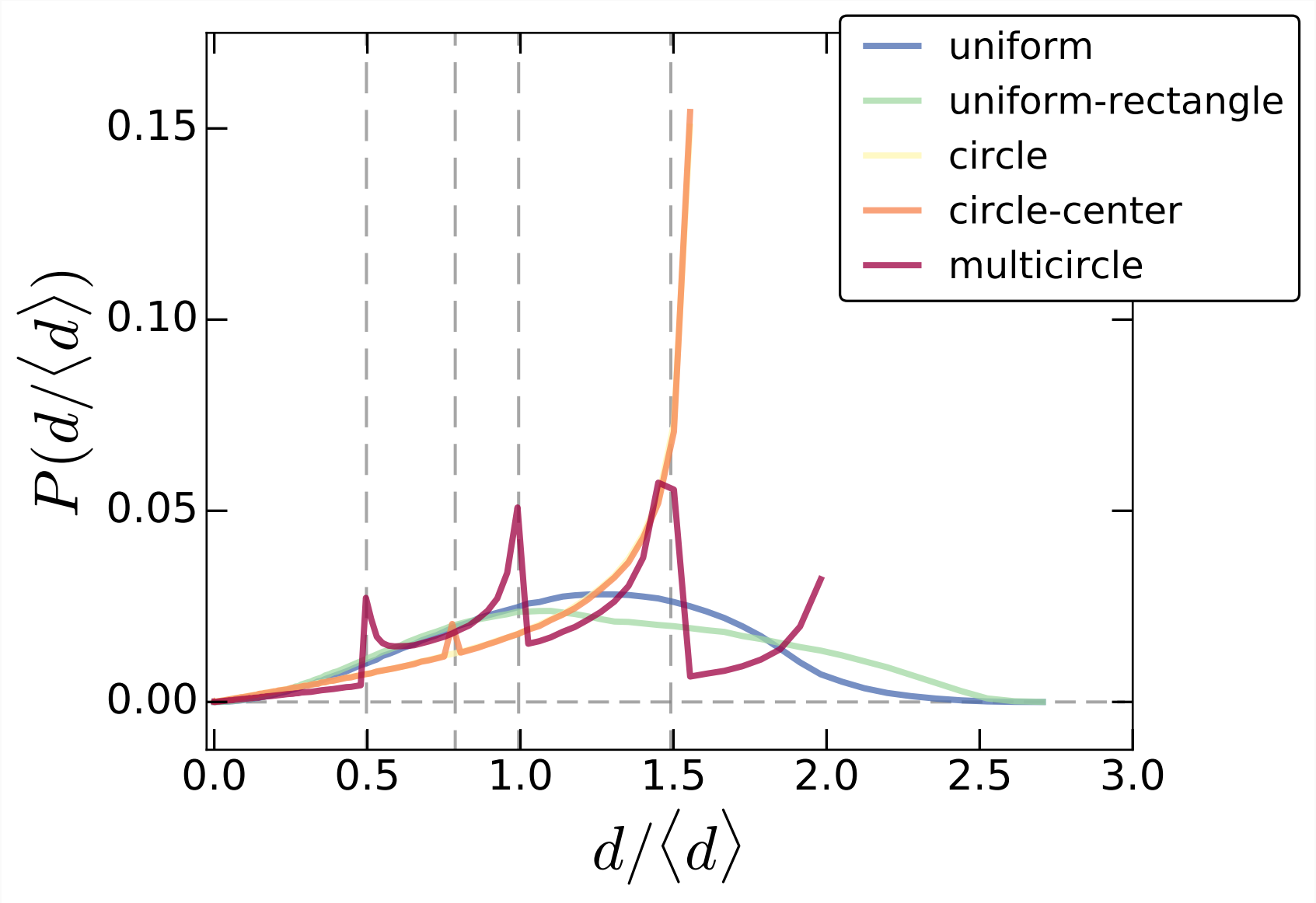

Strano E, Nicosia V, Latora V, Porta S and Barthélemy M 2012 *Sci. Rep.* **2** 1–8 https://doi.org/10.1038/srep00296

Extra contents


1.0
0.8
0.6
0.4
0.2
0.0
$\langle E_{\rm int} \rangle$
$N = 100$
$N = 200$
$N = 300$
$N = 400$
(a)
10^{-1}
10^{0}
10^{1}
10^{2}
$L_{\rm tot}$
(b)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
L'


0.25
0.00
−0.25
−0.50
ΔE_{glob}
−0.45
−0.30
−0.15
0.00
ΔE_{loc}
(a)
1.0
0.5
0.0
E_{int}
0
2
4
6
8
10
12
14
16
L'
(b)
UK Flights
Cities
Latium Vetus
Etruria
Trains
Hispania roads
Data age
ancient
recent


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.5
2.0


uniform
uniform-rectangle
circle
circle-center
multicircle
0.15
0.10
0.05
0.00
$P(d/\langle d \rangle)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$d/\langle d \rangle$