Characterization of interactions' persistence in time-varying networks

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Francisco Bauzá

Mario Floría

Alex Arenas

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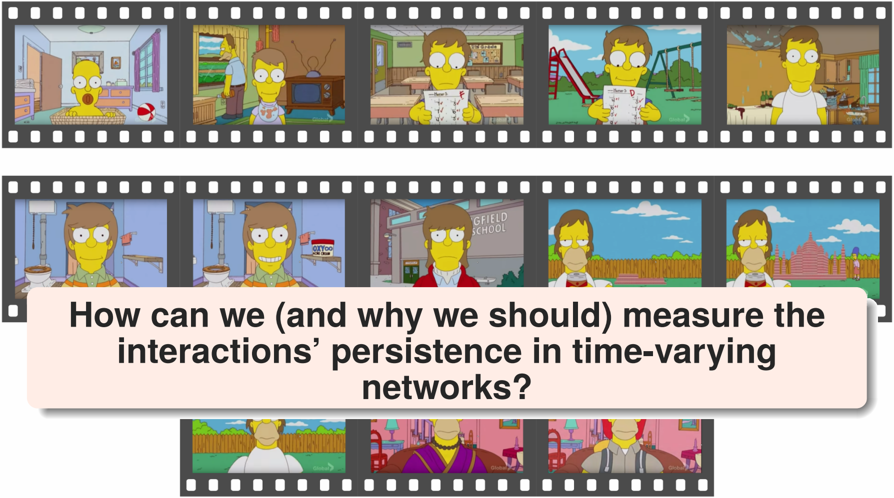
https://www.youtube.com/watch?v=faWaqRyR8nY



How can we (and why we should) measure the interactions' persistence in time-varying networks?

Main questions:

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4. What are the effects of changing the time **resolution** (coarse-graining)?

Measuring the persistence: Temporality

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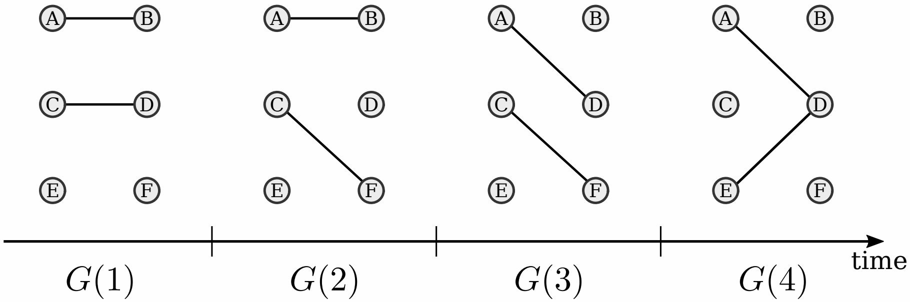
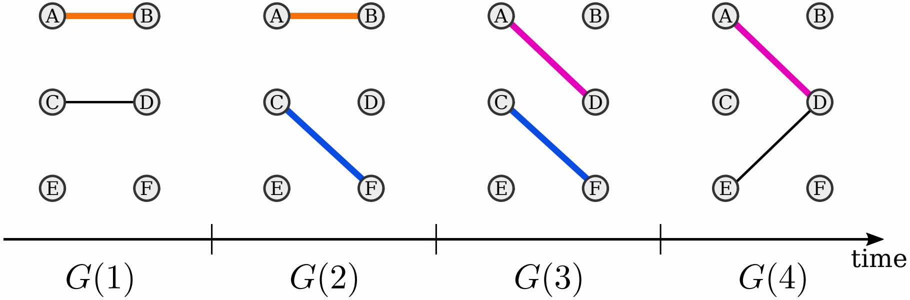


Measuring the persistence: Temporality



Measuring the persistence: Temporality

Definition

$$\mathcal{T}_{m,n} = \frac{\sum_{i,j=1}^{N} \left| a_{i,j}(m) - a_{i,j}(n) \right|}{\sum_{i,j} \max\left\{ a_{i,j}(m), a_{i,j}(n) \right\}}.$$

$a_{i,j}(m) \longrightarrow$ (i,j)-th element of the adjacency matrix of snapshot $G(m)$.

$N \longrightarrow$ Number of nodes.

- A. Li, *et al.* *Evolution of cooperation on temporal networks*. Nat. Comms., **11**, 2259, (2020). DOI:10.1038/s41467-020-16088-w

Measuring the persistence: Temporality

Definition

$$\mathcal{T}_{m,n} = \frac{\bigcup_{m,n} - \bigcap_{m,n}}{\bigcup_{m,n}} = 1 - \frac{\bigcap_{m,n}}{\bigcup_{m,n}},$$

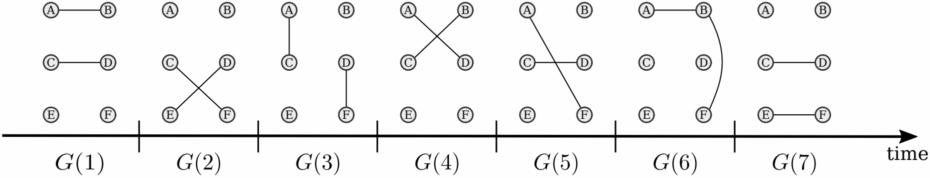
$\bigcup_{m,n} \longrightarrow$ Size of the union of the edges' sets, $\mathcal{E}_m$ and $\mathcal{E}_n$, of snapshots $G(m)$ and $G(n)$.

$\bigcap_{m,n} \longrightarrow$ Size of the intersection of the same sets.

Measuring the persistence: Temporality

$$\mathcal{T}_{m,n} = \begin{cases} 1 & \text{if} \quad \bigcap_{m,n} = 0 \\ 0 & \text{if} \quad \bigcap_{m,n} = \bigcup_{m,n} \end{cases}$$

Measuring the persistence: Temporality



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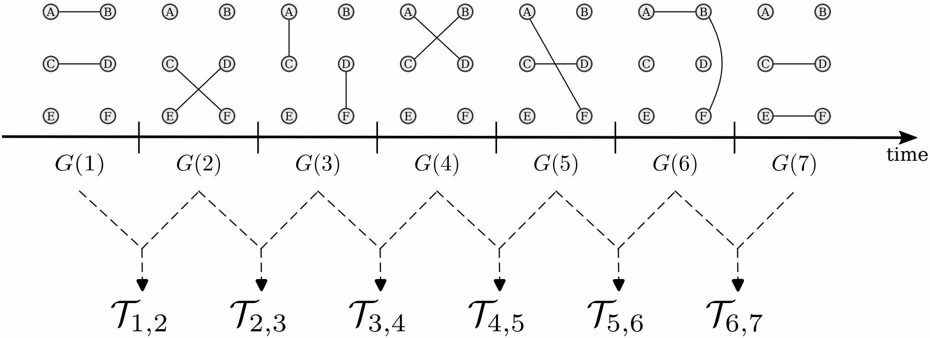

Measuring the persistence: Temporality

Average temporality

$$\overline{\mathcal{T}} = \frac{1}{N_s - 1} \sum_{m=1}^{N_s-1} \mathcal{T}_{m,m+1}$$

How *special* your network is: Null model

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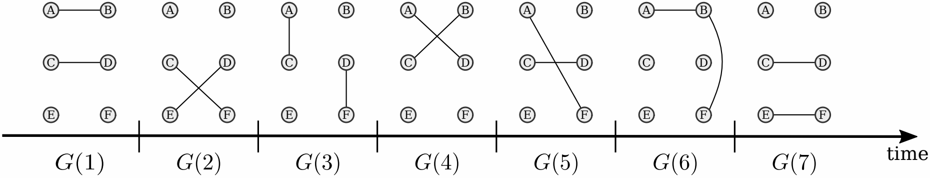

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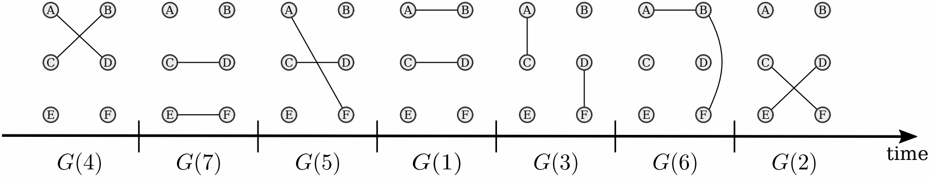

• L. Gauvin *et al.* *Randomized reference models for temporal networks*, arXiv:1806.04032 (2018).

DOI: 10.48550/arxiv.1806.04032

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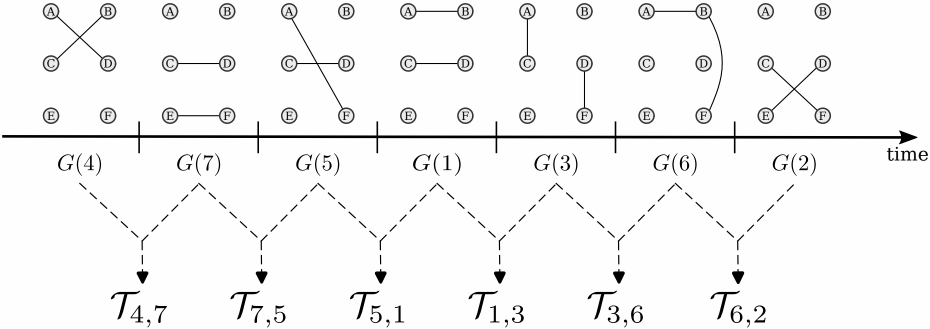

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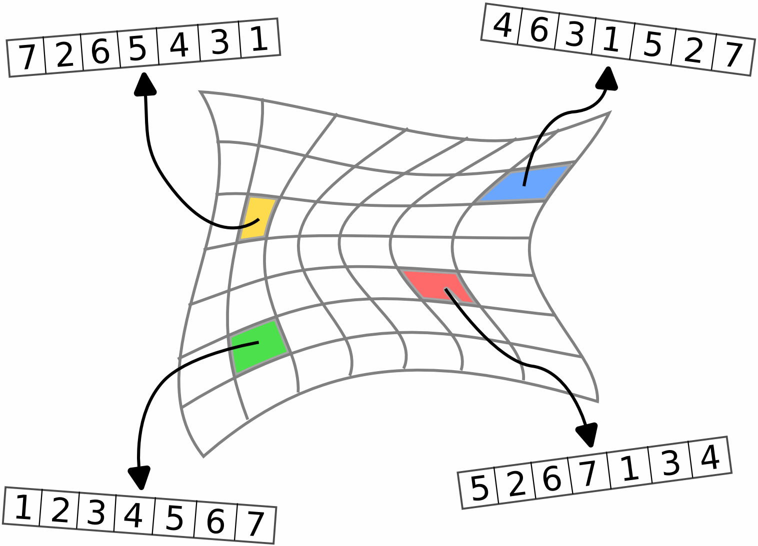

How *special* your network is: Null model

$$\overline{\mathcal{T}}_{\text{th}} = 1 - \frac{1}{N_s - 1} \sum_{m=1}^{N_s-1} \frac{\bigcap_{m,m+1}}{\bigcup_{m,m+1}} = $$

$$= 1 - \frac{\langle x_{ij}^2 \rangle}{2\langle x_{ij} \rangle - \langle x_{ij}^2 \rangle}$$

$x_{ij} \longrightarrow$ Probability that edge (i, j) exists in any of the snapshots.

Data

Data

- 5 face-to-face networks (from the SocioPatterns repository).
- 1 transportation network of US domestic flights.
- 1 social network of e-mail exchange.
- 1 functional brain network.
- 3 (star-like) trade networks (from the UN COMMTRADE database).

Good to know

The data on trade and US domestic flights (and code to download them) will be available (soon) at:
https://cardillo.web.bifi.es/data.html

Data

Dataset	N	N_s	Δt	K_{TOT}	$\overline{\mathcal{T}}$	$\langle \rho \rangle (\times 10^{-4})$
			FACE-TO-FACE			
Baboons	13	40845★	20 s	78	0.592	287.47
Malawi	86	43437★		347	0.436	8.51
High School	327	7374		5818	0.512	4.80
SFHH Conference	403	3508		9565	0.510	2.47
Hospital	75	9452		1139	0.532	12.36
			TRADE			
Trade - carpets Turkey	207	52	1 year	206	0.301	34.29
Trade - guns Italy	156	116	1 month	155	0.287	59.92
Trade - cereals Italy	157	108		156	0.286	60.41
			OTHER			
Brain	16	396	$\frac{1}{200}$ s	120	0.452	395.99
E-mails	1890	19380	1 s	4383	0.940	0.01
US domestic flights	1677	371	1 month	25890	0.248	24.16

Temporality of empirical networks

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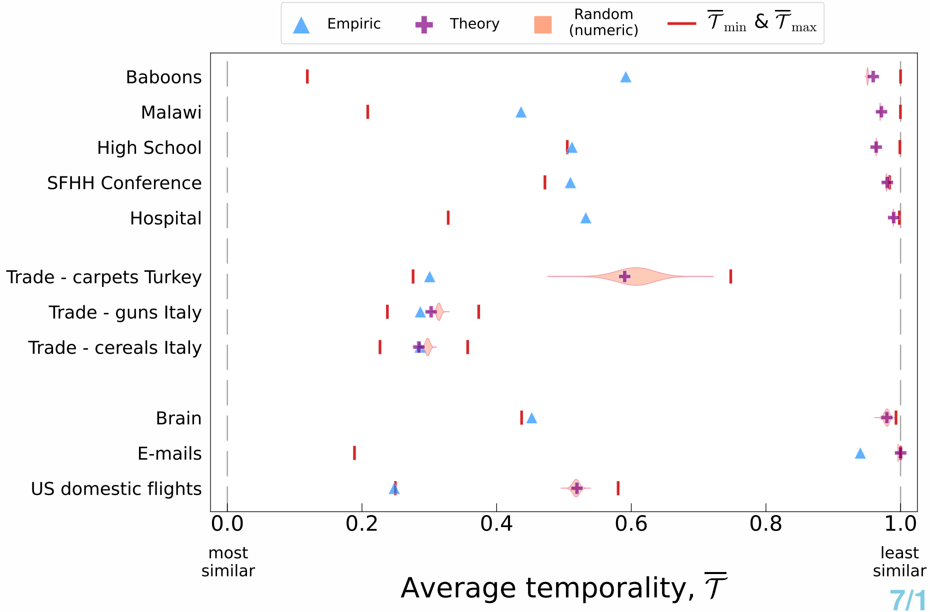

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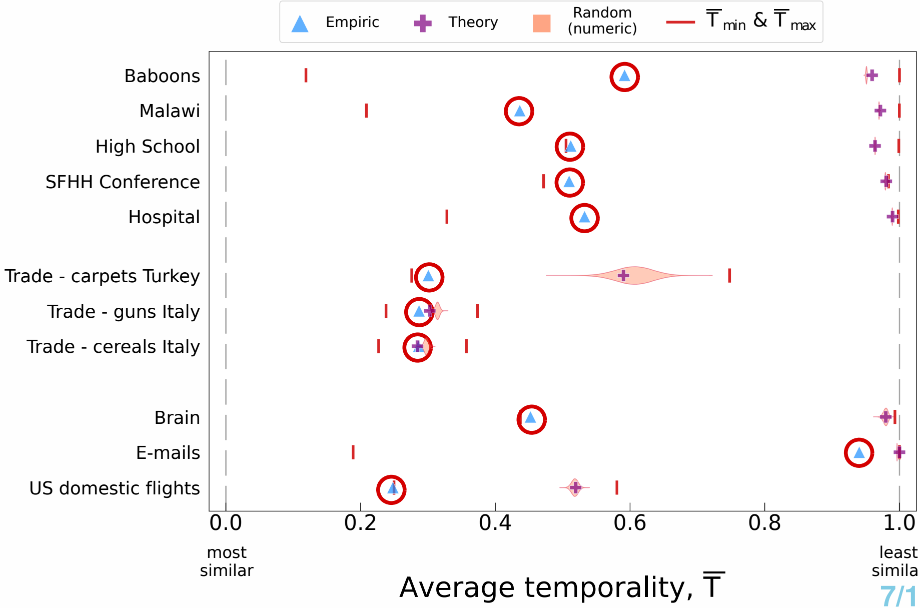

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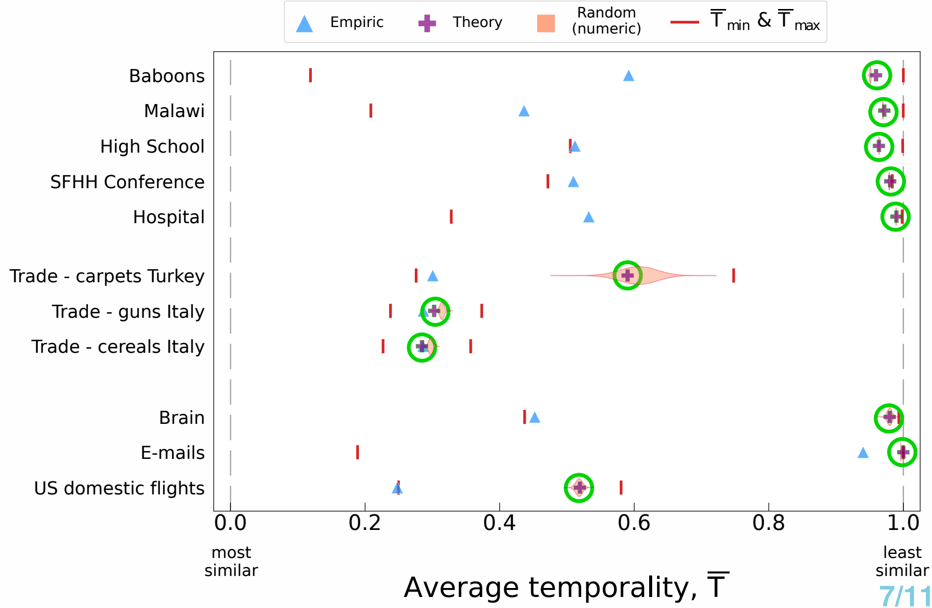

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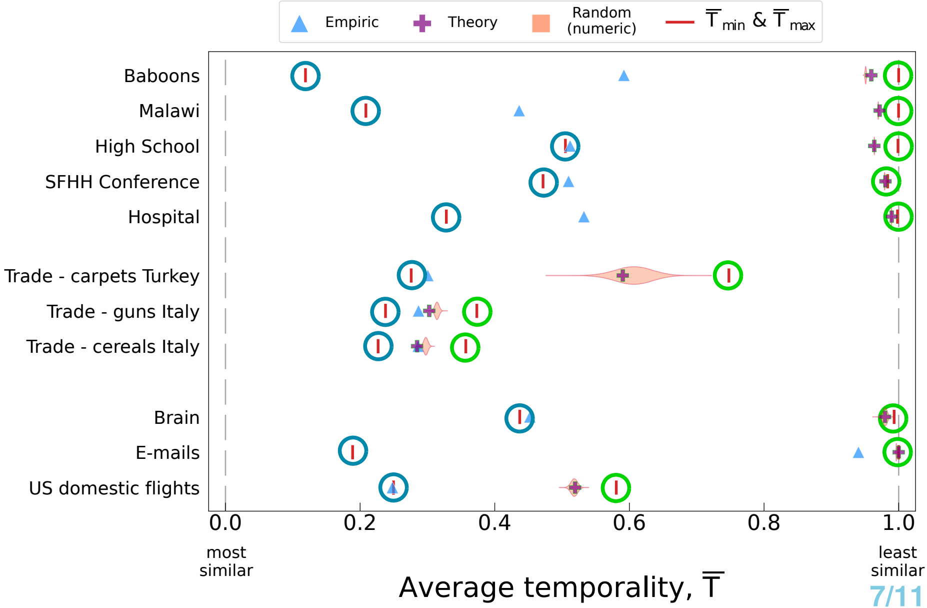

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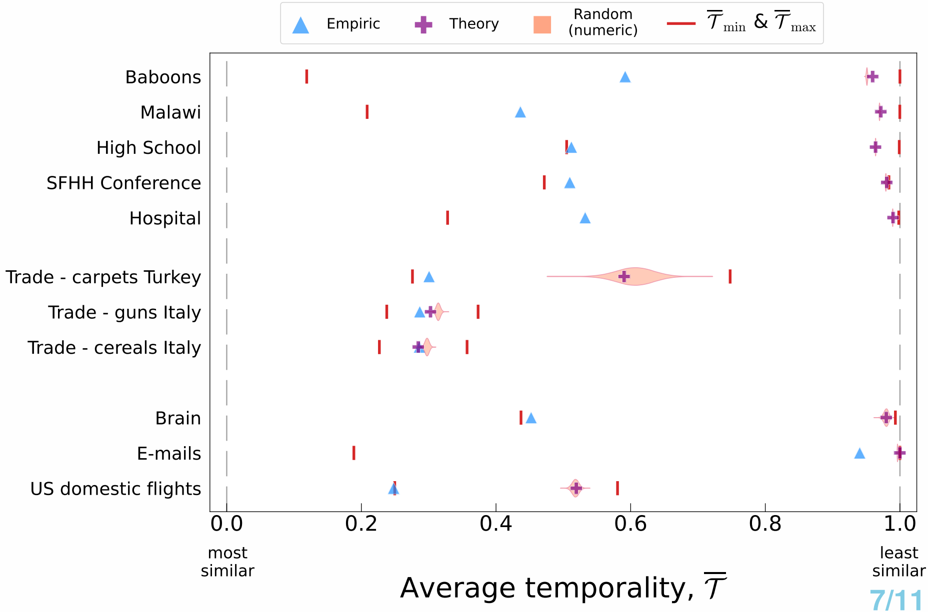

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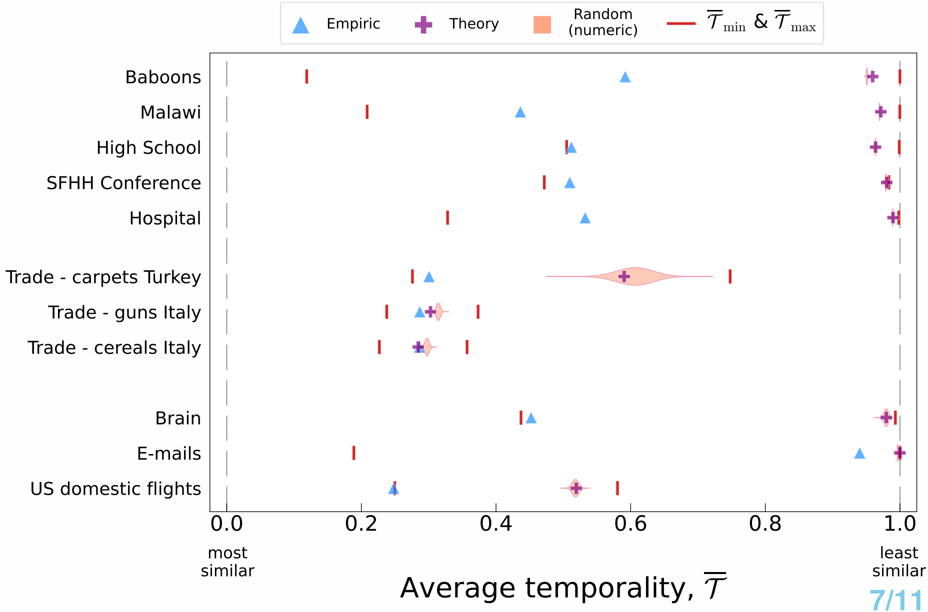

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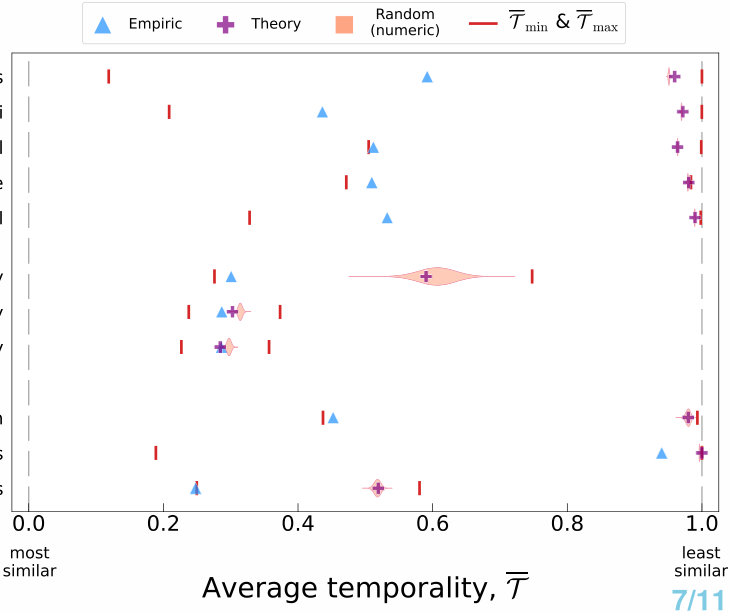

Changing the time resolution

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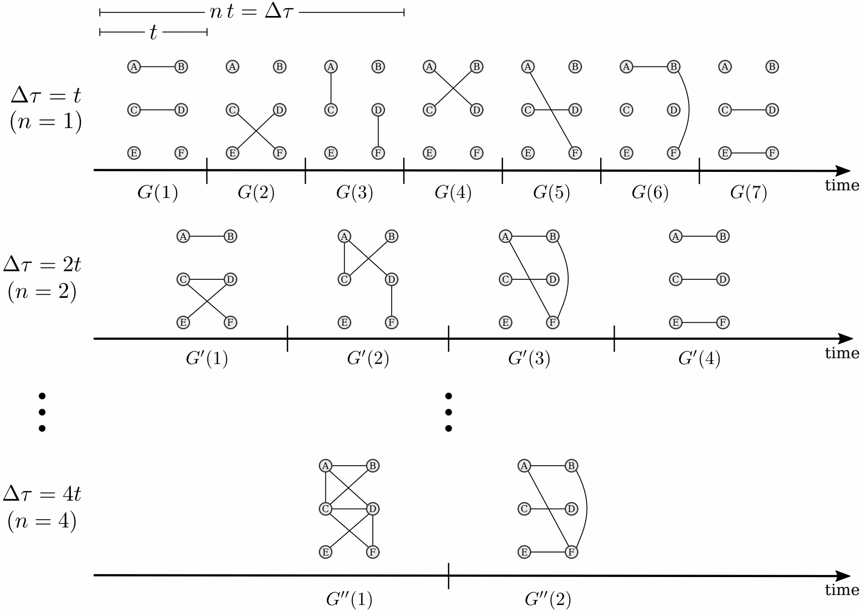

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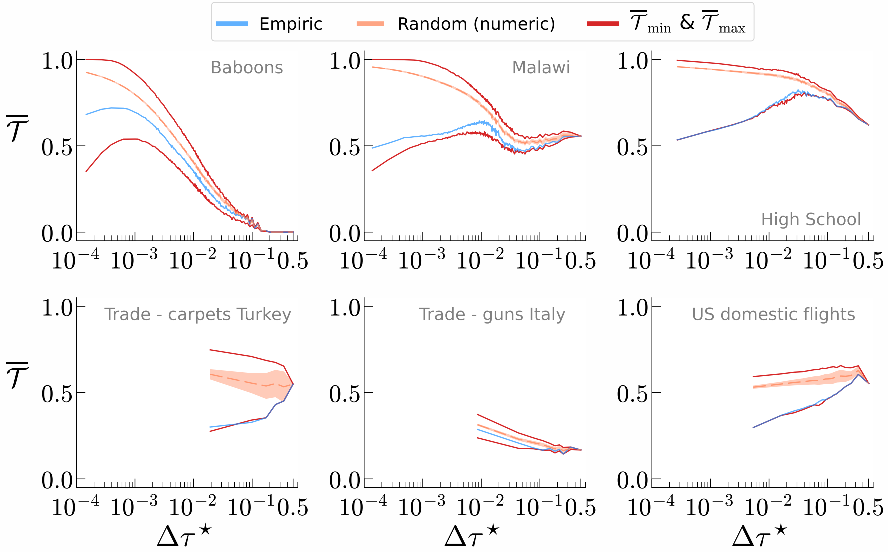
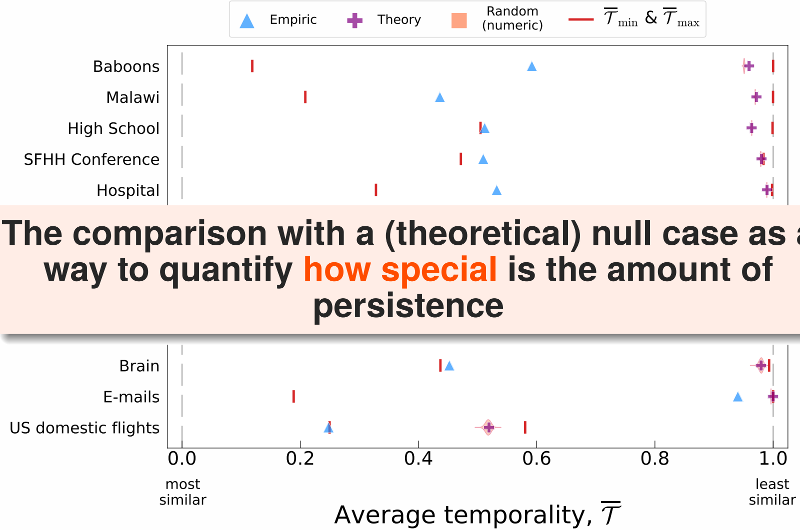


Summing up ...

Take home messages

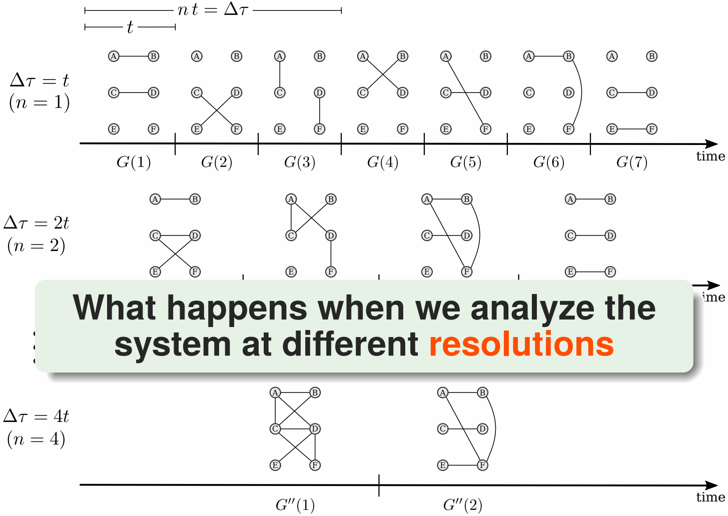
Temporality as a metric to quantify the persistence of the interactions in time-varying networks

Take home messages



The comparison with a (theoretical) null case as a way to quantify how special is the amount of persistence

Take home messages



What happens when we analyze the system at different resolutions

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[Submitted on 30 May 2022]

Characterization of interactions' persistence in time-varying networks

Francisco Bauzá Mingueza, Mario Floría, Jesús Gómez-Gardeñes, Alex Arenas, Alessio Cardillo

Many complex networked systems exhibit volatile dynamic interactions among their vertices, whose order and persistence reverberate on the outcome of dynamical processes taking place on them. To quantify and characterize the similarity of the snapshots of a time-varying network -- a proxy for the persistence,-- we present a study on the persistence of the interactions based on a descriptor named \emph{temporality}. We use the average value of the temporality, $\overline{\mathcal{T}}$, to assess how ``\emph{special}'' is a given time-varying network within the configuration space of ordered sequences of snapshots. We analyze the temporality of several empirical networks and find that empirical sequences are much more similar than their randomized counterparts. We study also the effects on $\overline{\mathcal{T}}$ induced by the (time) resolution at which interactions take place.

DOI: 10.48550/arXiv.2205.15435

https://cardillo.web.bifi.es/data.html

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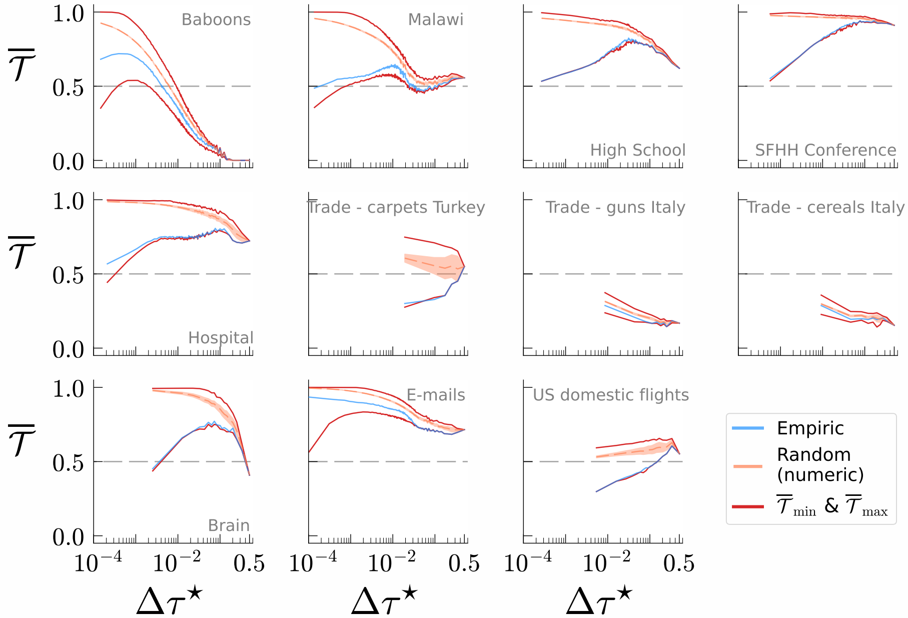
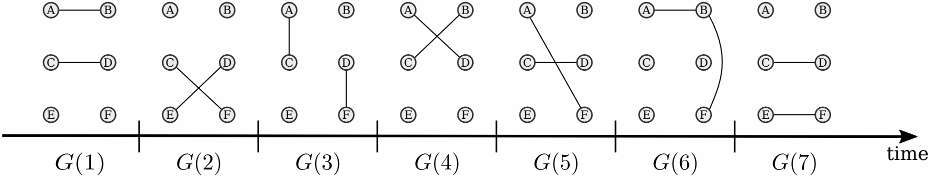
Extra contents



Baboons
Malawi
High School
SFHH Conference
Hospital
Trade - carpets Turkey
Trade - guns Italy
Trade - cereals Italy
Brain
E-mails
US domestic flights
Empiric
Random (numeric)
$\overline{\mathcal{T}}_{\min}$ & $\overline{\mathcal{T}}_{\max}$
$\overline{\mathcal{T}}$
$\Delta\tau^\star$
1.0
0.5
0.0
10^{-4}
10^{-2}
0.5

Finding the optimal order



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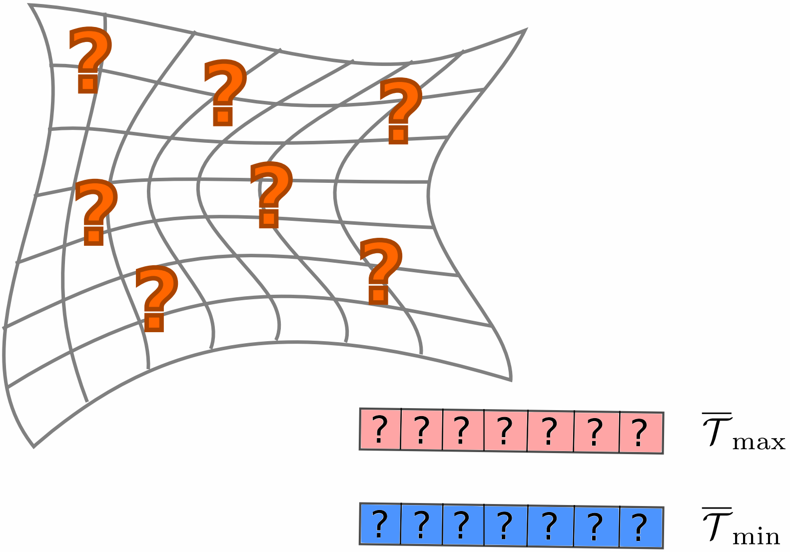

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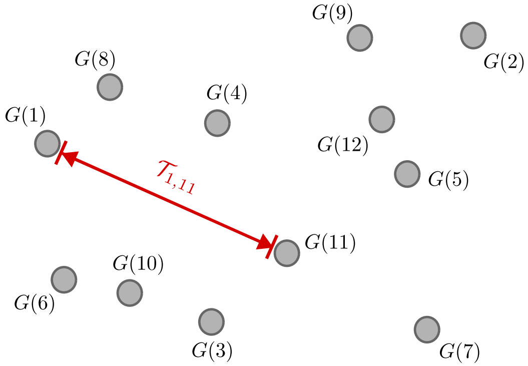
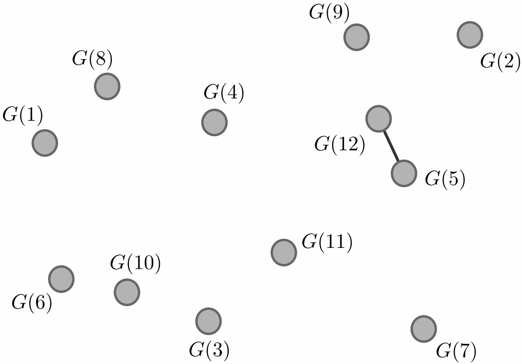


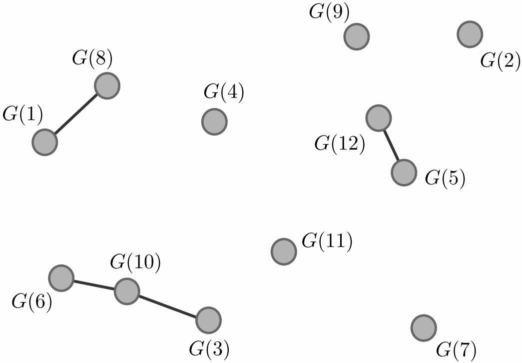
1. Embed snapshots in a metric space with $d_{m,n} = \mathcal{T}_{m,n}$

Finding the optimal order



1. Embed snapshots in a metric space with $d_{m,n} = \mathcal{T}_{m,n}$
2. Compute all the temporalities between snapshots and sort them in ascending (desceinding) order.

Finding the optimal order



1. Embed snapshots in a metric space with $d_{m,n} = \mathcal{T}_{m,n}$
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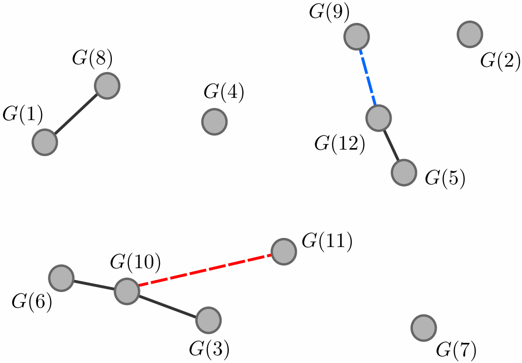
3. Add edges to the Spanning Tree (chain).

Finding the optimal order



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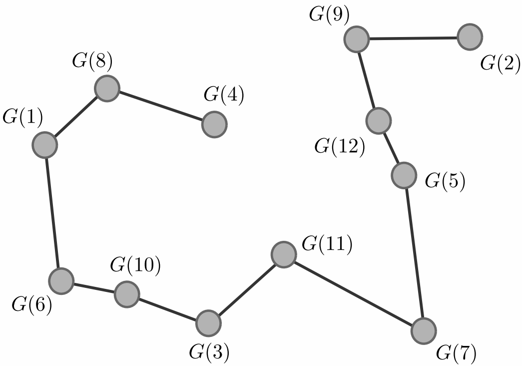
3. Add edges to the Spanning Tree (chain).
4. Edges can be added only if the graph forms a chain and the nodes can have at most degree 2.

Finding the optimal order



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4. Edges can be added only if the graph forms a chain and the nodes can have at most degree 2.
5. Repeat steps 3 and 4 until getting an open chain.