Coevolution of Synchronization and Cooperation in Costly Networked Interactions

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CHARIOTS OF FIRE

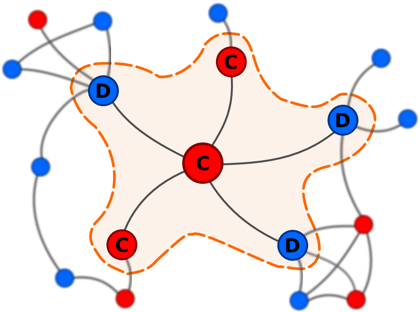
Kuramoto's Dilemma

Kuramoto's Dilemma

What happens to the synchronization when the interactions are regulated by the cost/benefit ratio?

Agents have strategies:
(*e.g.* cooperation and defection).

Agents play in a pairwise manner, and accumulate a payoff p according to the payoff matrix of the game.

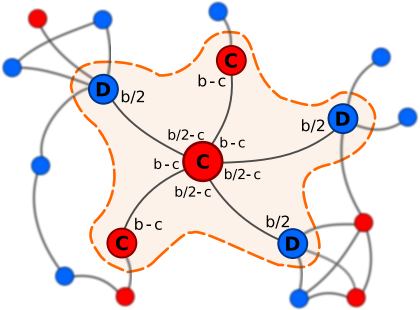


- Roca, C. P., *et al.* (2009). Phys. of Life Rev., **6**, 208.
- Szabó, G., & Fáth, G. (2007). Evolutionary games on graphs. Phys. Rep., **446**, 97–216.

Prisoner's Dilemma-like game

benefit: $b > 0$; cost: $c > 0$ ($b < 2c$)

	COOPERATION	DEFECTION
COOPERATION	**b - c**	**b/2 - c**
DEFECTION	**b/2**	**0**



• Roca, C. P., *et al.* (2009). Phys. of Life Rev., **6**, 208.
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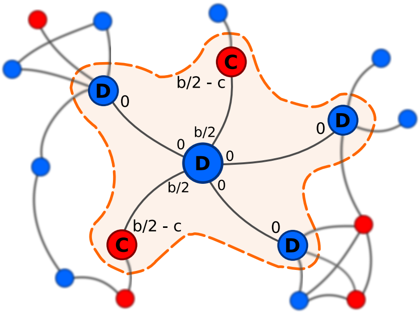
Agents update their strategies according to some rule.

$$P_{l \to m} = \frac{1}{1 + e^{-\beta(p_m - p_l)}} .$$



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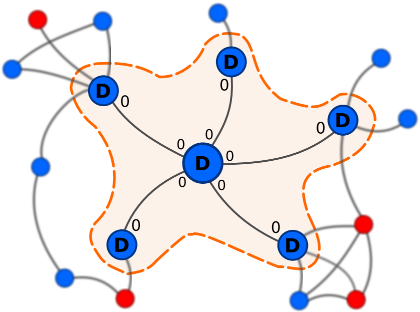
Repeat until stationary state



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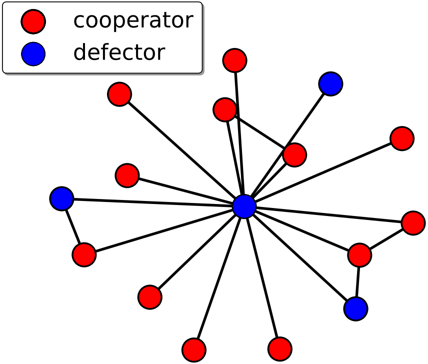
The model

The model



Strategy

$$s_l = \begin{cases} 1 & \text{if } l \text{ is cooperator} \\ 0 & \text{if } l \text{ is defector} \end{cases}$$

Phase

$$\theta_l \in [0, 2\pi]$$

Kuramoto

$$\dot{\theta}_l = \omega_l + \overbrace{s_l \, \lambda}^{\text{interaction}} \sum_{j=1}^{N} a_{lj} \sin(\theta_l - \theta_j) \, .$$

- Kuramoto, Y. (1984). Progress of Theoretical Physics Supplement, **79**, 223–240.
- Arenas, A. *et al.* (2008). Physics Reports, **469**, 93–153.

Payoff

$$p_l = \underbrace{r_{L_l}}_{\text{benefit}} - \underbrace{\alpha \frac{c_l}{2\pi}}_{\text{cost}}$$

$$\alpha \in \left]0, \infty\right[.$$

Payoff

$$p_l = \underbrace{r_{L_l}}_{\color{blue}{\text{benefit}}} - \underbrace{\alpha \frac{c_l}{2\pi}}_{\color{red}{\text{cost}}}$$

$$\alpha \in \,]0, \infty[\,.$$

Benefit

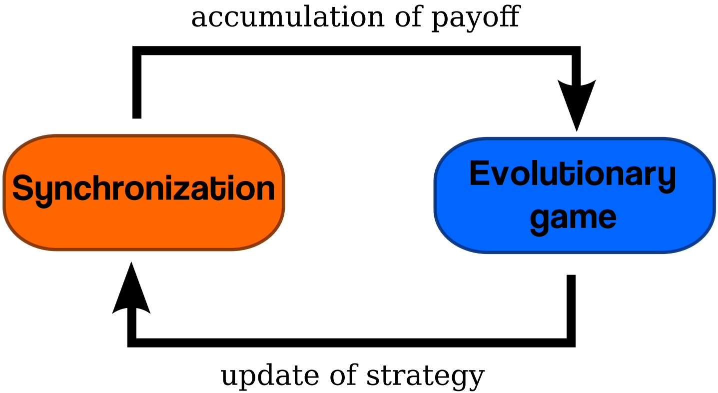
$$r_{L_l} = \frac{1}{k_l} \sum_{j=1}^{N} a_{lj} \frac{|e^{i\theta_l} + e^{i\theta_j}|}{2}$$

$$r_L \in [0, 1]\,,$$

Cost

$$c_l = \Delta\dot{\theta}_l = \left|\dot{\theta}_l(t) - \dot{\theta}_l(t-1)\right|$$

The model


accumulation of payoff
Synchronization
Evolutionary game
update of strategy

Question:

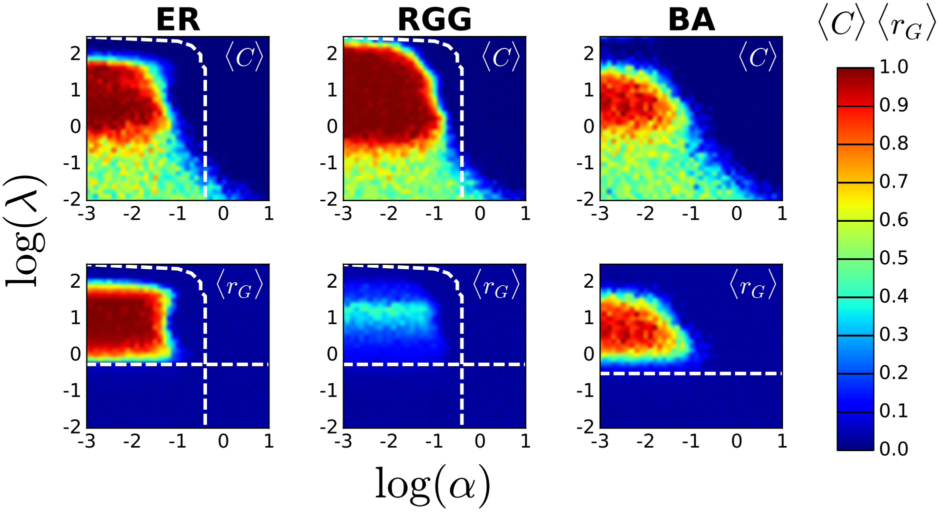
How the underlying topology of the interactions affects the **emergence** of cooperation/synchronization?

Answer:

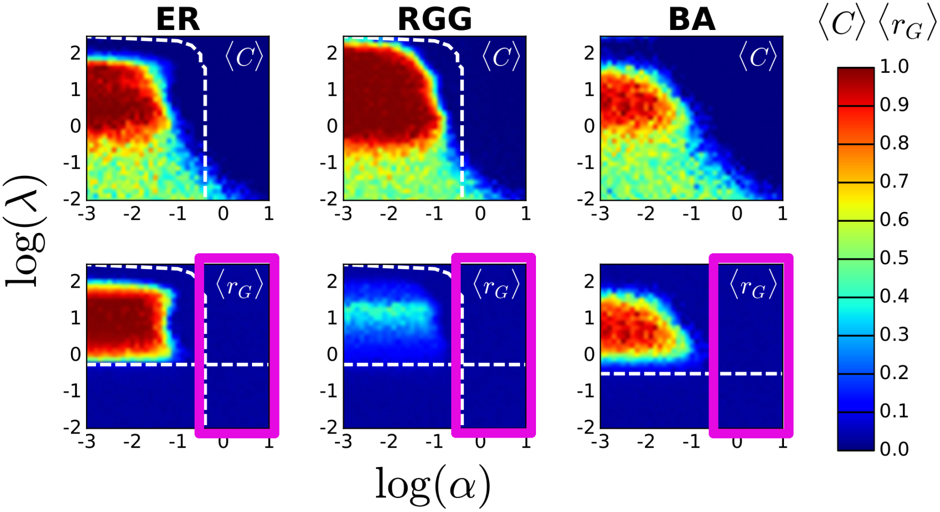
We consider three different topologies:

- ER $\Rightarrow$ Erdős-Rényi random graphs
- RGG $\Rightarrow$ Random Geometric Graph
- BA $\Rightarrow$ Barabási-Albert scale-free

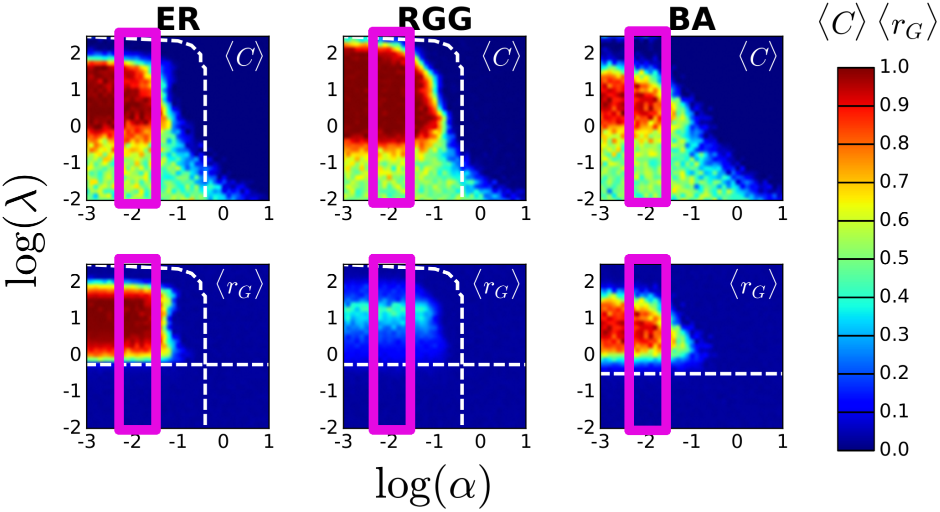
Macroscopic Behavior



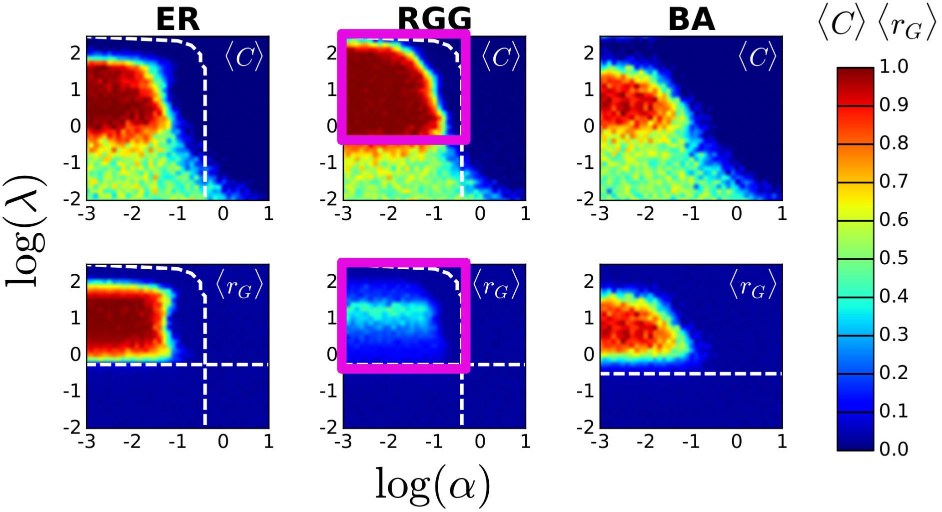
Macroscopic Behavior



Macroscopic Behavior

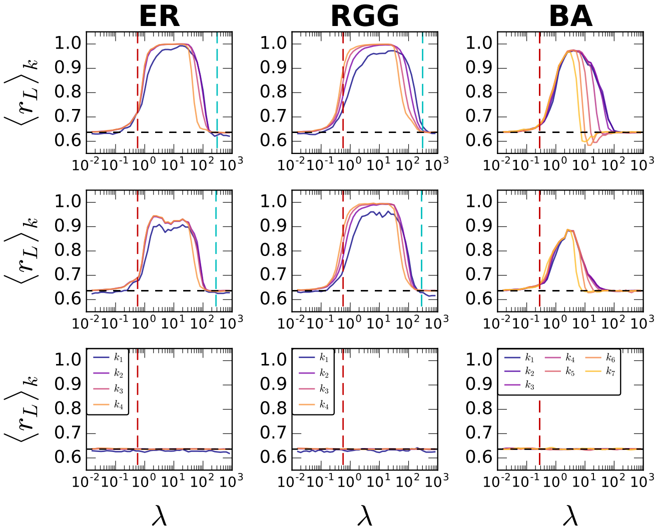


Macroscopic Behavior



Microscopic Behavior

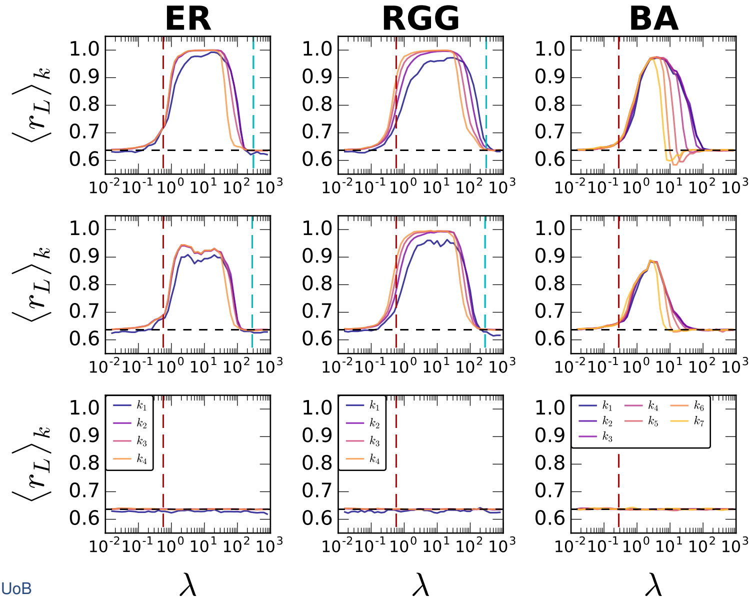
Microscopic Behavior



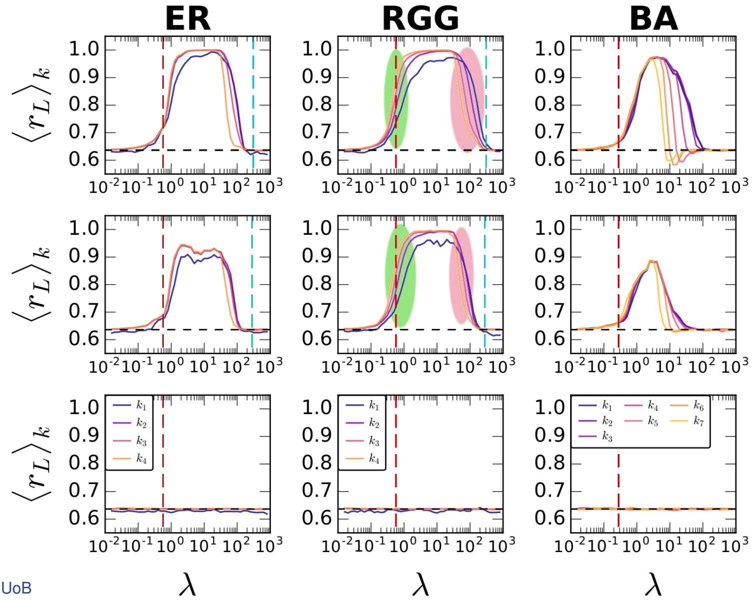
Three regimes of relative cost α:

- 10^{-3} Cheap
- $10^{-1.4}$ Medium
- 10^{0} Expensive

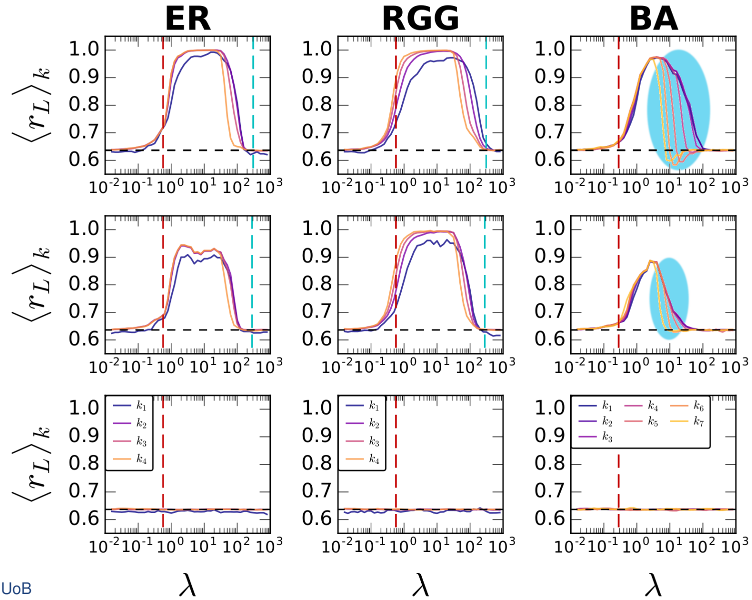
Microscopic Behavior



Microscopic Behavior

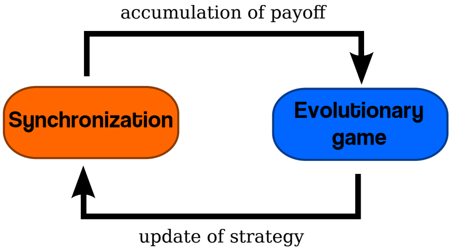


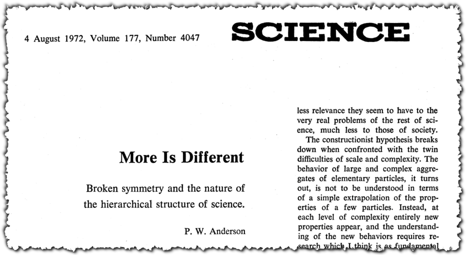
Microscopic Behavior



Conclusions

Coevolutionary model (Evolutionary Kuramoto's Dilemma) combining synchronization & evolutionary game theory.




4 August 1972, Volume 177, Number 4047

SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

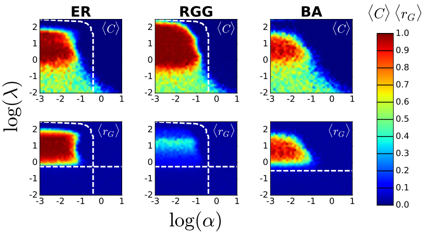
less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

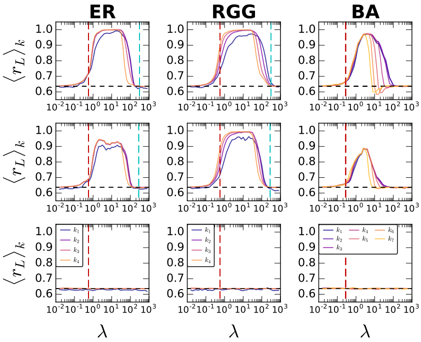
The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental

• Anderson, P. W. (1972). More Is Different. Science, **177**, 393–396.

Take home messages

Role of the underlying topology in the emergence of cooperation/synchronization.





Acknowledgements

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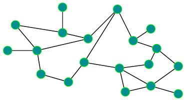
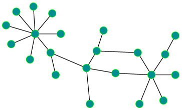
Extra Contents

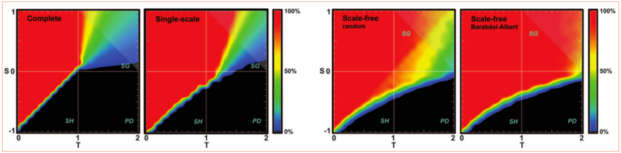
Erdős Rényi

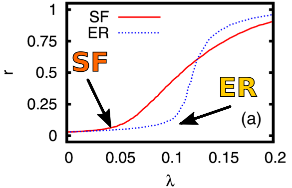




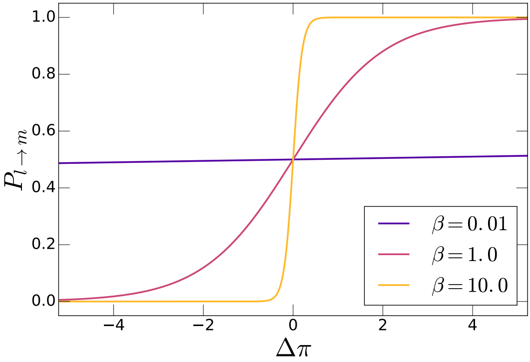
Scale Free

ER **SF**





- Santos, F., *et al.* (2006). Proceedings of the National Academy of Sciences, **103**, 3490–3494.
- Gómez-Gardeñes, J., *et al.* (2007). Physical Review Letters, **98**, 34101.



Fermi's Rule

$$P_{l\to m} = \frac{1}{1 + e^{-\beta(\pi_m - \pi_l)}}\,.$$

Lower bound

$$\lambda_c = \lambda_c^{MF} \frac{\langle k \rangle}{\langle k^2 \rangle}$$

Upper bound

$$\frac{\Delta b}{\Delta c} = \frac{b_{Coop} - b_{Def}}{c} > \langle k \rangle$$

$$\frac{\sqrt{2\left[1 + \sin(\varepsilon\lambda)\right]} - \sqrt{2}}{\varepsilon\lambda\langle k \rangle}\pi > \alpha \, .$$

- Arenas, A., *et al.* (2008). Physics Reports, **469**, 93–153.
- Ohtsuki, H. *et al.* (2006). Nature, **441**, 502–505.

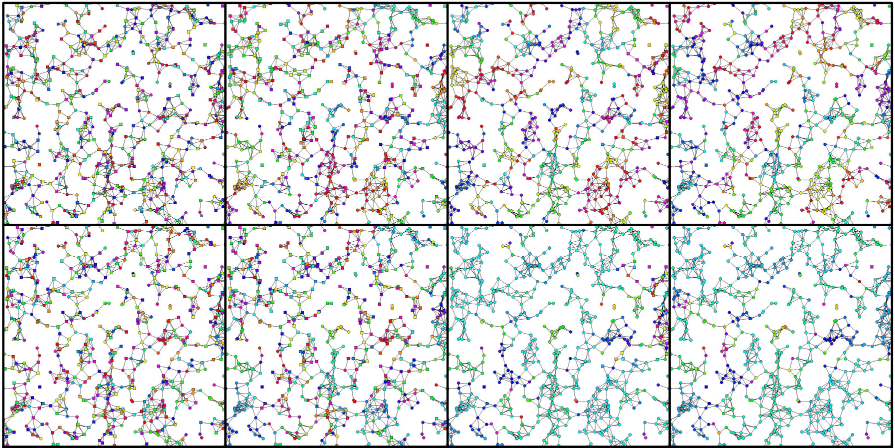
average pairwise order parameter

$$\overline{r_{lm}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\left\|1 + e^{i\theta}\right\|}{2}\, d\theta =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\|1 + \cos\theta + i\sin\theta\|}{2}\, d\theta =$$

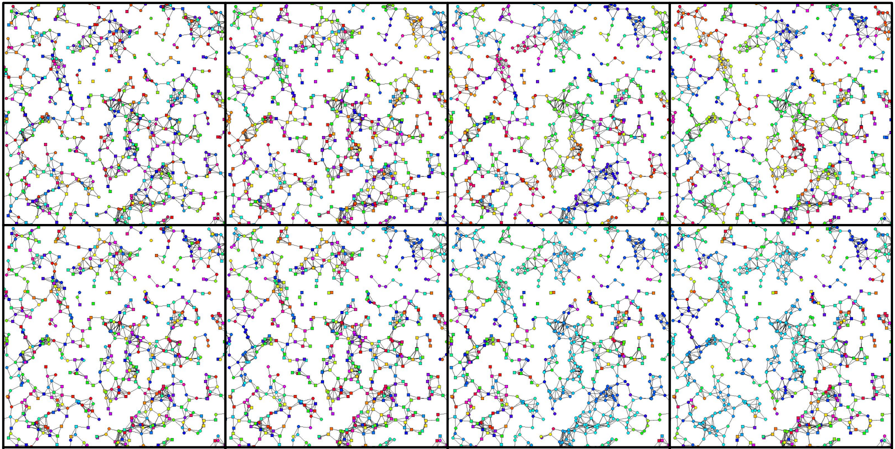
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\sqrt{[1+\cos\theta]^2 + \sin^2\theta}}{2}\, d\theta = \frac{4}{2\pi} = \frac{2}{\pi} \sim 0.6366 \,.$$

Microscopic Behaviour RGG



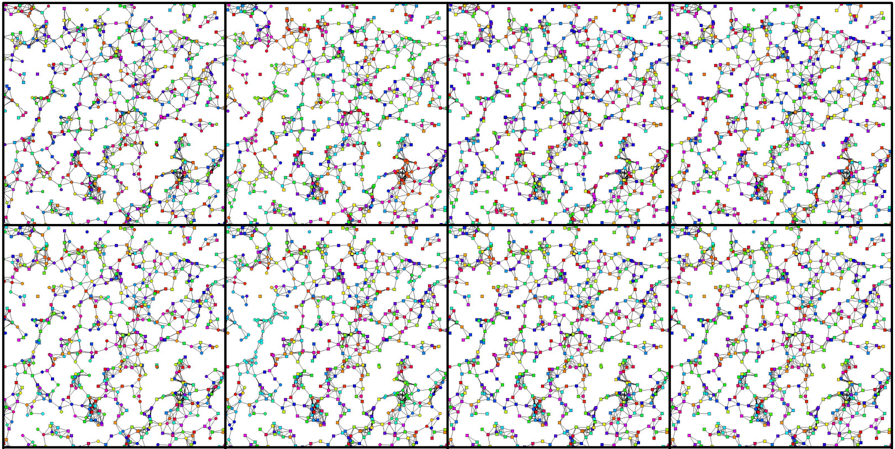
t = 0	t = 200	t = 2000	t = 5000
r_G = 0.011	r_G = 0.054	r_G = 0.114	r_G = 0.070
r_L = 0.640	r_L = 0.787	r_L = 0.913	r_L = 0.918
coop = 0.500	coop = 0.565	coop = 0.937	coop = 0.998

Microscopic Behaviour RGG



t = 0	t = 200	t = 2000	t = 5000
r_G = 0.039	r_G = 0.097	r_G = 0.042	r_G = 0.094
r_L = 0.641	r_L = 0.773	r_L = 0.840	r_L = 0.822
coop = 0.500	coop = 0.479	coop = 0.690	coop = 0.604

Microscopic Behaviour RGG



$t = 0$	$t = 200$	$t = 2000$	$t = 5000$
$r_G = 0.033$	$r_G = 0.048$	$r_G = 0.053$	$r_G = 0.042$
$r_L = 0.631$	$r_L = 0.738$	$r_L = 0.647$	$r_L = 0.635$
coop = 0.500	coop = 0.322	coop = 0.017	coop = 0.008

Synchronous Imitation of the best