# Tipping point in evolutionary games on networks triggered by zealots

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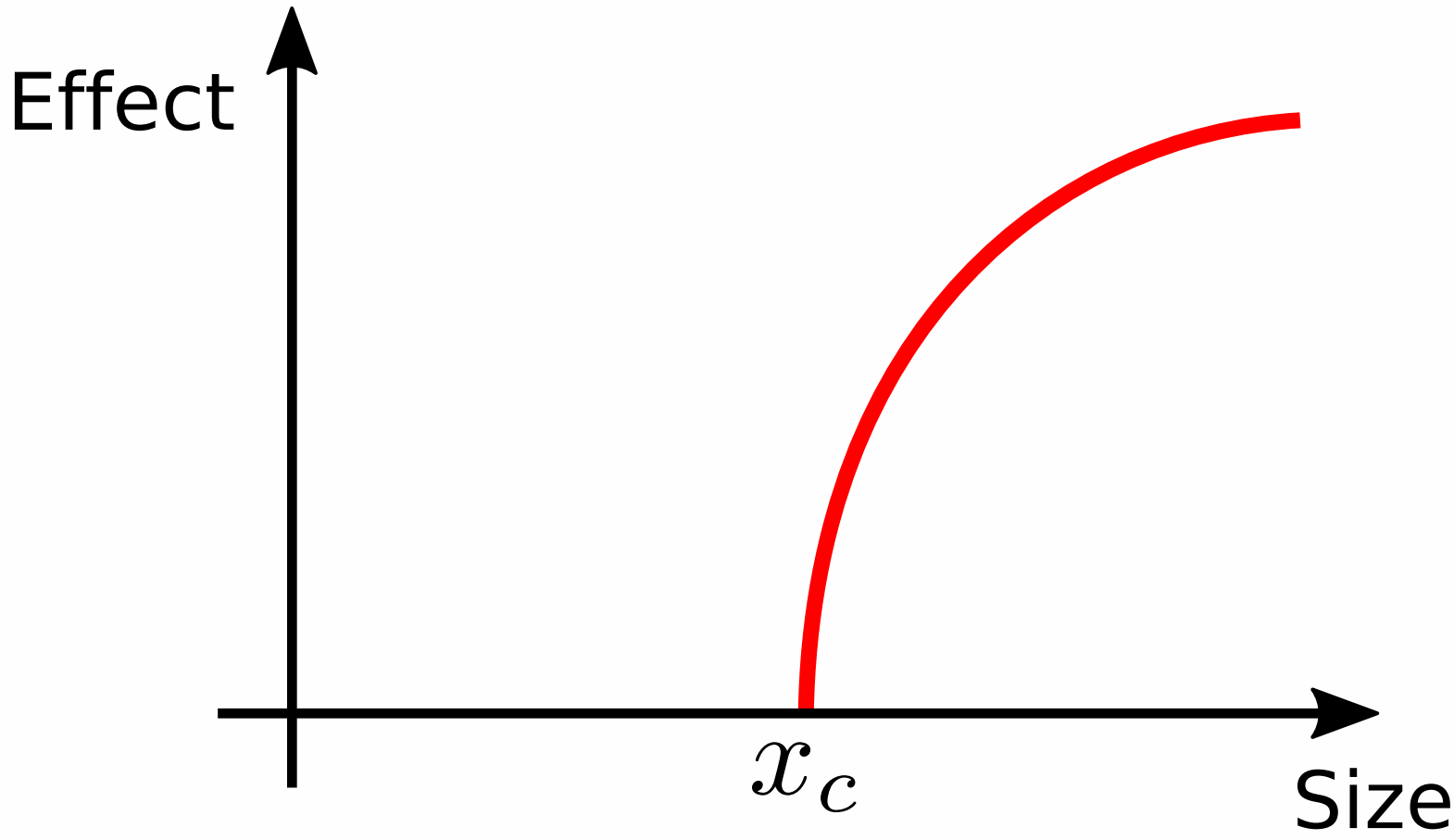
Why do
1 in 100
children
have
AUTISM





- M. Mobilia, Phys. Rev. Lett. **91**, 028701 (2003).
- J. Xie *et al.* , Phys. Rev. E, **84**, 011130 (2011).
- D. Centola *et al.* , Science, **360**, 1116 (2018).

## Main questions:

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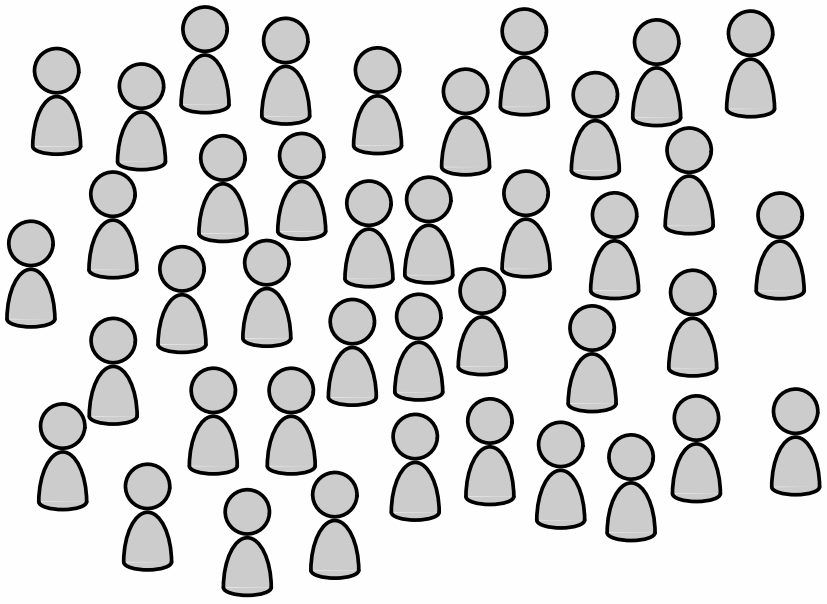
2. Which **factors affect** the presence of such an effect?
3. What about **networked interactions**?

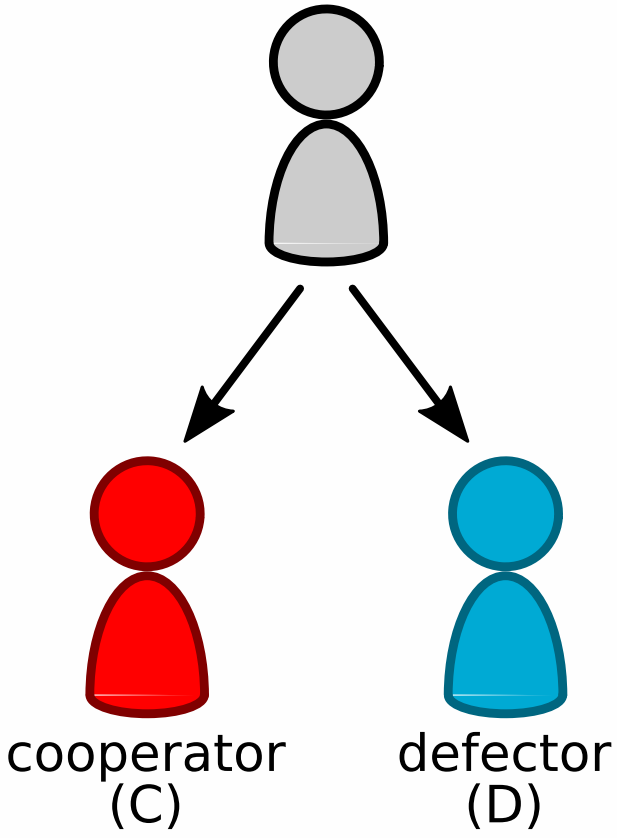
# The model

- Population of $N$ agents



# The model

- Population of $N$ agents
- Two strategies: **cooperation** (C) and **defection** (D)



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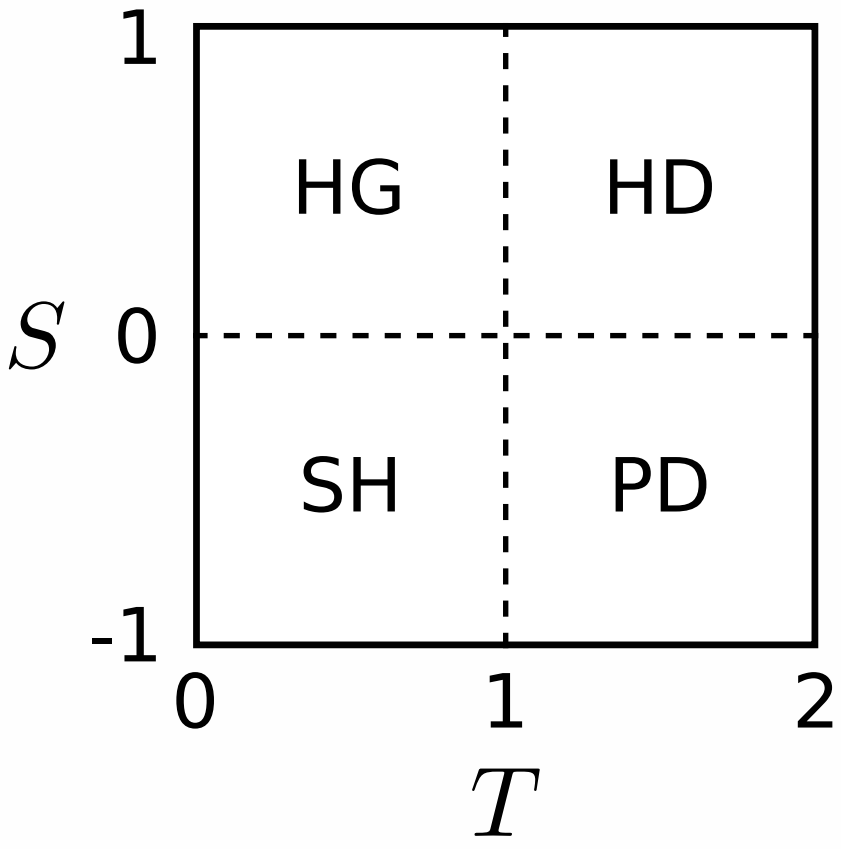
$$\begin{array}{c} \\ \text{C} \\ \text{D} \end{array} \begin{array}{c} \begin{array}{cc} \text{C} & \text{D} \end{array} \\ \begin{pmatrix} 1 & S \\ T & 0 \end{pmatrix} \end{array} \quad \text{with} \quad \begin{cases} T \in [0, 2] \\ S \in [-1, 1] \end{cases}$$



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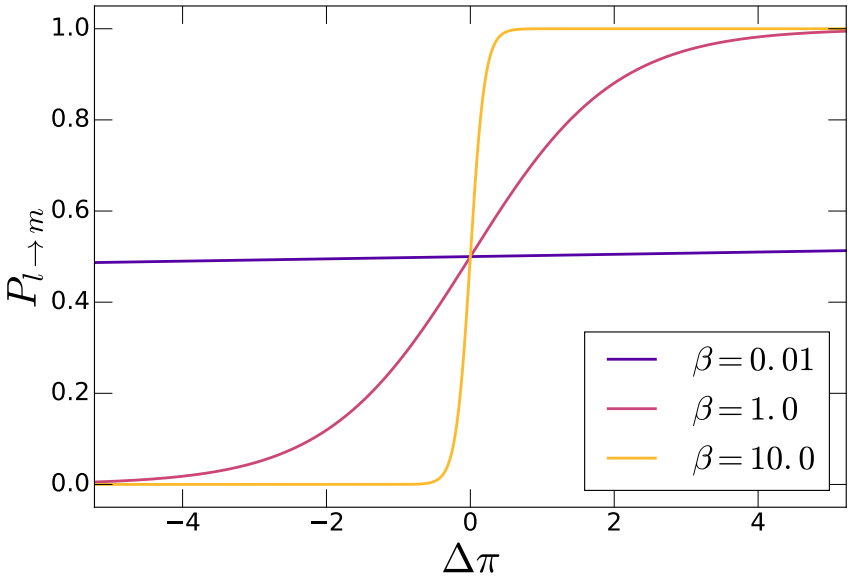
- Strategies evolve according to the **Fermi rule**

$$P_{\mathrm{X} \leftarrow \mathrm{Y}} = \frac{1}{1 + e^{-\beta(\pi_{\mathrm{Y}} - \pi_{\mathrm{X}})}} \quad \beta \in [0, \infty[$$





- G. Szabó, & C. Tőke, Phys. Rev. E, **58**, 69 (1998).

# The model

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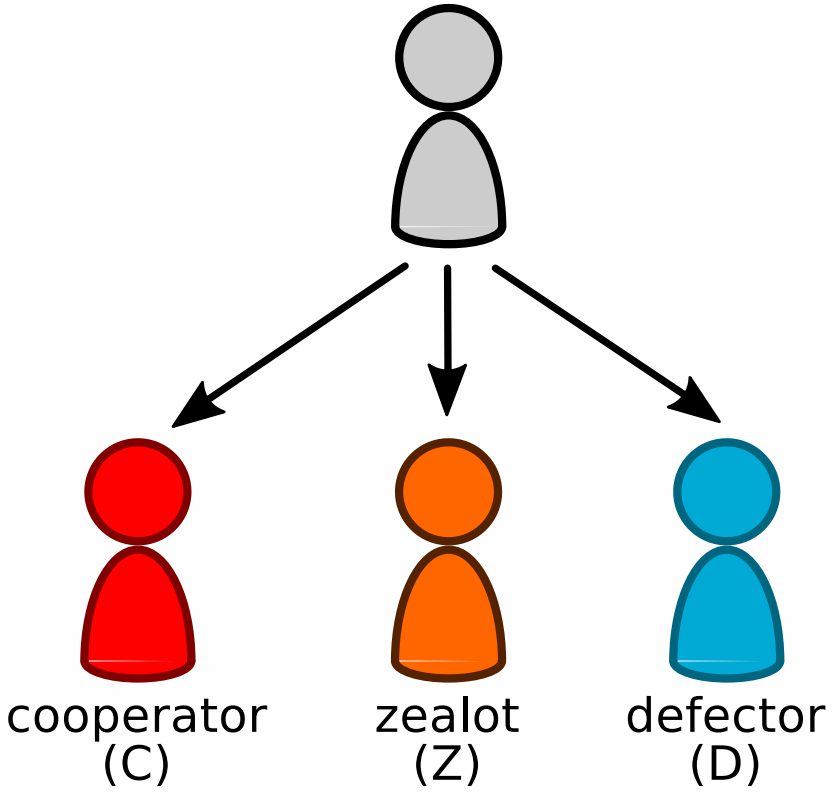

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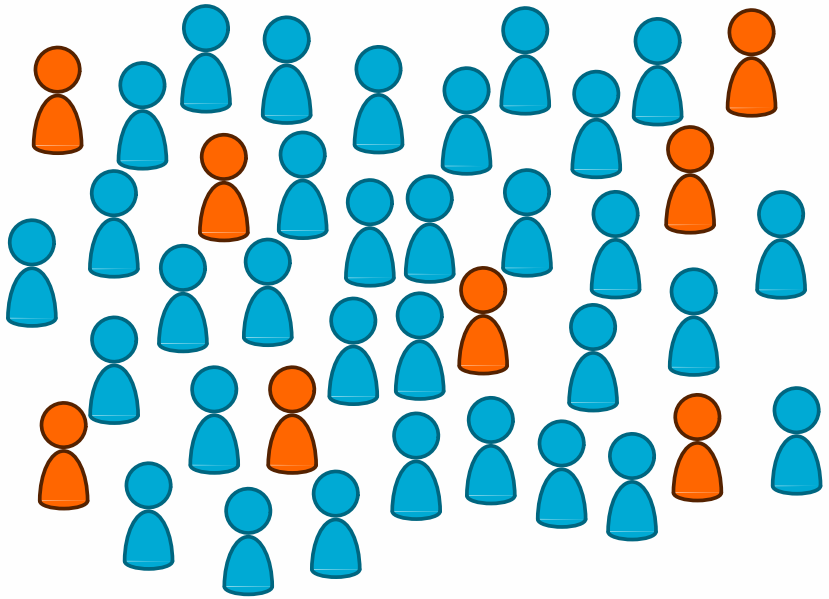
- New type of player: (cooperative) **zealot**.
- Population fully made of defectors & replace a fraction $f_Z \in \left[0, \frac{1}{2}\right]$ of agents with zealots.



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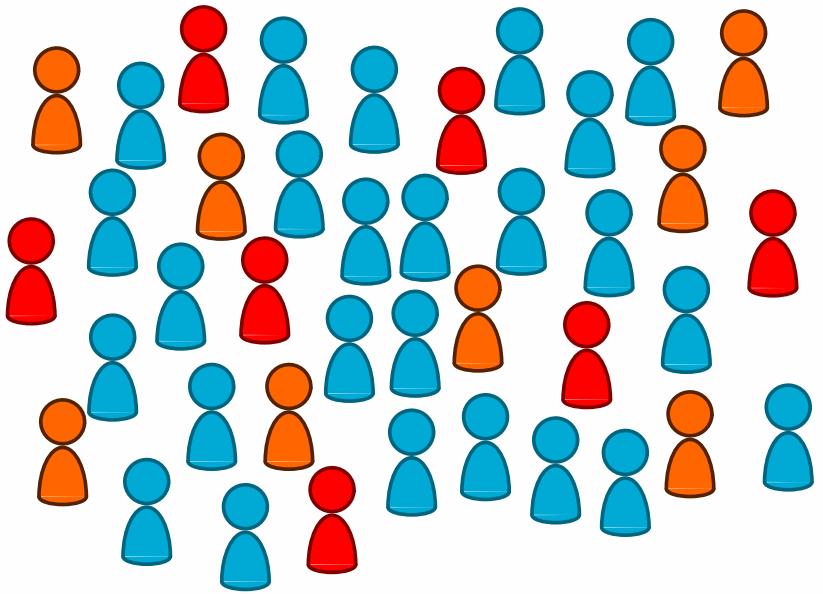

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- Compute fraction of cooperators **among normal agents**, $f_C$

$$f_C = \frac{N_C}{(1 - f_Z)\, N} \qquad f_C \in [0, 1]$$

# Results in mean field populations

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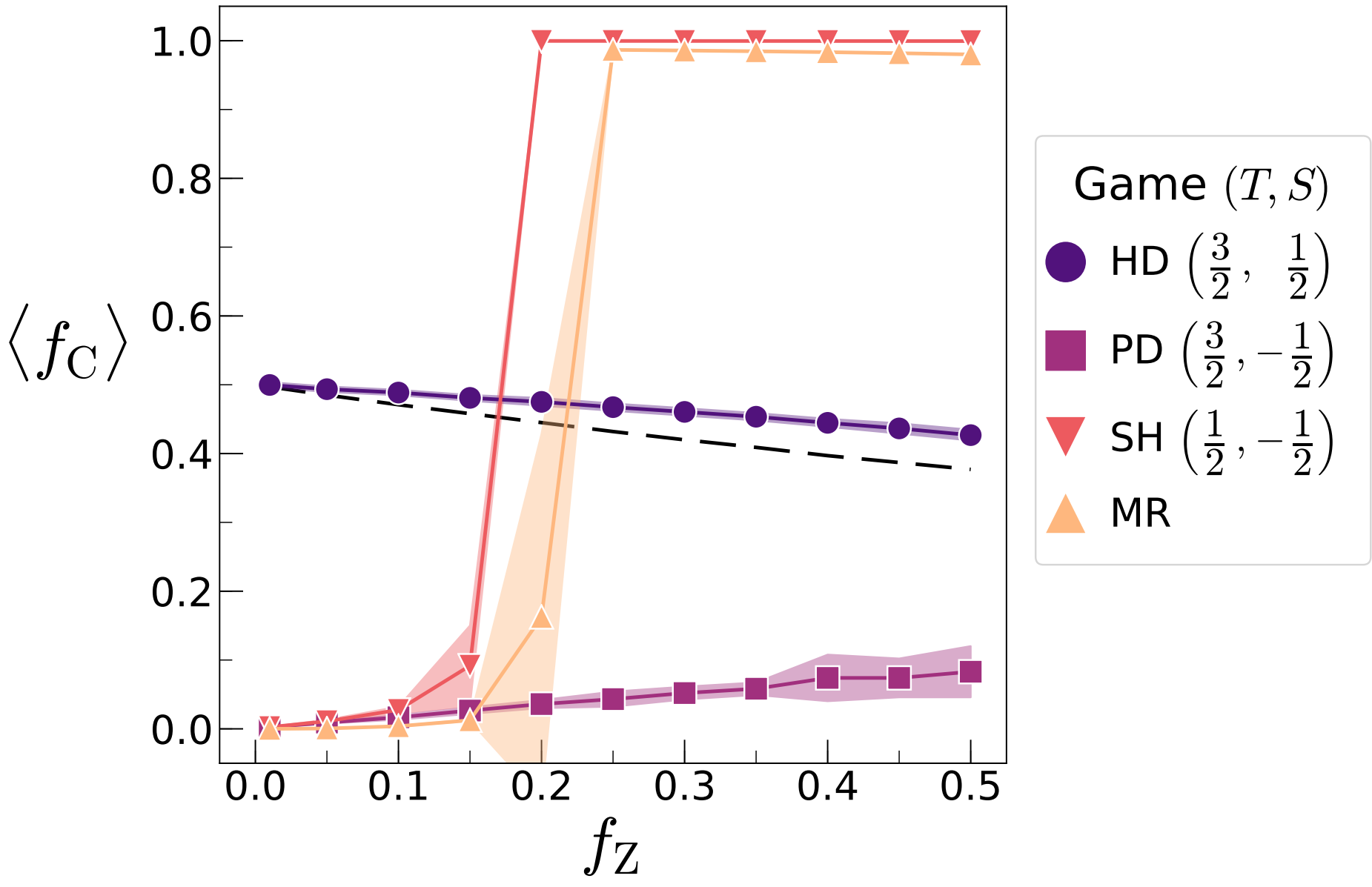

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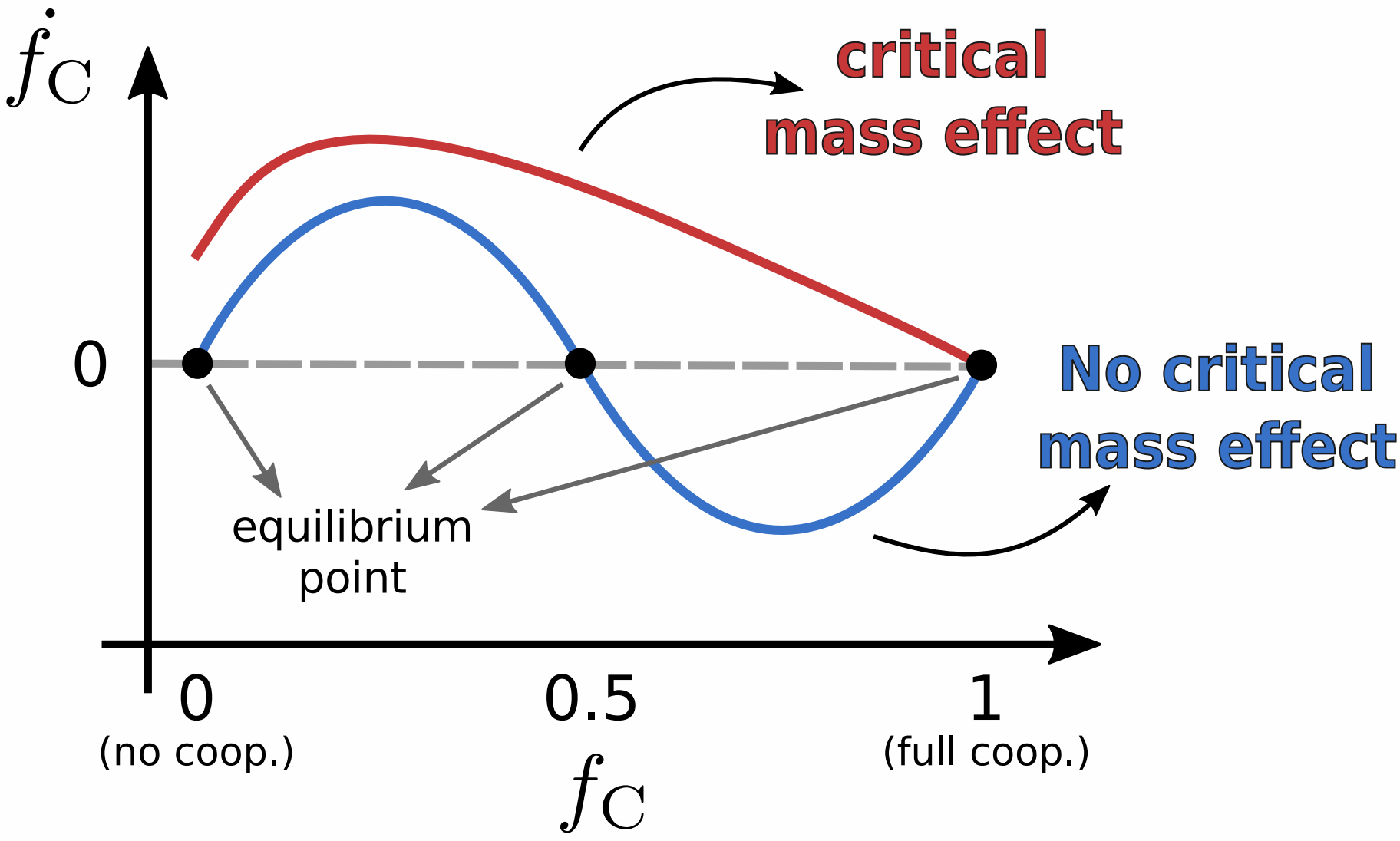

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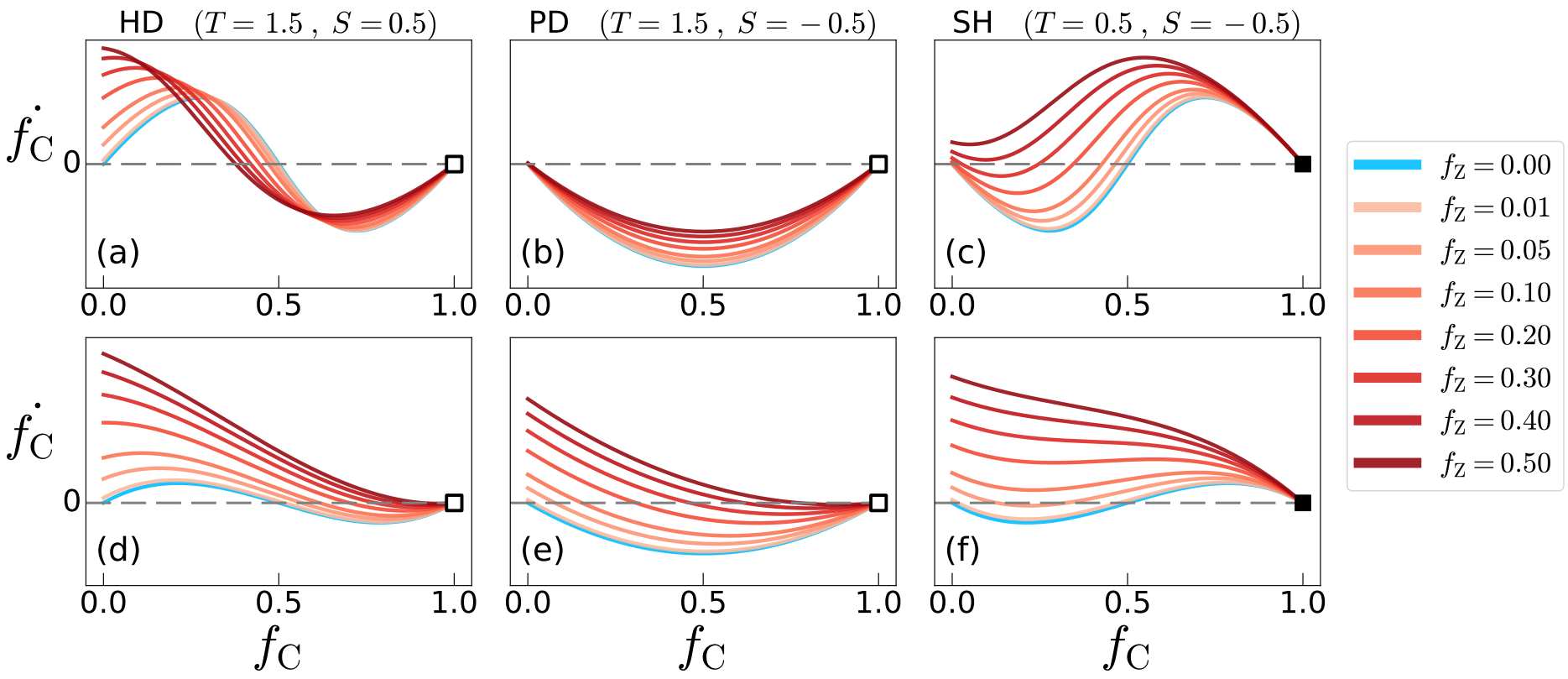

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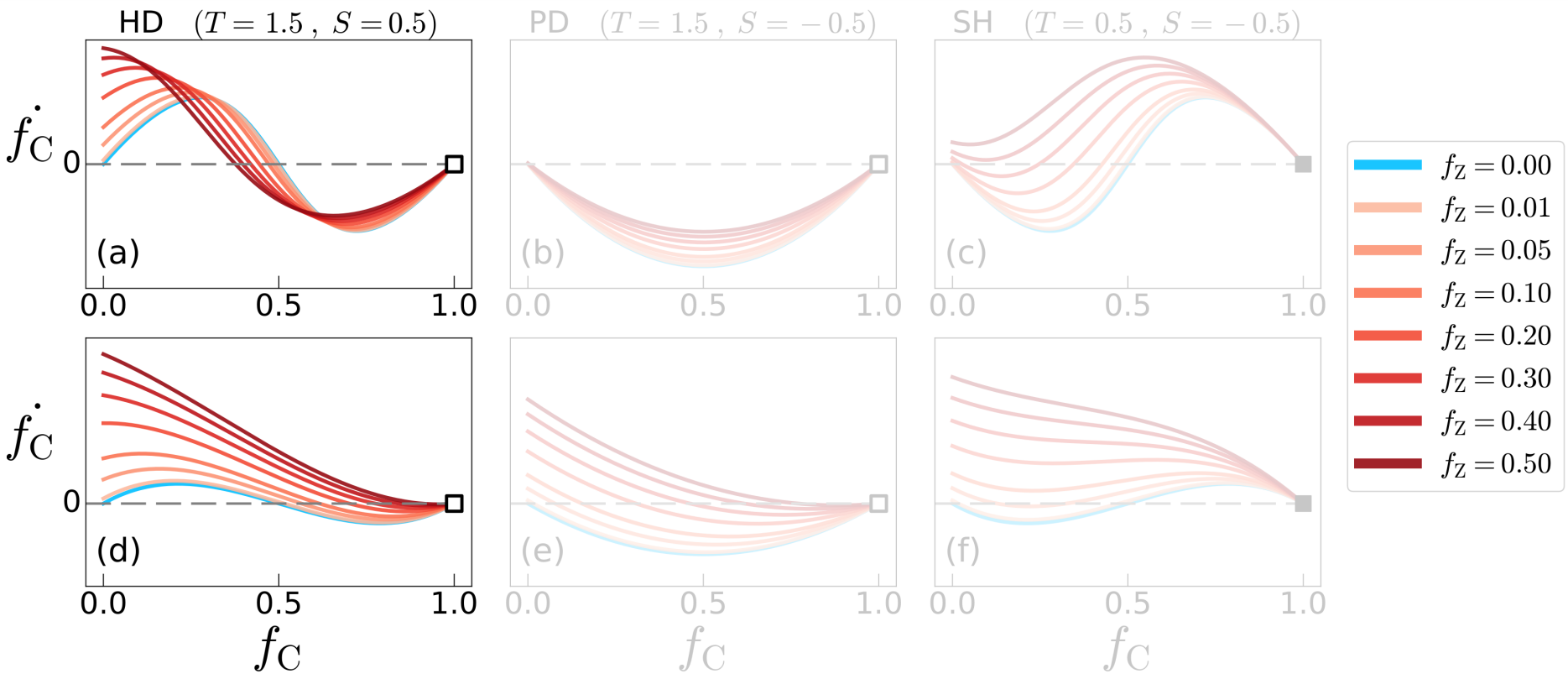

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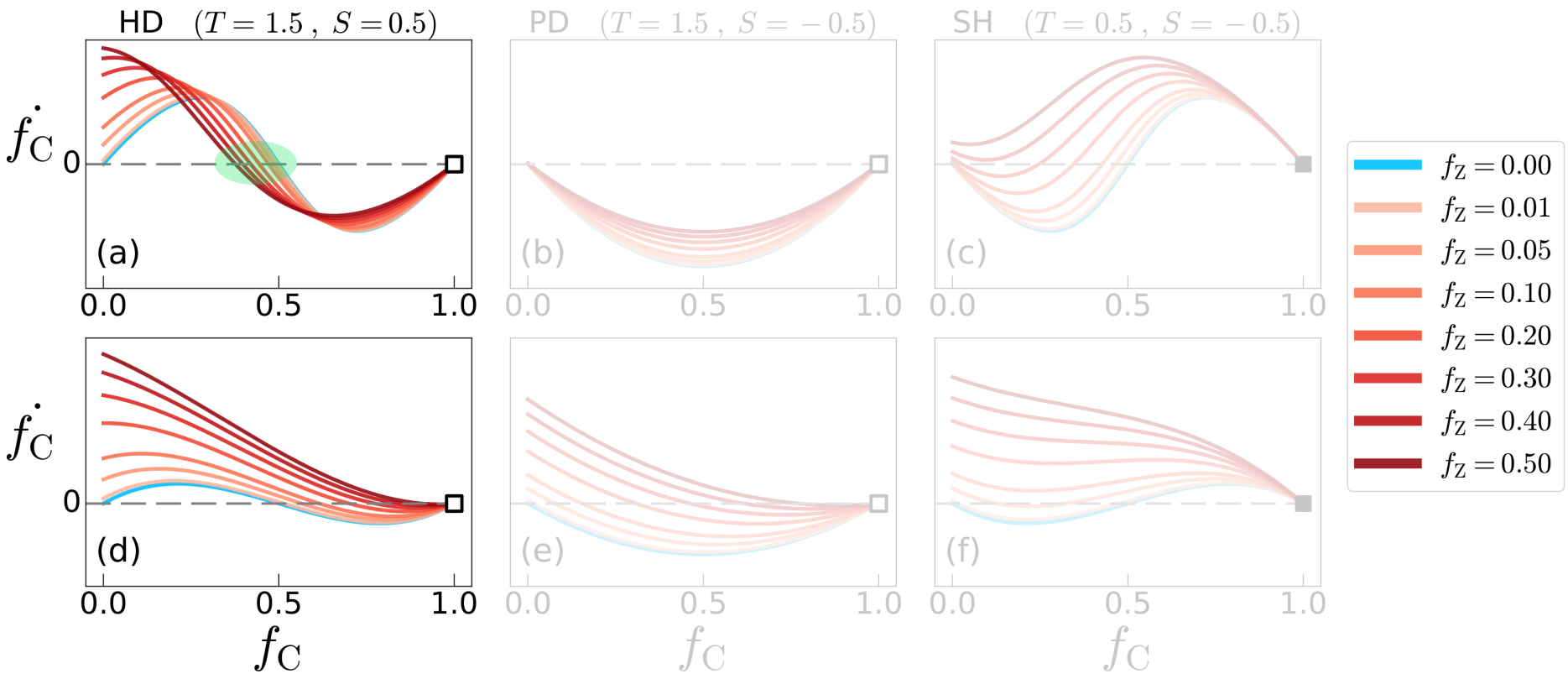

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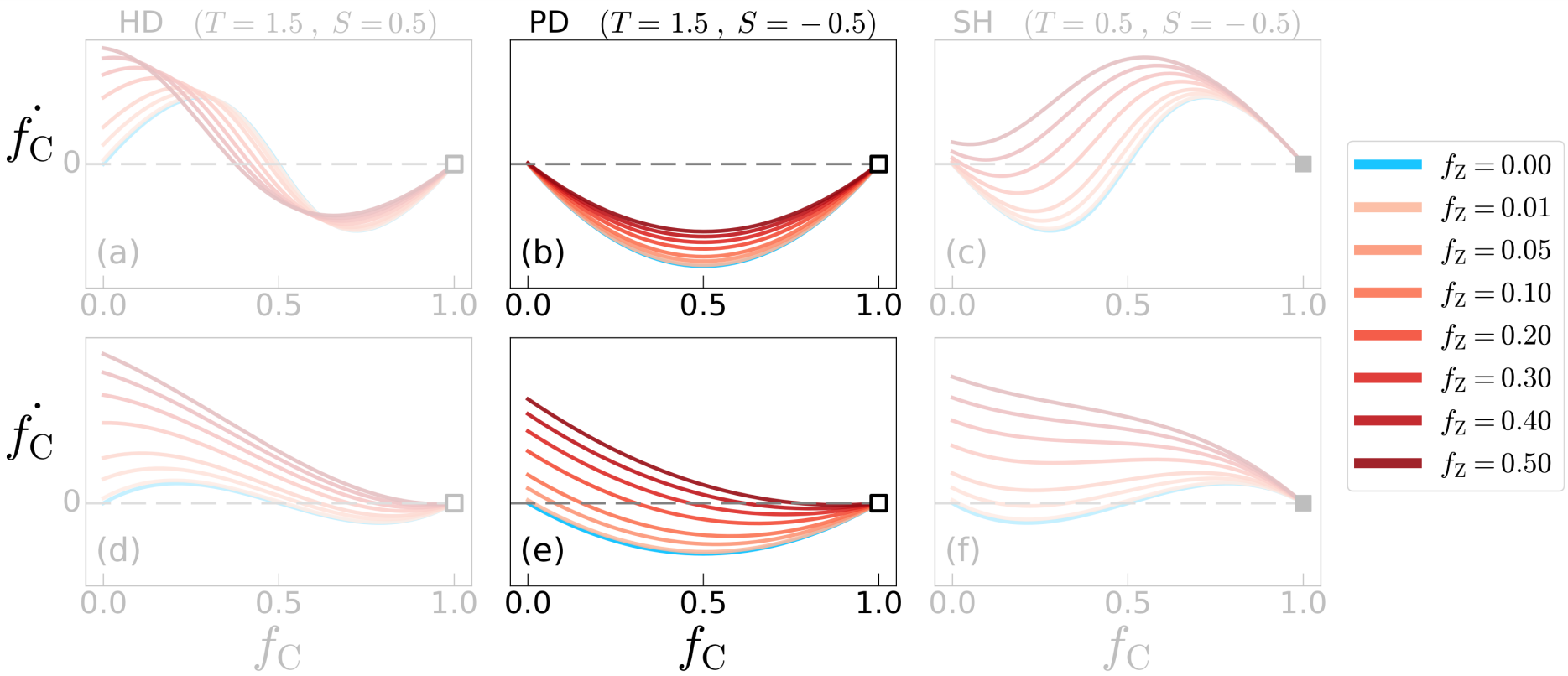

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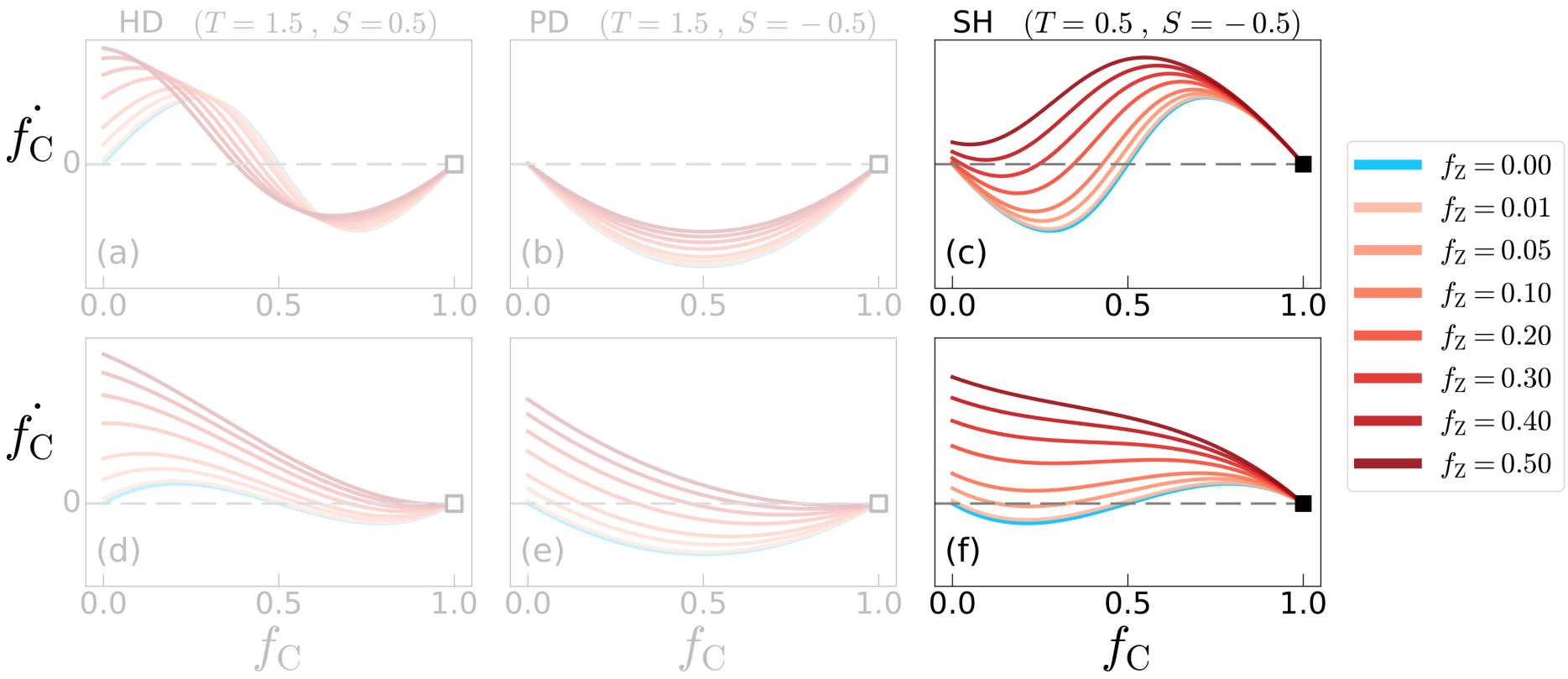

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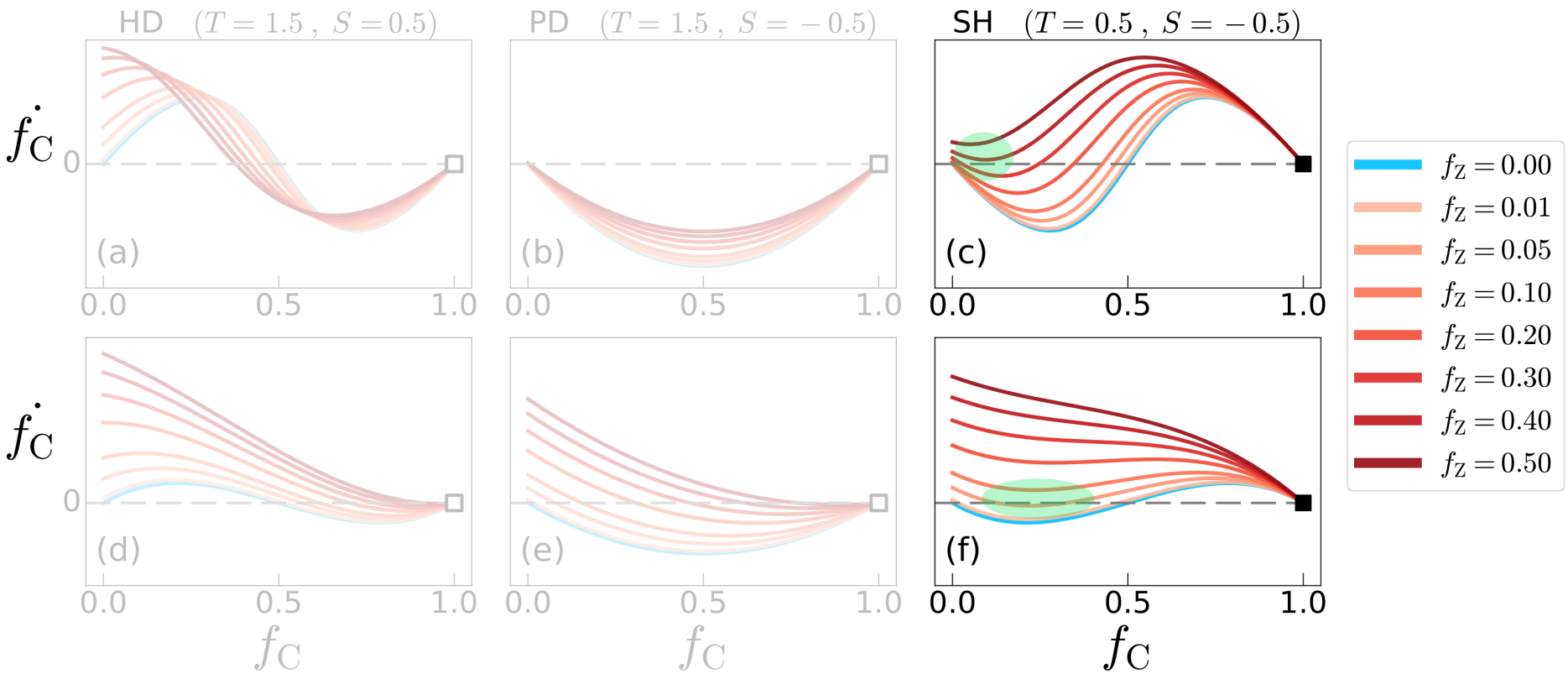

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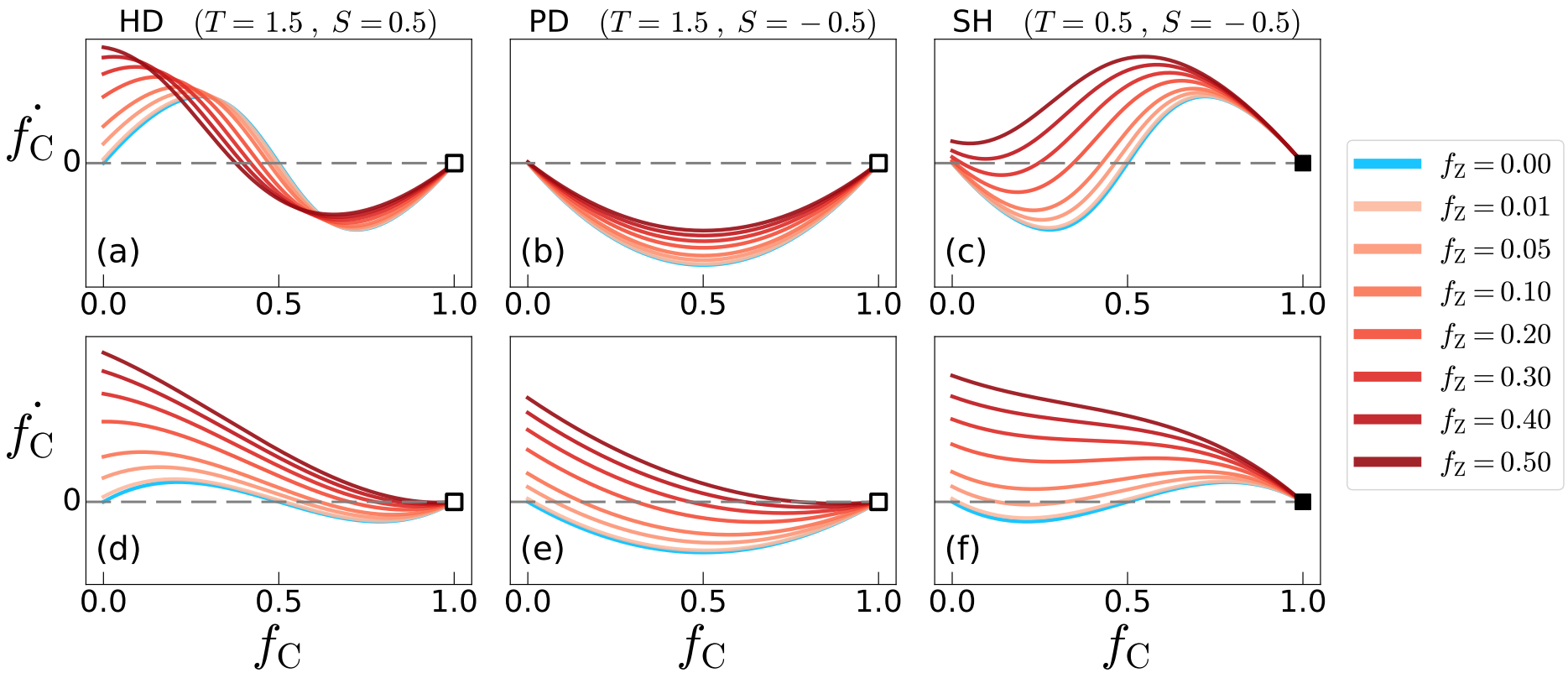

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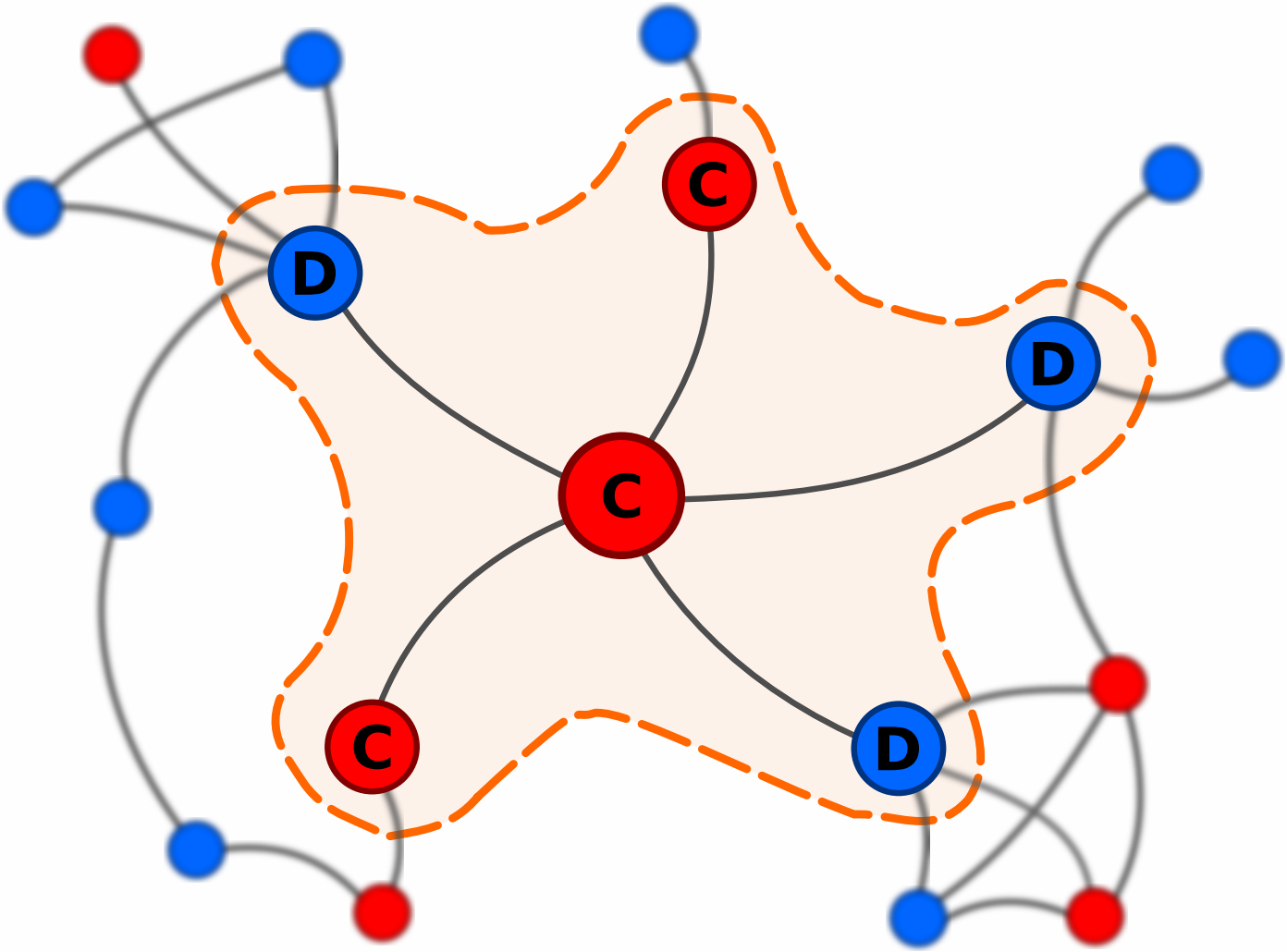

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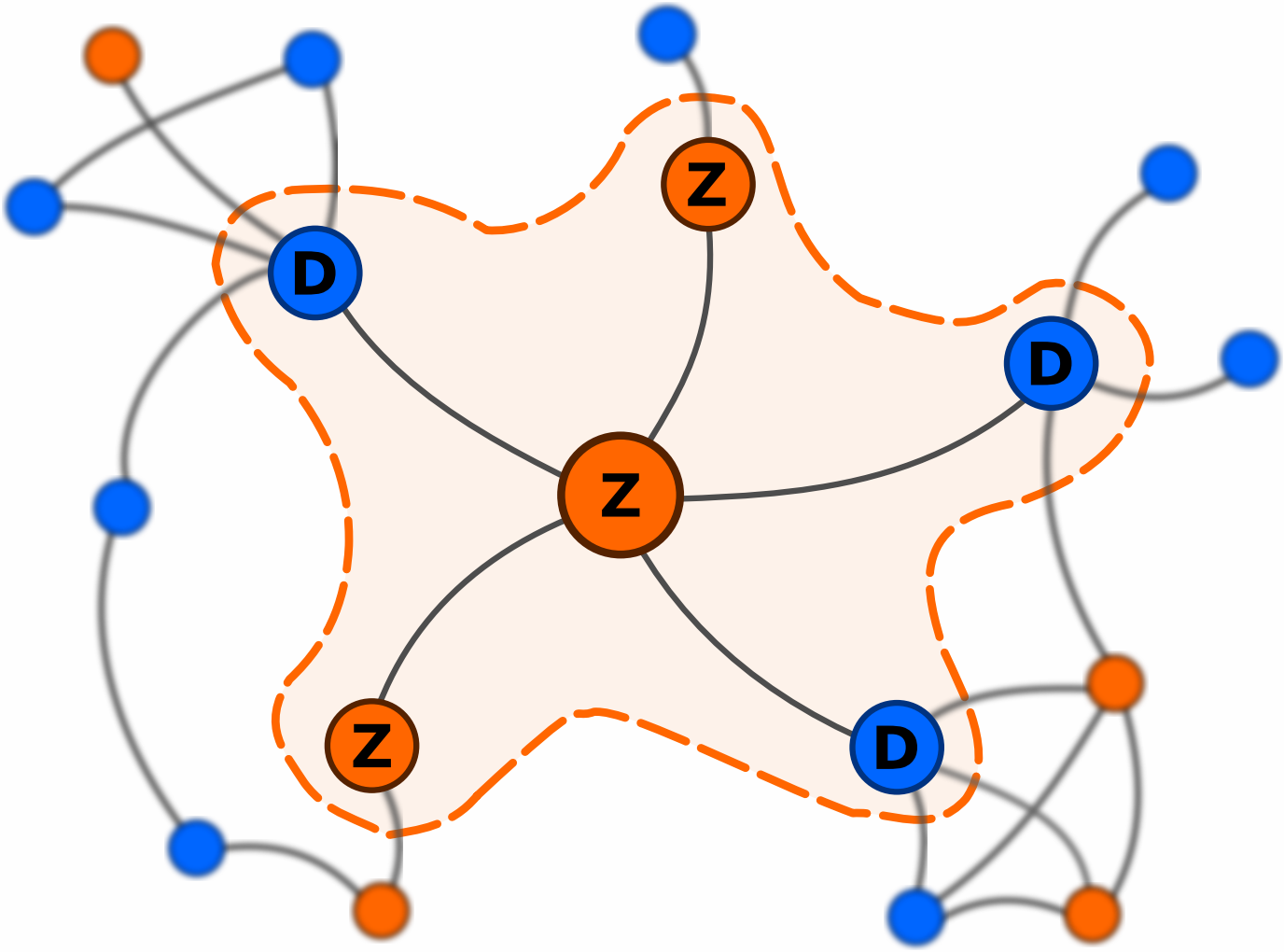

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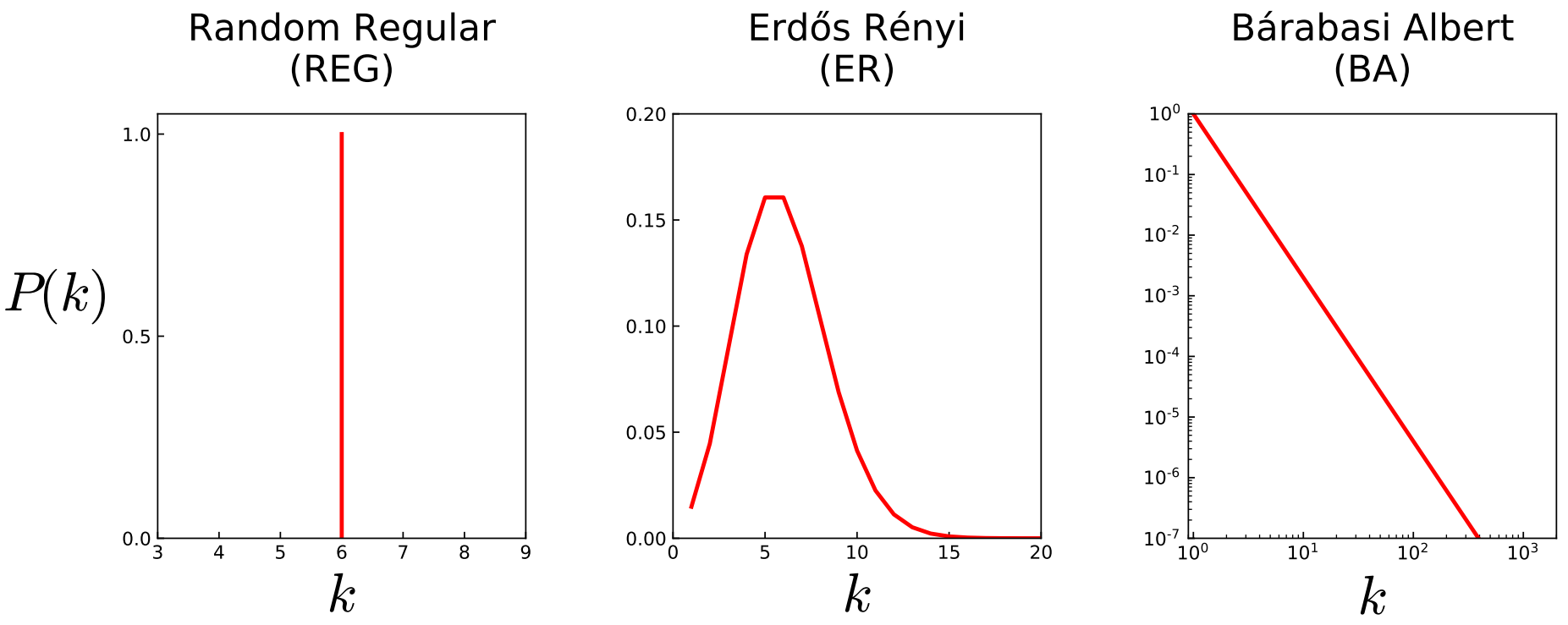


**Note:**

All nets have $N = 1000$ and $\langle k \rangle = 6$

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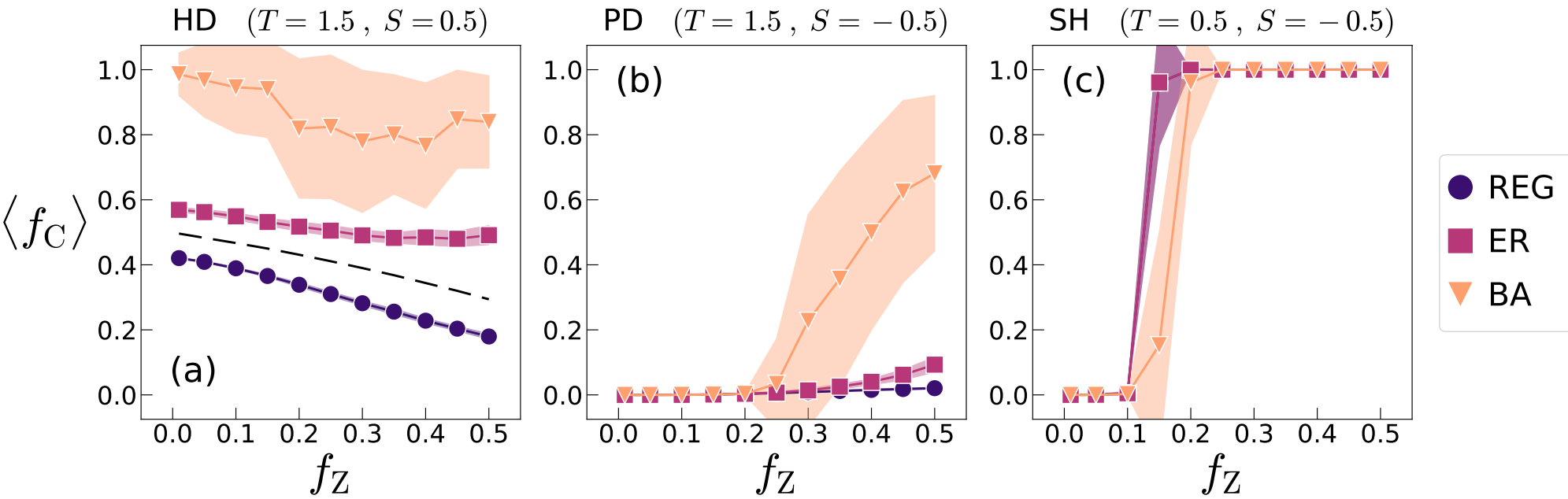

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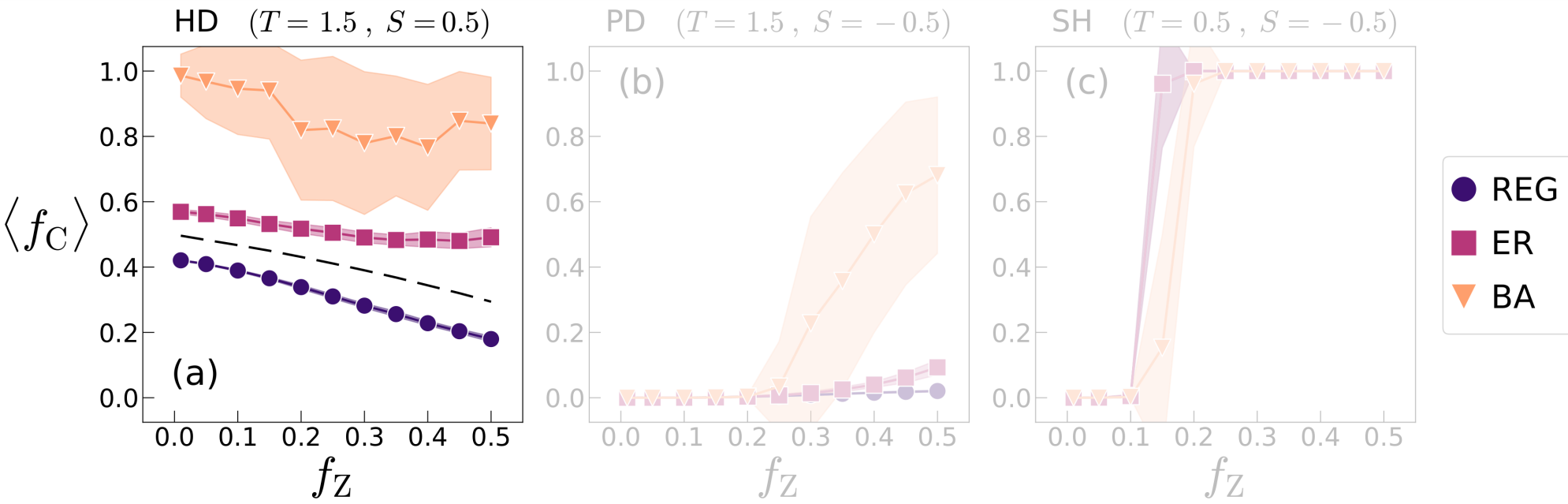

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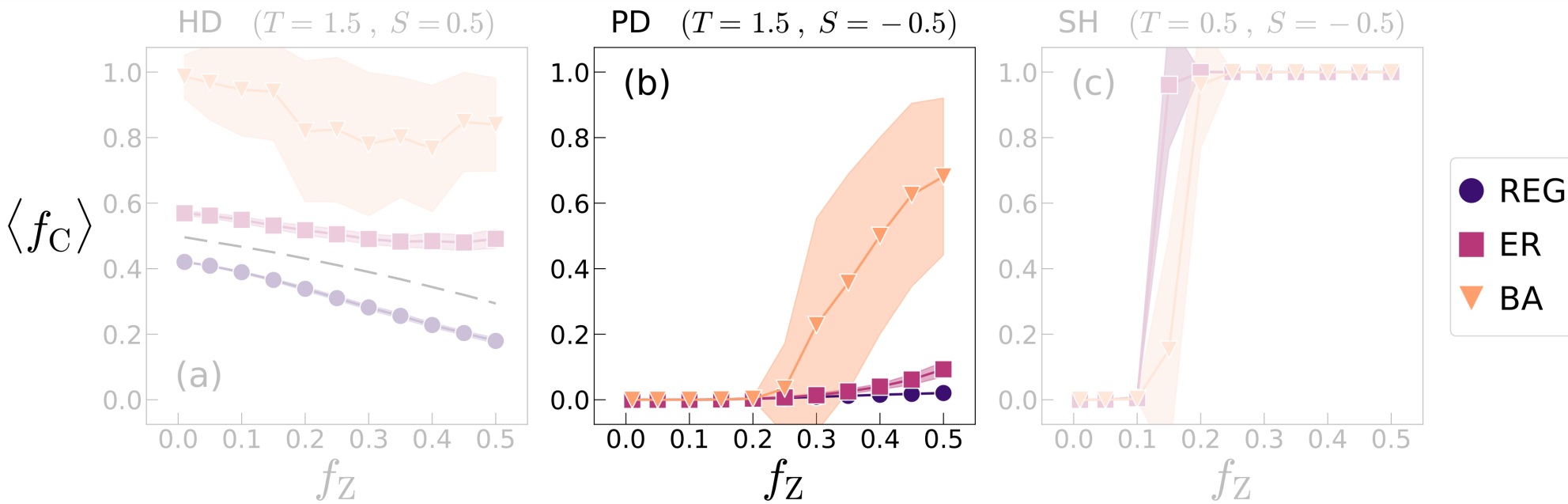

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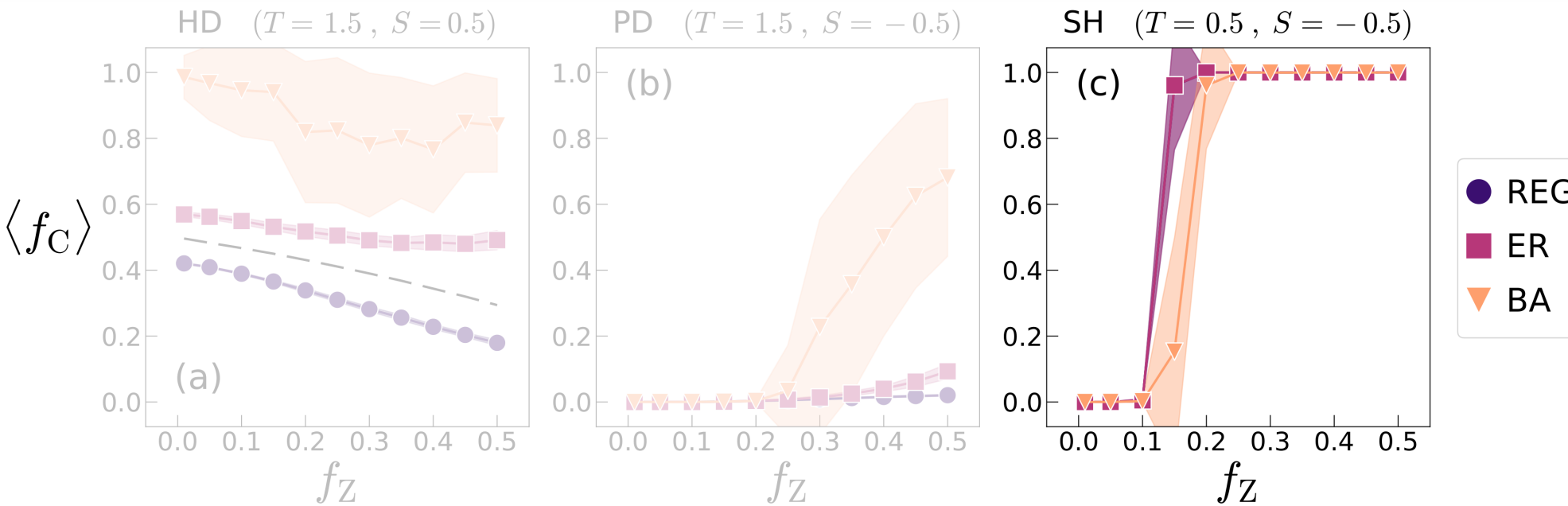

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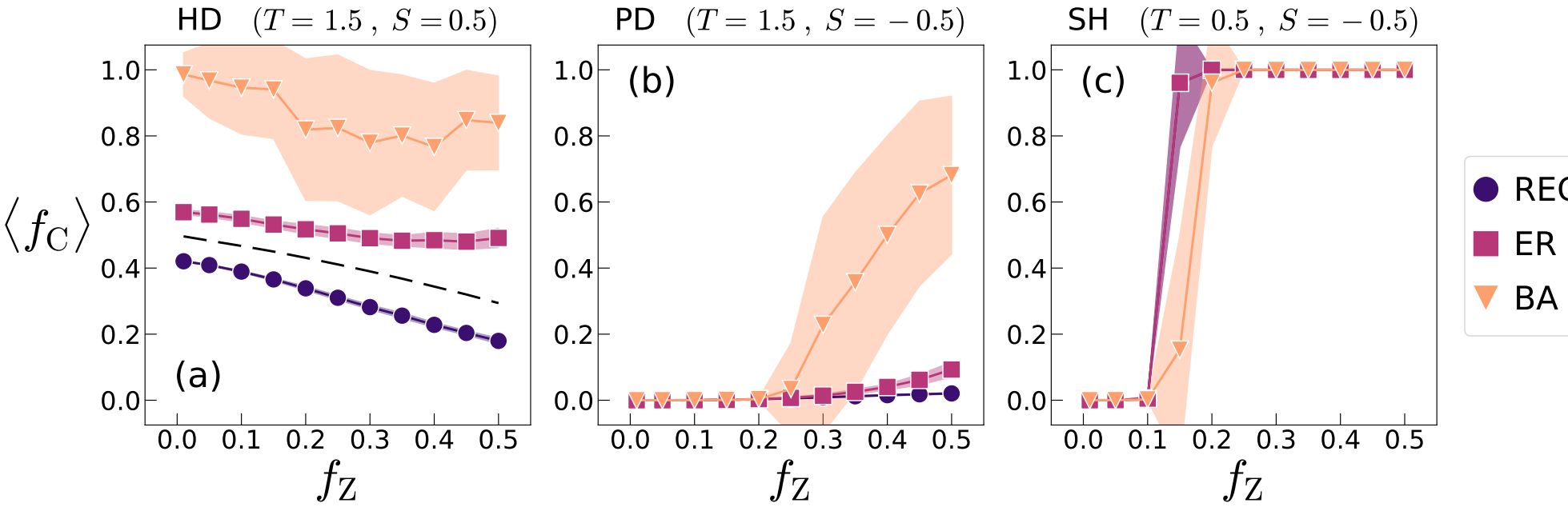
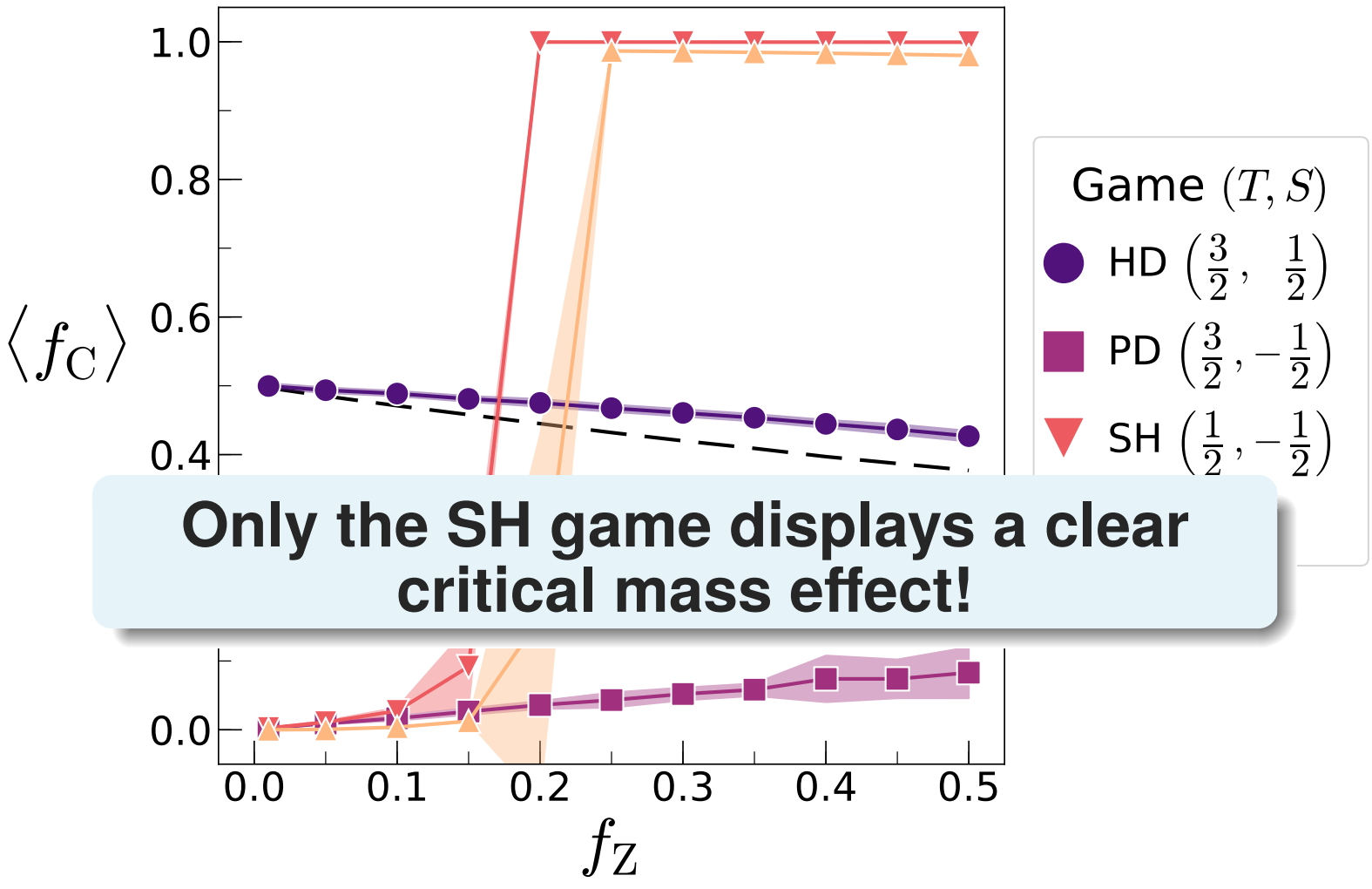



**Remark:**

Zealots nodes are placed **at random**!

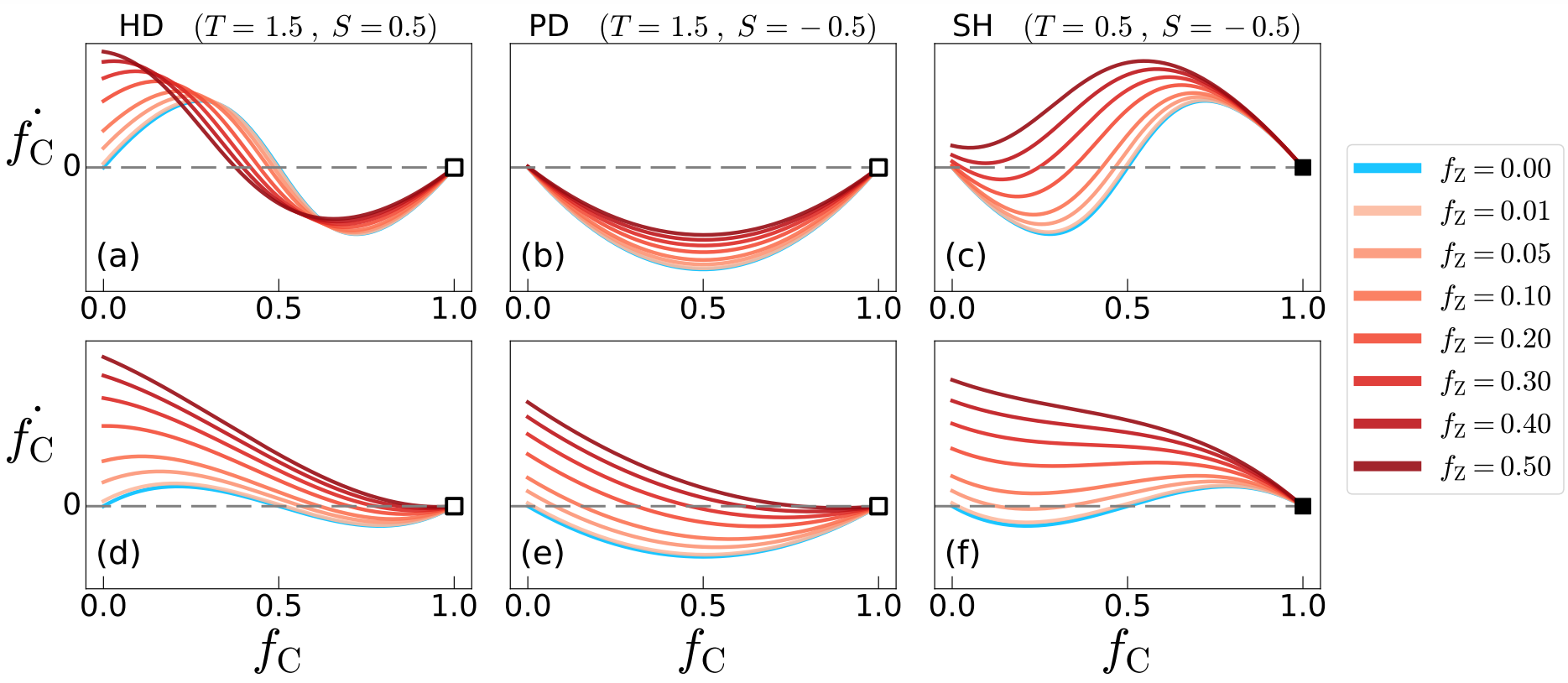
# Summing up ...

# Take home messages



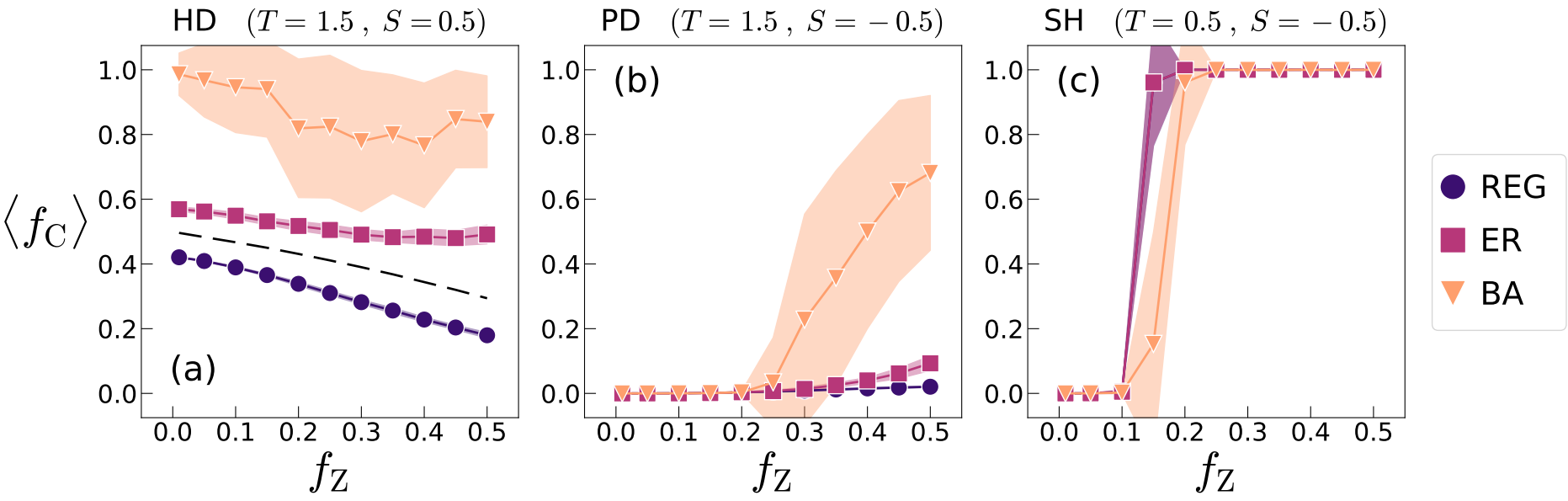

**Only the SH game displays a clear critical mass effect!**

# Take home messages




Reducing the selection pressure (or changing the update rule) can trigger the appearance of a critical mass effect

# Take home messages




**The topology of the interactions plays a crucial role!**

# Acknowledgements

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# Acknowledgements


10.1103/PhysRevResearch.2.023305

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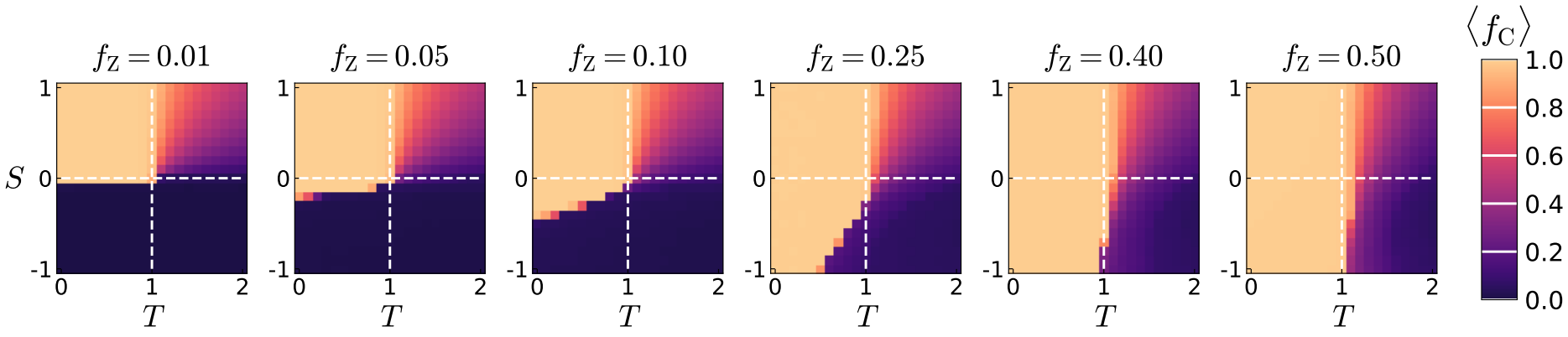
# Extra contents


$f_{\mathrm{Z}} = 0.01$
$f_{\mathrm{Z}} = 0.05$
$f_{\mathrm{Z}} = 0.10$
$f_{\mathrm{Z}} = 0.25$
$f_{\mathrm{Z}} = 0.40$
$f_{\mathrm{Z}} = 0.50$
$S$
$T$
1
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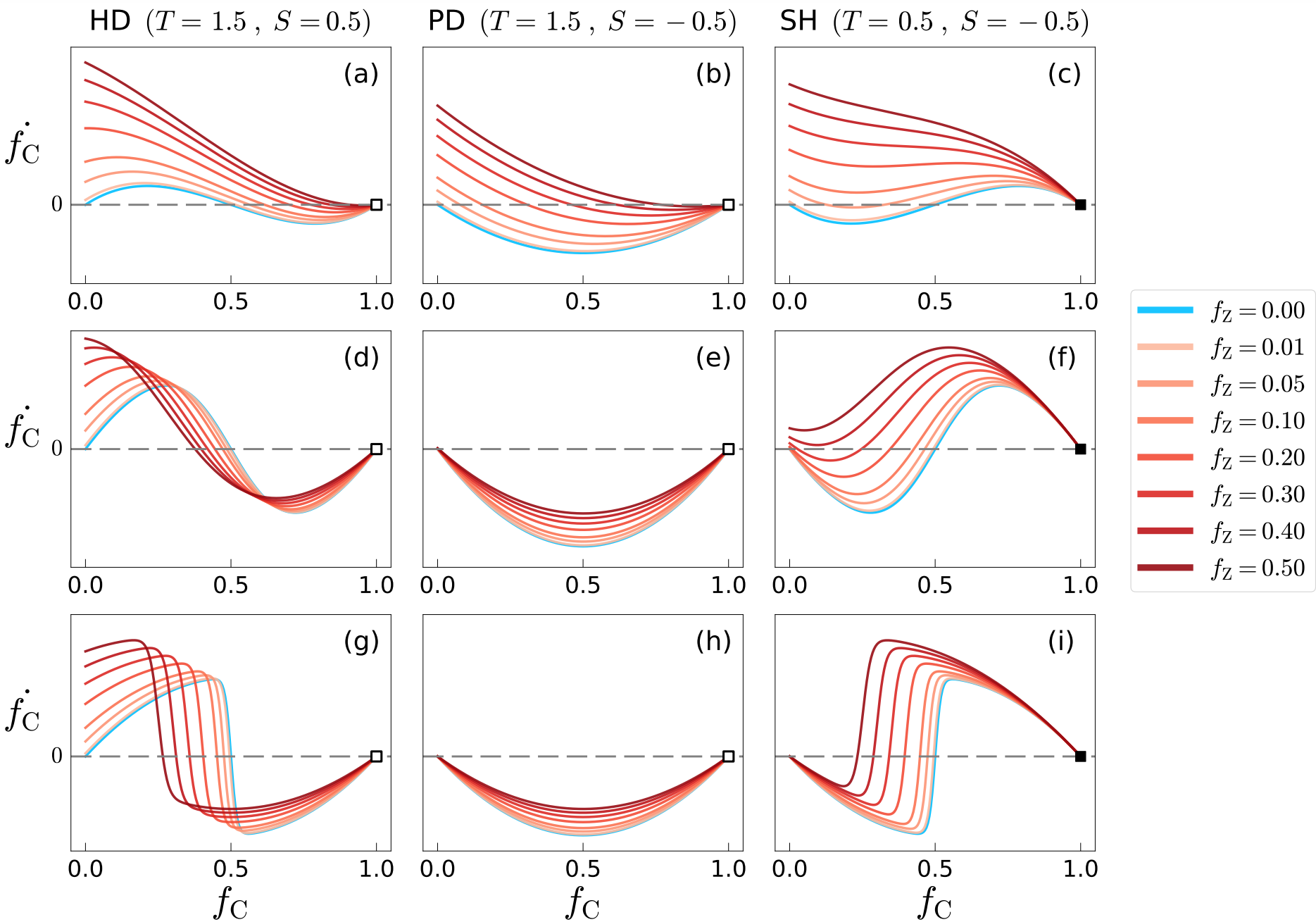
-1
0
1
2
$\langle f_{\mathrm{C}} \rangle$
1.0
0.8
0.6
0.4
0.2
0.0


HD $(T = 1.5\ ,\ S = 0.5)$
PD $(T = 1.5\ ,\ S = -0.5)$
SH $(T = 0.5\ ,\ S = -0.5)$
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
$\dot{f}_{\mathrm{C}}$
0
0.0
0.5
1.0
$f_{\mathrm{C}}$
$f_{\mathrm{Z}} = 0.00$
$f_{\mathrm{Z}} = 0.01$
$f_{\mathrm{Z}} = 0.05$
$f_{\mathrm{Z}} = 0.10$
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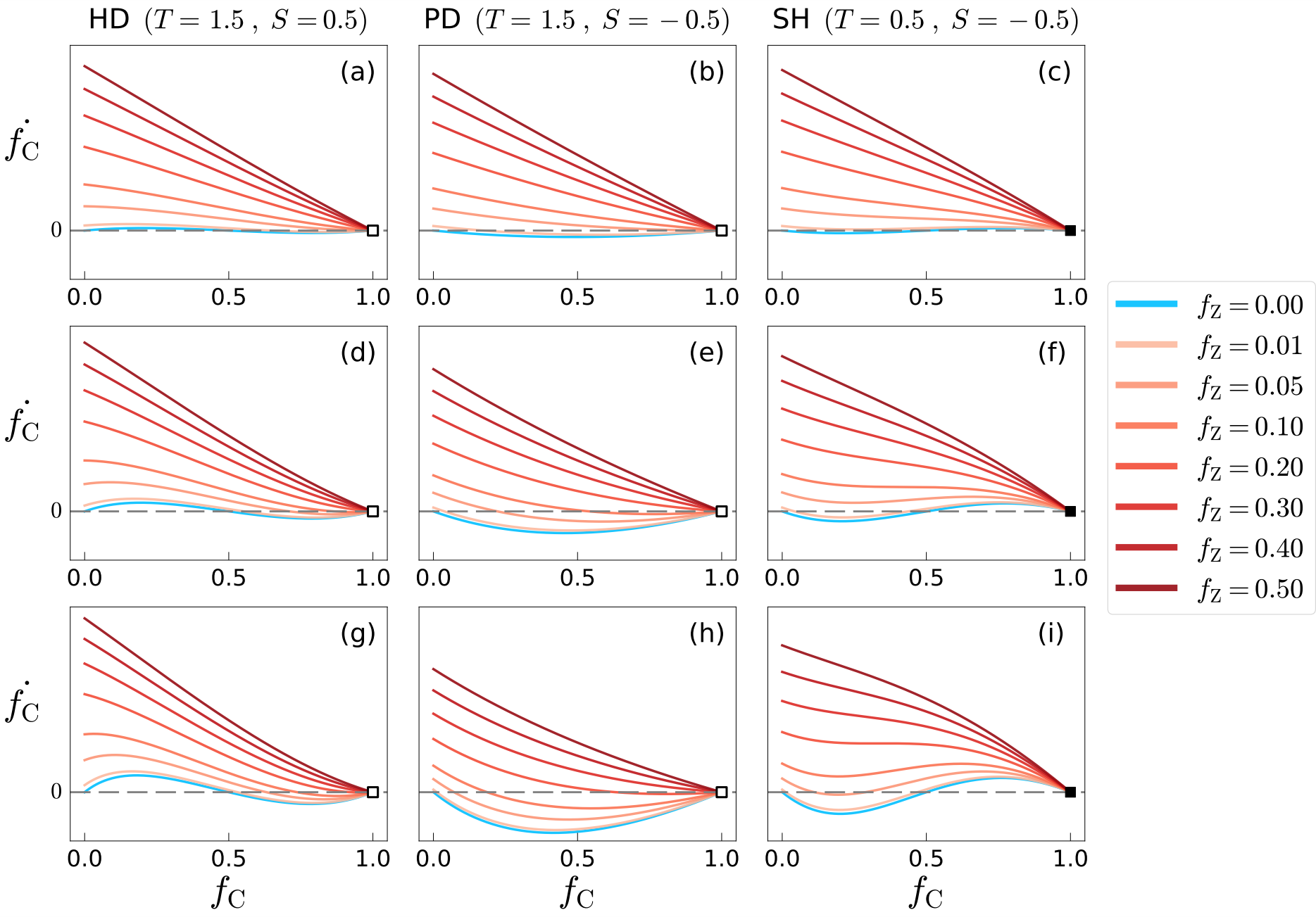
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$\dot{f}_{\mathrm{C}}$
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$f_{\mathrm{Z}} = 0.00$
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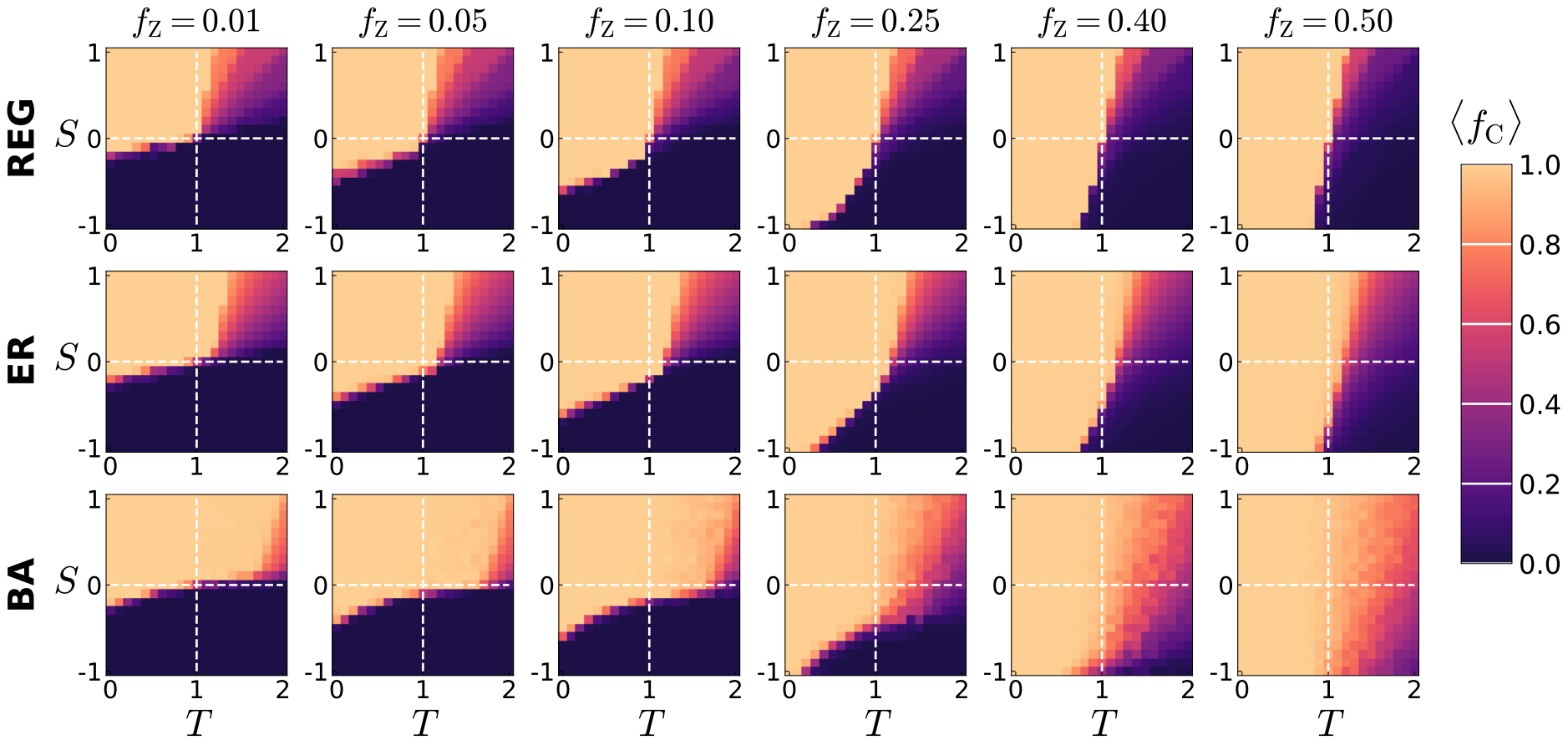
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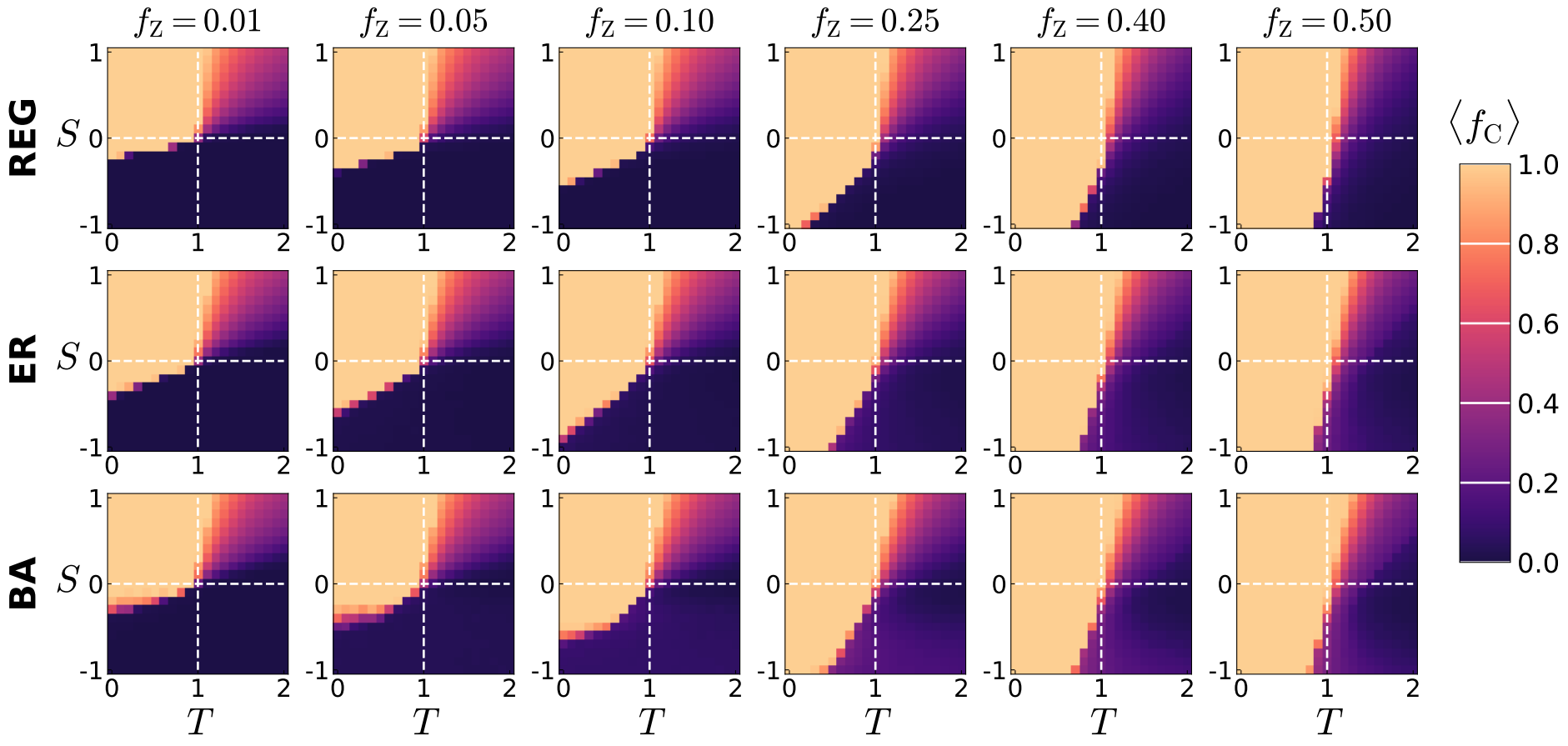
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