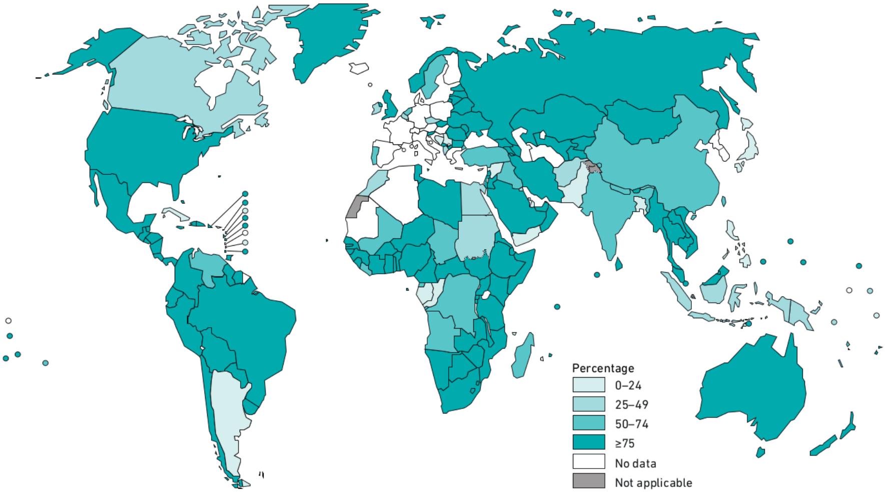
Emergence of synergistic and competitive pathogens in a coevolutionary spreading model

Alessio Cardillo (@a_cardillo)

Internet Interdisciplinary Institute (IN3) – Universitat Oberta de Catalunya, Barcelona (Spain)

Conference on Complex Systems (CCS) 2022 — Palma de Mallorca
Tuesday, October 18th 2021

Percentage of new and relapse TB cases with documented HIV status, 2017[a]



- WHO Global Tuberculosis report 2017. Available at:
https://www.who.int/teams/global-tuberculosis-programme/tb-reports


Welcome

Main questions:

1. Can we describe the emergence of coinfection/cross-immunity as the outcome of an evolutionary process?

Main questions:

1. Can we describe the emergence of coinfection/cross-immunity as the outcome of an evolutionary process?
2. What happens to a double-SIR model when we introduce cooperative/defective strains?

Main questions:

1. Can we describe the emergence of coinfection/cross-immunity as the outcome of an evolutionary process?
2. What happens to a double-SIR model when we introduce cooperative/defective strains?
3. What happens when you combine spreading dynamics and (evolutionary) game theory?

Multiple SIR with cooperative/defective strains

Multiple SIR with cooperative/defective strains

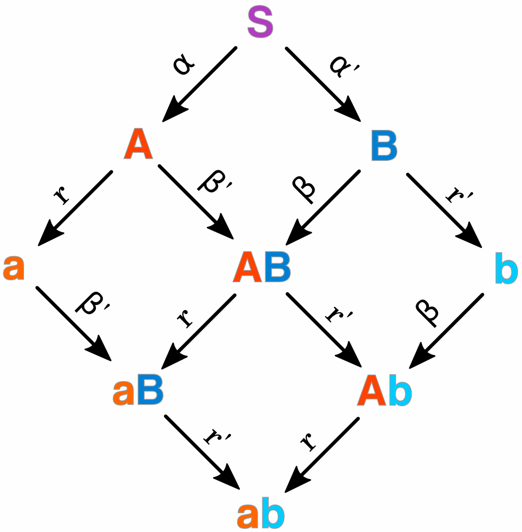
$$
\begin{tikzcd}
\mathbf{S} \arrow[r, "\alpha"] \arrow[d] & \mathbf{I} \arrow[r, "r"] \arrow[d] & \mathbf{R} \arrow[d] \\
\mathbf{S} & \mathbf{A} & \mathbf{a}
\end{tikzcd}
$$

Simple SIR

- 1 pathogen (*A*)
- 3 states
- 2 parameters

• Anderson, R. M., & May, R. M. "*Infectious diseases of humans: Dynamics and control*". (Oxford University Press, Oxford, 1991).

Multiple SIR with cooperative/defective strains

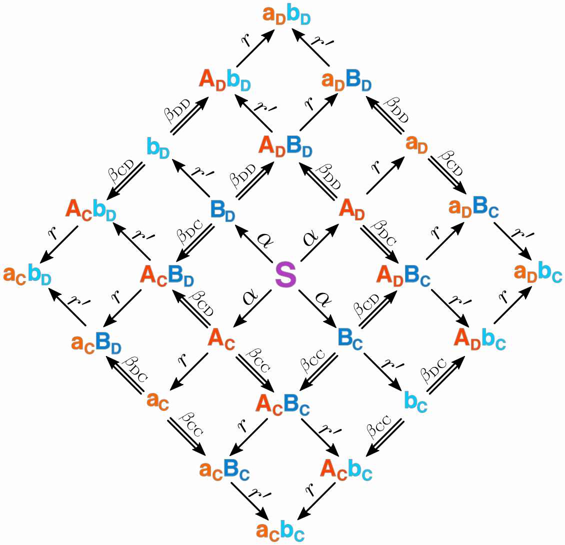


Double SIR

- 2 pathogens (A and B)
- 9 states
- 6 parameters

• Chen, L., *et al.* Europhys. Lett., **104**, 50001 (2013). DOI: `10.1209/0295-5075/104/50001`

Multiple SIR with cooperative/defective strains



Multiple SIR (strategies)

- 2 pathogens (A and B)
- 2 strategies (C and D)
- 4 species (A_C, A_D, B_C, B_D)
- 25 states
- $>$ 7 parameters

Multiple SIR with cooperative/defective strains

Let's simplify a bit . . .

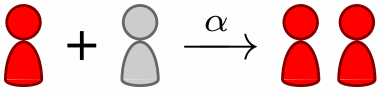
- Set all recovery rates equal to one (*i.e.* $r = 1$).

Multiple SIR with cooperative/defective strains

Let's simplify a bit . . .

- Set all recovery rates equal to one (*i.e.* $r = 1$).
- **Single pathogen infection** independent of pathogen's strategy (*i.e.* $\alpha_C = \alpha_D = \alpha$).

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

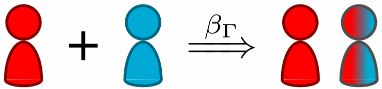


Multiple SIR with cooperative/defective strains

Let's simplify a bit . . .

- Set all recovery rates equal to one (*i.e.* $r = 1$).
- **Single pathogen infection** independent of pathogen's strategy (*i.e.* $\alpha_C = \alpha_D = \alpha$).
- **Multiple pathogens infection** dependent only on host's strategy (*i.e.* $\beta_{CC} = \beta_{DC} = \beta_C$ and $\beta_{CD} = \beta_{DD} = \beta_D$).

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_\Gamma} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda\mathbf{Y}_\Gamma$$



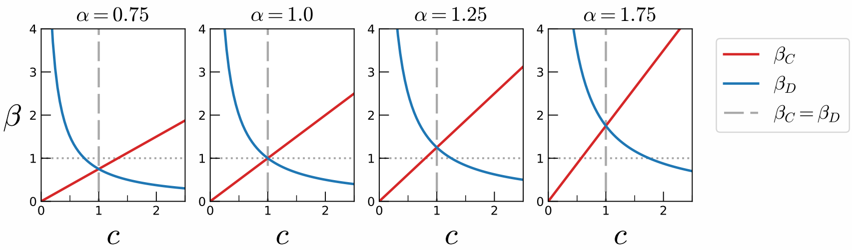
Multiple SIR with cooperative/defective strains

Let's simplify a bit ...

- Set all recovery rates equal to one (*i.e.* $r = 1$).
- **Single pathogen infection** independent of pathogen's strategy (*i.e.* $\alpha_C = \alpha_D = \alpha$).
- **Multiple pathogens infection** dependent only on host's strategy (*i.e.* $\beta_{CC} = \beta_{DC} = \beta_C$ and $\beta_{CD} = \beta_{DD} = \beta_D$).
- Easier to infect a host occupied by a cooperator pathogen than a defector one (*i.e.* $\beta_C > \beta_D$).

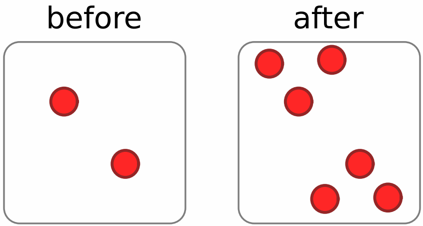
Multiple SIR with cooperative/defective strains

$$\beta_C = c\,\alpha \qquad \text{with} \qquad \alpha \in \,]0, +\infty[$$
$$\beta_D = \frac{\alpha}{c} \qquad\qquad\qquad c \in \,]0, +\infty[$$



Crash course on evolutionary dynamics

Crash course on evolutionary dynamics

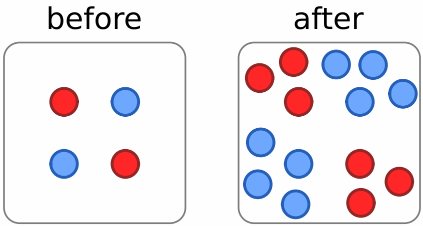


Evolutionary theory

Replication The ability of an organism to reproduce.

- Nowak, M. A. *Evolutionary dynamics: exploring the equations of life*. (Belknap Press, Harvard, 2007).

Crash course on evolutionary dynamics



Evolutionary theory

Replication The ability of an organism to reproduce.

Selection The ability of a species to replicate faster than another.

- Nowak, M. A. *Evolutionary dynamics: exploring the equations of life*. (Belknap Press, Harvard, 2007).

Crash course on evolutionary dynamics

Replicator equation

$$\rho_i^{T+1} = \rho_i^T \left[1 + \underbrace{\Pi_i^T - \overline{\Pi}^T}_{\text{selection}} \right]$$

T Time (discrete)

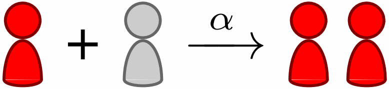
ρ_i Density of species i

$\Pi_i(\rho)$ Fitness (payoff) of species i

$\overline{\Pi}$ Average fitness (whole population)

- Nowak, M. A. *Evolutionary dynamics: exploring the equations of life*. (Belknap Press, Harvard, 2007).

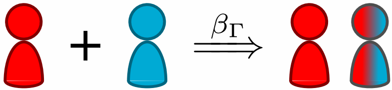
Crash course on evolutionary dynamics



Single pathogen infection

$$\pi_{\mathbf{x}_\Lambda} = 1$$

Crash course on evolutionary dynamics



Multiple pathogens infection

$$
\left.\begin{array}{l} \pi_{\mathbf{X}_\Lambda} \\ \pi_{\mathbf{Y}_\Gamma} \end{array}\right\} =
\begin{array}{c} \\ C \\ D \end{array}
\begin{array}{c} \begin{array}{cc} \quad C \quad & \quad D \quad \end{array} \\ \begin{pmatrix} \frac{1}{2} & \gamma \\ 1-\gamma & -\frac{1}{2} \end{pmatrix} \end{array}
$$

with $\gamma \in \left[0, \frac{1}{2}\right]$

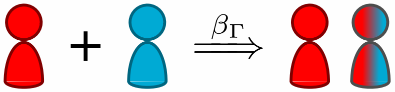
- Nowak, M. A. *Evolutionary dynamics: exploring the equations of life*. (Belknap Press, Harvard, 2007).

Crash course on evolutionary dynamics

Multiple pathogens infection

$$
\left.\begin{array}{l}\pi_{\mathbf{X}_\Lambda}\\ \pi_{\mathbf{Y}_\Gamma}\end{array}\right\} = \begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & \gamma \\ 1-\gamma & -\frac{1}{2} \end{pmatrix} \end{array}
$$

with $\gamma \in \left[0, \frac{1}{2}\right]$



Note

This payoff matrix corresponds to the so-called **Hawk and Dove** game.

• Nowak, M. A. *Evolutionary dynamics: exploring the equations of life*. (Belknap Press, Harvard, 2007).

The coevolutionary model

The coevolutionary model

Phase 1 (within season T)

Coupled SIR (25 states) dynamics

Single pathogen infection

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

Multiple pathogens infection

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_{\Lambda\Gamma}} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda\mathbf{Y}_\Gamma$$

Accumulation of Payoff

Single pathogen infection

$$\pi_{\mathbf{X}_\Lambda} = 1$$

Multiple path. infection (HD game)

Infecting $\pi_{\mathbf{X}_\Lambda}$

$$\begin{array}{c|cc} & C & D \\ \hline C & \frac{1}{2} & \gamma \\ D & 1-\gamma & -\frac{1}{2} \end{array}$$

Infected $\pi_{\mathbf{Y}_\Gamma}$

$$\begin{array}{c|cc} & C & D \\ \hline C & \frac{1}{2} & 1-\gamma \\ D & \gamma & -\frac{1}{2} \end{array}$$

Evaluation of fitness

$$\rho_i^{T+1}\Big|_{t_0} = \rho_i^T\Big|_{t_0}\left[1 + \Pi_i^T\Big|_{t_\infty} - \overline{\Pi}^T\Big|_{t_\infty}\right]$$

New epidemic season

Phase 2: Evolution of concentrations/strategies (between season T and $T+1$)

The coevolutionary model

Phase 1 (within season T)

Coupled SIR (25 states) dynamics

Single pathogen infection

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

Multiple pathogens infection

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_{\Lambda\Gamma}} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda\mathbf{Y}_\Gamma$$

Accumulation of Payoff

Single pathogen infection

$$\pi_{\mathbf{X}_\Lambda} = 1$$

Multiple path. infection (HD game)

Infecting $\pi_{\mathbf{X}_\Lambda}$

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & \gamma \\ 1-\gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Infected $\pi_{\mathbf{Y}_\Gamma}$

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & 1-\gamma \\ \gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Evaluation of fitness

$$\rho_i^{T+1}\Big|_{t_0} = \rho_i^T\Big|_{t_0}\left[1 + \Pi_i^T\Big|_{t_\infty} - \overline{\Pi}^T\Big|_{t_\infty}\right]$$

Phase 2: Evolution of concentrations/strategies (between season T and $T+1$)

New epidemic season

The coevolutionary model

Phase 1 (within season T)

Coupled SIR (25 states) dynamics

Single pathogen infection

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

Multiple pathogens infection

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_{\Lambda\Gamma}} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda \mathbf{Y}_\Gamma$$

Accumulation of Payoff

Single pathogen infection

$$\pi_{\mathbf{X}_\Lambda} = 1$$

Multiple path. infection (HD game)

Infecting $\pi_{\mathbf{X}_\Lambda}$

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & \gamma \\ 1-\gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Infected $\pi_{\mathbf{Y}_\Gamma}$

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & 1-\gamma \\ \gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Evaluation of fitness

$$\rho_i^{T+1}\Big|_{t_0} = \rho_i^{T}\Big|_{t_0}\left[1 + \Pi_i^T\Big|_{t_\infty} - \overline{\Pi}^T\Big|_{t_\infty}\right]$$

New epidemic season

Phase 2: Evolution of concentrations/strategies (between season T and $T+1$)

The coevolutionary model

Phase 1 (within season T)

Coupled SIR (25 states) dynamics

Single pathogen infection

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

Multiple pathogens infection

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_{\Lambda\Gamma}} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda\mathbf{Y}_\Gamma$$

Accumulation of Payoff

Single pathogen infection

$$\pi_{\mathbf{X}_\Lambda} = 1$$

Multiple path. infection (HD game)

Infecting $\pi_{\mathbf{X}_\Lambda}$

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & \gamma \\ 1-\gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Infected $\pi_{\mathbf{Y}_\Gamma}$

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & 1-\gamma \\ \gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Evaluation of fitness

$$\rho_i^{T+1}\Big|_{t_0} = \rho_i^T\Big|_{t_0}\left[1 + \Pi_i^T\Big|_{t_\infty} - \overline{\Pi}^T\Big|_{t_\infty}\right]$$

New epidemic season

Phase 2: Evolution of concentrations/strategies (between season T and $T+1$)

The coevolutionary model

Phase 1 (within season T)

Coupled SIR (25 states) dynamics

Single pathogen infection

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

Multiple pathogens infection

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_{\Lambda\Gamma}} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda \mathbf{Y}_\Gamma$$

Accumulation of Payoff

Single pathogen infection

$$\pi_{\mathbf{X}_\Lambda} = 1$$

Multiple path. infection (HD game)

Infecting $\pi_{\mathbf{X}_\Lambda}$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & \gamma \\ 1-\gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Infected $\pi_{\mathbf{Y}_\Gamma}$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & 1-\gamma \\ \gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Evaluation of fitness

$$\rho_i^{T+1}\Big|_{t_0} = \rho_i^{T}\Big|_{t_0}\left[1 + \Pi_i^{T}\Big|_{t_\infty} - \overline{\Pi}^{T}\Big|_{t_\infty}\right]$$

Phase 2: Evolution of concentrations/strategies (between season T and $T+1$)

New epidemic season

Results

Pathogens’ perspective

Pathogens' perspective

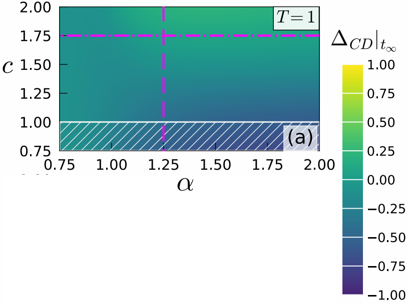
Species' prevalence

$$\Delta_{\mathrm{CD}}\Big|_{t_\infty} = \left(a_{\mathrm{C}} + b_{\mathrm{C}} + a_{\mathrm{C}} b_{\mathrm{C}}\right) - \left(a_{\mathrm{D}} + b_{\mathrm{D}} + a_{\mathrm{D}} b_{\mathrm{D}}\right)$$

Pathogens' perspective

Species' prevalence

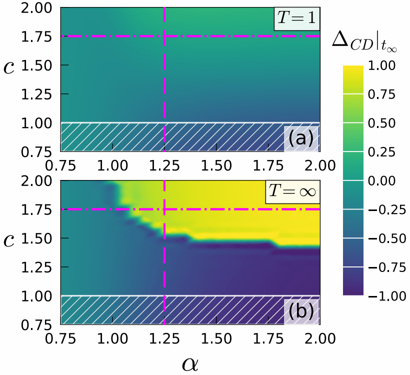
$$\Delta_{\mathrm{CD}}\Big|_{t_\infty} = \left(a_{\mathrm{C}} + b_{\mathrm{C}} + a_{\mathrm{C}}b_{\mathrm{C}}\right) - \left(a_{\mathrm{D}} + b_{\mathrm{D}} + a_{\mathrm{D}}b_{\mathrm{D}}\right)$$



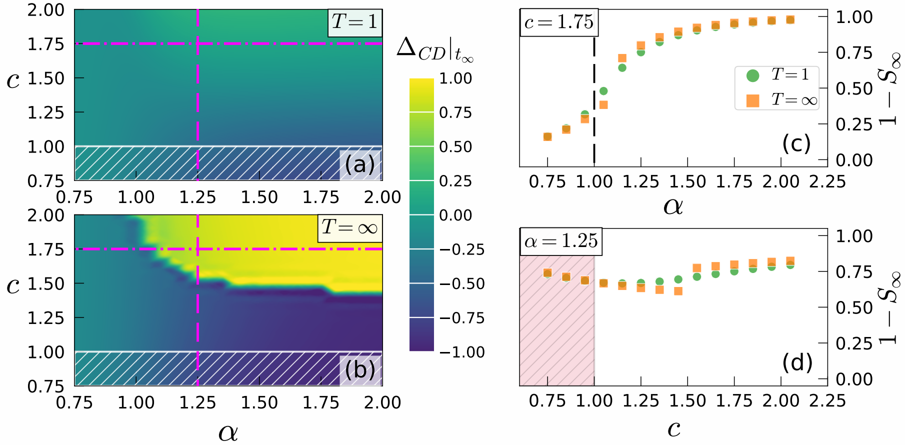
Pathogens' perspective

Species' prevalence

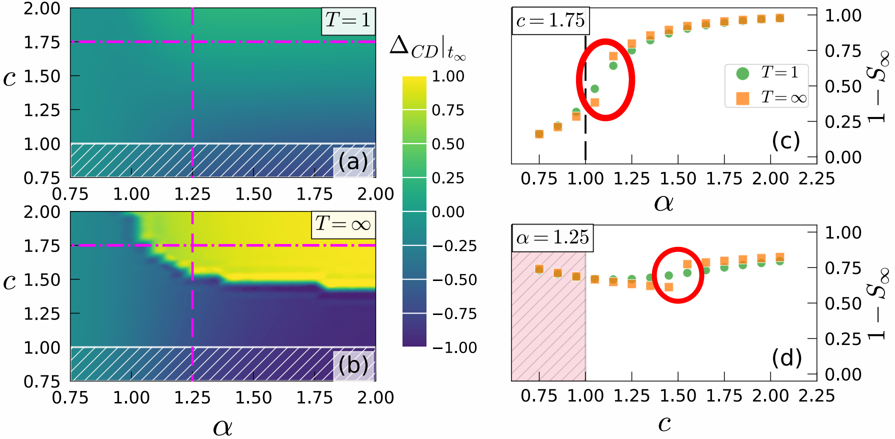
$$\Delta_{\mathrm{CD}}\Big|_{t_\infty} = \left(a_{\mathrm{C}} + b_{\mathrm{C}} + a_{\mathrm{C}}b_{\mathrm{C}}\right) - \left(a_{\mathrm{D}} + b_{\mathrm{D}} + a_{\mathrm{D}}b_{\mathrm{D}}\right)$$



Pathogens' perspective



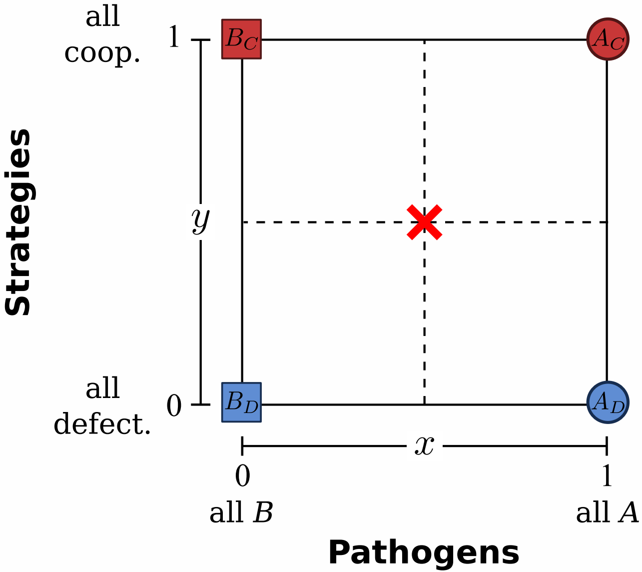
Pathogens' perspective



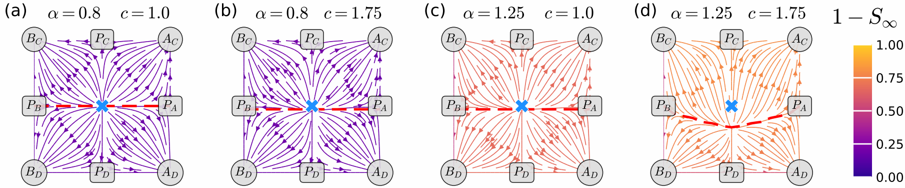
- Chen, L., *et al.* Europhys. Lett., **104**, 50001 (2013). DOI: `10.1209/0295-5075/104/50001`
- Cai, W., *et al.* Nat. Phys., **11**, 936–940 (2015). DOI: `10.1038/nphys3457`

Species' evolution across seasons

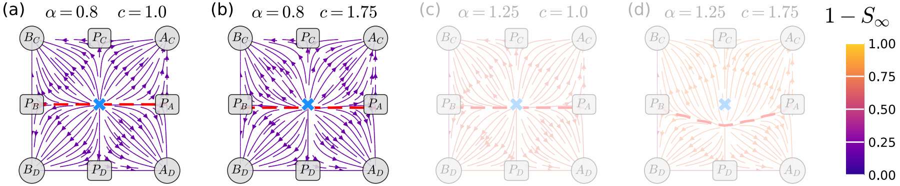
Species’ evolution across seasons



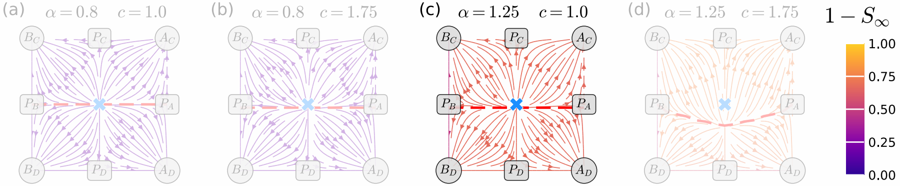
Species' evolution across seasons



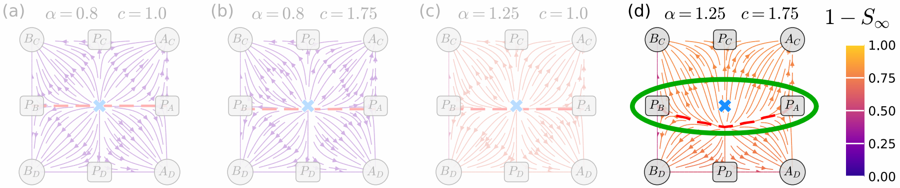
Species' evolution across seasons



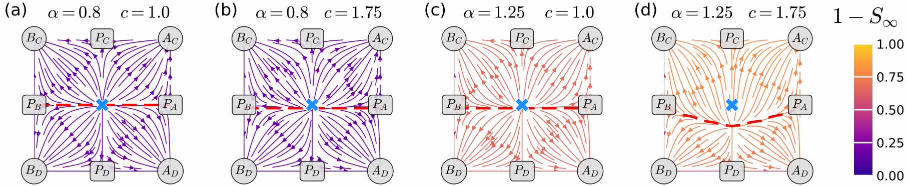
Species' evolution across seasons



Species’ evolution across seasons



Species' evolution across seasons



Summing up . . .

Take home messages

Phase 1 (within season T)

Coupled SIR (25 states) dynamics

Single pathogen infection

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

Multiple pathogens infection

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_{\Lambda\Gamma}} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda \mathbf{Y}_\Gamma$$

Accumulation of Payoff

Single pathogen infection

$$\pi_{\mathbf{X}_\Lambda} = 1$$

Multiple path. infection (HD game)

Infecting $\pi_{\mathbf{X}_\Lambda}$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & \gamma \\ 1-\gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Infected $\pi_{\mathbf{Y}_\Gamma}$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \frac{1}{2} & 1-\gamma \\ \gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Evaluation of fitness

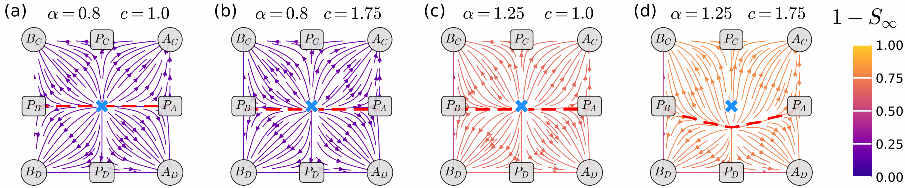
$$\rho_i^{T+1}\Big|_{t_0} = \rho_i^T\Big|_{t_0}\left[1 + \Pi_i^T\Big|_{t_\infty} - \overline{\Pi}^T\Big|_{t_\infty}\right]$$

Phase 2: Evolution of concentrations/strategies (between season T and $T+1$)

New epidemic season

A proposal to include evolutionary processes in multiple (SIR) disease spreading

Take home messages



The outcome of the dynamics is neither the expected one for epidemics, nor for games (more is different)

Acknowledgements

Fakhteh Ghanbarnejad

Philipp Hövel

Kai Seegers

Acknowledgements

GOBIERNO DE ESPAÑA
MINISTERIO DE CIENCIA E INNOVACIÓN

Grant no.
IJCI-2017-34300

European Research Council
Established by the European Commission

ERC-2018-STG
ID: 803860

Acknowledgements

PHYSICAL REVIEW E
covering statistical, nonlinear, biological, and soft matter physics

Highlights Recent Accepted Collections Authors Referees Search Press

Emergence of synergistic and competitive pathogens in a coevolutionary spreading model

Fakhteh Ghanbarnejad, Kai Seegers, Alessio Cardillo, and Philipp Hövel
Phys. Rev. E **105**, 034308 – Published 21 March 2022

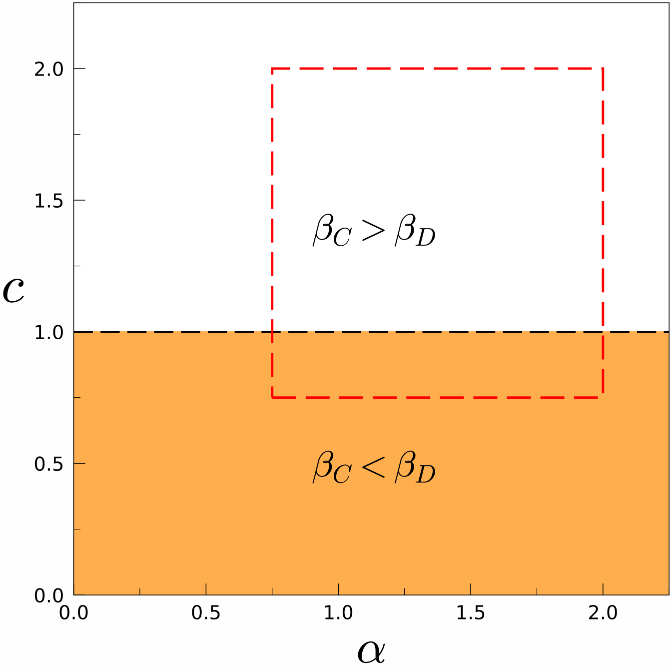
DOI: 10.1103/PhysRevE.105.034308

acardillo@uoc.edu

https://cardillo.web.bifi.es/

@a_cardillo

Extra contents


2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
c
α
$\beta_C > \beta_D$
$\beta_C < \beta_D$

Seed conservation

$$\rho_{A_C}\big|_{t=0} + \rho_{A_D}\big|_{t=0} + \rho_{B_C}\big|_{t=0} + \rho_{B_D}\big|_{t=0} = \omega$$

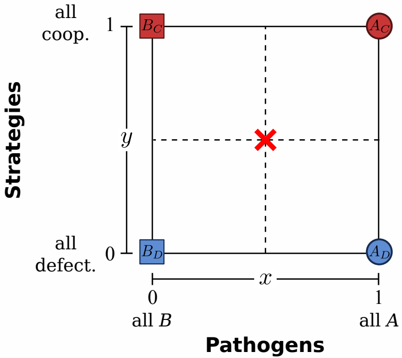
4D projection

$$\rho_{A_C} = [A_C]\Big|_{t=0} = \omega\, x\, y\,,$$

$$\rho_{A_D} = [A_D]\Big|_{t=0} = \omega\, x\,(1-y)\,,$$

$$\rho_{B_C} = [B_C]\Big|_{t=0} = \omega\,(1-x)\,y\,,$$

$$\rho_{B_D} = [B_D]\Big|_{t=0} = \omega\,(1-x)\,(1-y)\,.$$



Hawk and Dove's equilibria

Three fixed points

$$x^\star = \begin{cases} 0 & \longrightarrow \text{unstable} \\ 0 < x^\star < 1 & \longrightarrow \text{stable} \\ 1 & \longrightarrow \text{unstable} \end{cases}$$