The evolutionary Kuramoto’s dilemma

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A synchronized world...
Motivation

Question:

What happens to the synchronization when the interactions are regulated by the cost/benefit ratio?
The model

Strategy

\[ s_i = \begin{cases} 
1 & \text{if } i \text{ is cooperator} \\
0 & \text{if } i \text{ is defector}
\end{cases} \]

Phase

\[ \theta_i \in [0, 2\pi] \]
The model

Kuramoto

\[ \dot{\theta}_l = \omega_l + s_l \lambda \sum_{j=1}^{N} a_{lj} \sin(\theta_l - \theta_j). \]

The model

Evolutionary game

\[ p_l \propto \frac{r_{L_l}}{\text{benefit}} - \alpha \frac{c_l}{\text{cost}}. \]

\[ r_{L_l} = \frac{1}{k_l} \sum_{j=1}^{N} a_{lj} \frac{|e^{i\theta_l} + e^{i\theta_m}|}{2}, \quad r_L \in [0, 1], \]

\[ c_l \propto \Delta \dot{\theta}_l = \left| \dot{\theta}_l(t) - \dot{\theta}_l(t-1) \right|. \]

The model

Synchronization

Evolutionary game

accumulation of payoff

update of strategy
Question:

How the underlying topology of the interactions affects the emergence of cooperation/synchronization?
Macroscopic behavior

\[ \langle C \rangle \langle r_G \rangle \]

\( \log(\lambda) \)

\( \log(\alpha) \)

ER

RGG

BA
Macroscopic behavior

Microscopic Behavior

For different models (ER, RGG, BA), the graph shows the behavior of $\langle r_L \rangle_k$ as a function of $\lambda$.

- In the ER model, the behavior is characterized by a peak at a specific value of $\lambda$.
- In the RGG model, the peak is more pronounced and shifts slightly to the right compared to the ER model.
- In the BA model, the peak is less pronounced and shifts further to the right compared to the RGG model.

Overall, the graphs demonstrate how the microscopic behavior changes with different network models.
Microscopic Behavior

Low cost

- ER
- RGG
- BA

\[ \langle r_L \rangle_k \]

\[ \lambda \]
Microscopic Behavior

Medium cost

\[ \langle r_L \rangle_k \]

ER

\[ \lambda \]

RGG

\[ \lambda \]

BA

\[ \lambda \]

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Microscopic behavior

High cost

ER

RGG

BA

\( \langle r_L \rangle_k \)

\( \lambda \)

\( \lambda \)

\( \lambda \)
Conclusions
Coevolutionary model based on synchronization and evolutionary game theory.
Role of the **underlying topology** in the emergence of cooperation/synchronization.
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Wanna know more?

Stay tuned on the arXiv . . .
Fermi’s Rule

\[ P_{l \rightarrow m} = \frac{1}{1 + e^{-\beta(p_m - p_l)}} \]