Beyond simple dynamics: coevolutionary dynamics

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Internet Interdisciplinary Institute (IN3) – Universitat Oberta de Catalunya, Barcelona (Spain)

IFISC seminar — IFISC – Palma de Mallorca
Wednesday, February 1st 2023

Universitat Oberta de Catalunya

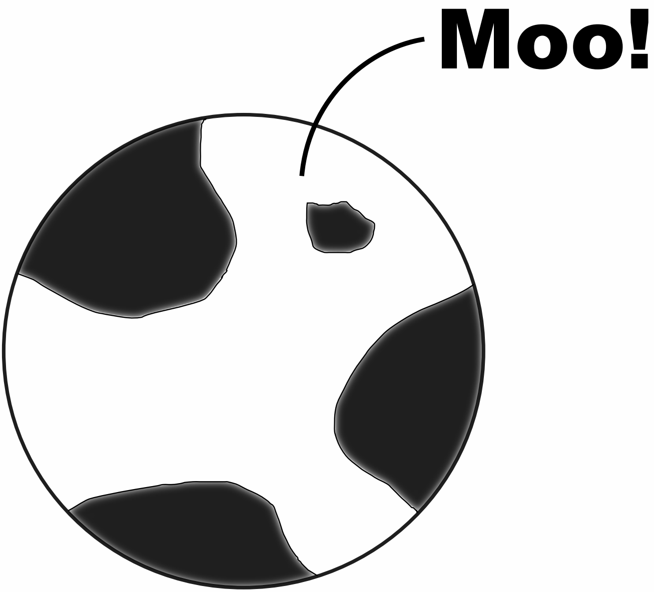
Synchronization

Cultural segregation

Cooperation

Disease spreading

- J. Acebrón, *et al.* Rev. Mod. Phys., **77**, 137, (2005). DOI: `10.1103/RevModPhys.77.137`
- C. Castellano, *et al.* Rev. Mod- Phys., **81**, 591, (2009). DOI: `10.1103/RevModPhys.81.591`
- R. Pastor-Satorras, *et al.* Rev. Mod. Phys., **87**, 925, (2015). DOI: `10.1103/RevModPhys.87.925`
- G. Szabó, and G. Fáth, Phys. Rep.,**446**, 97, (2007). DOI: `10.1016/j.physrep.2007.04.004`


Moo!

Main questions:

1. How can we **model** coevolutionary dynamics?

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2. What are the **properties** of the phenomenology **observed**? (examples of coevolutionary dynamics)

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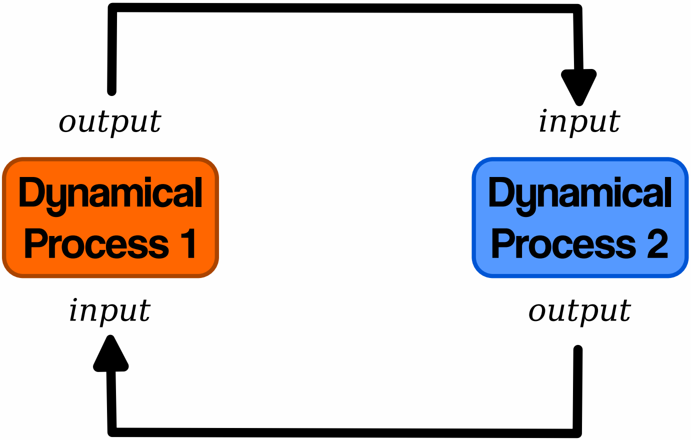
1. How can we **model** coevolutionary dynamics?
2. What are the **properties** of the phenomenology **observed**? (examples of coevolutionary dynamics)
3. What is the role of the **topology** of the interactions? (*i.e.* networks)

Outline

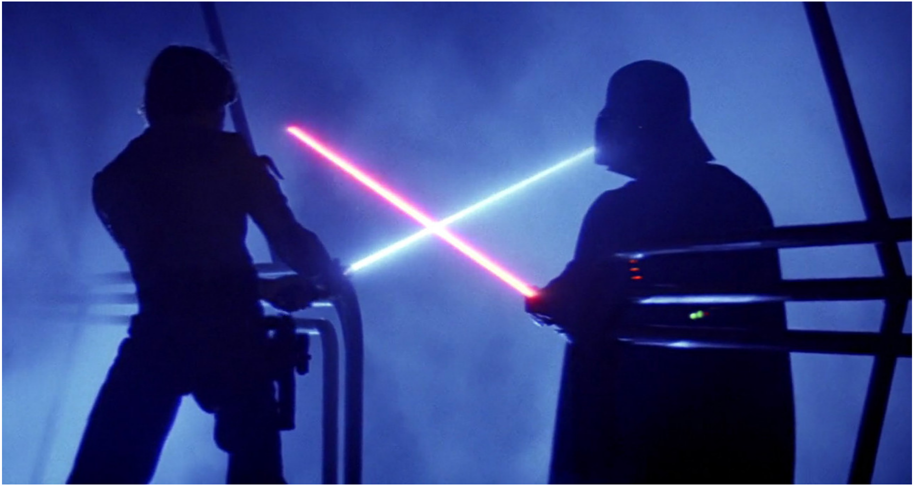
- ~~Motivation~~
- Generality of coevolutionary models
- Examples:
  - ★ Example 1: Spontaneous vaccination
  - ★ Example 2: Evolutionary synchronization
- Take home messages
- Questions

Coevolutionary dynamics

Coevolutionary dynamics



Coevolutionary dynamics



Coevolutionary dynamics

Crash Course on Epidemic spreading, Evolutionary dynamics and Synchronization on Networks

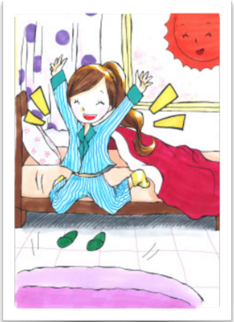
Intro on epidemic spreading

Intro on epidemic spreading

Compartmental Models

S - Susceptible (Healty)

I - Infected (and infectious)

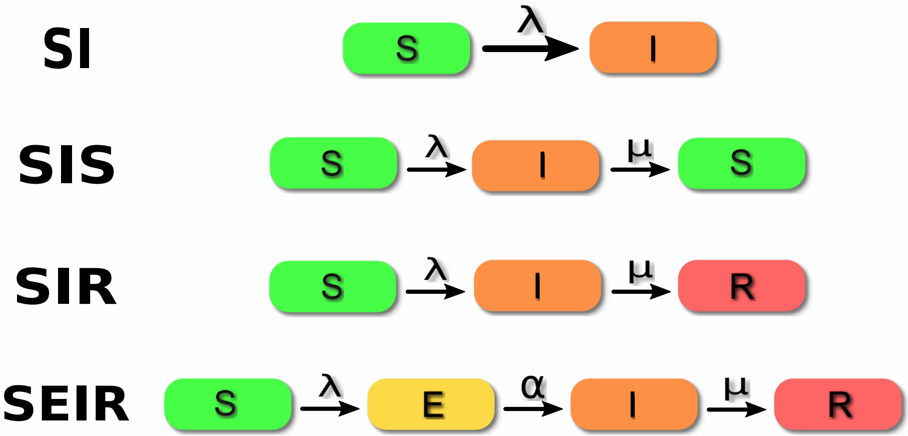


R - Recovered (immune/dead)

(From Petter Holme's blog)

• R. Pastor-Satorras, *et al.* "*Epidemic processes in complex networks*." Rev. Mod. Phys., **87**, 925–979, (2015). DOI: `10.1103/RevModPhys.87.925`

Intro on epidemic spreading



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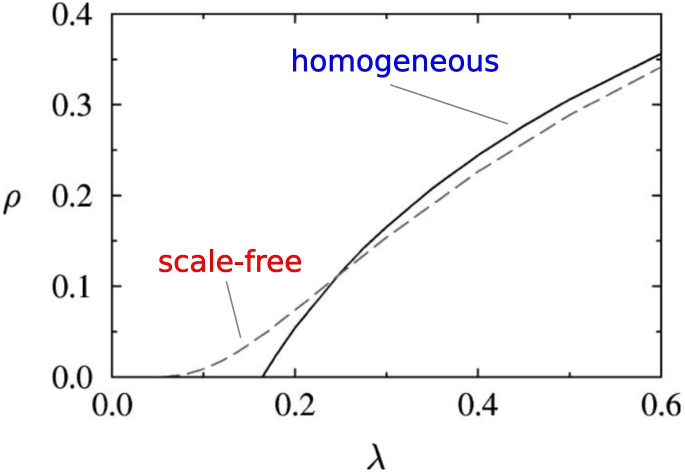
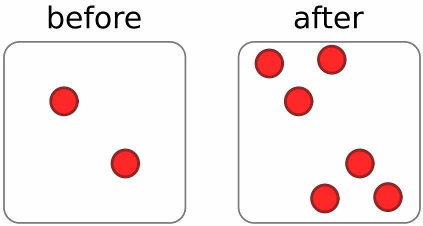


- R. Pastor-Satorras, and A. Vespignani, "*Epidemic dynamics and endemic states in complex networks*." Phys. Rev. E, **63**, 066117, (2001). DOI: `10.1103/PhysRevE.63.066117`

Intro on evolutionary dynamics

Intro on evolutionary dynamics

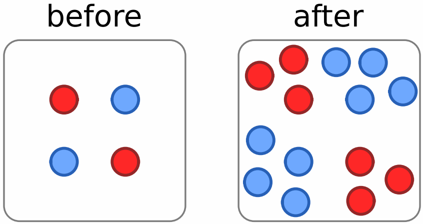


Evolutionary theory

Replication The ability of an organism to reproduce.

- M. A. Nowak, "*Evolutionary dynamics: exploring the equations of life*." (Belknap Press, Harvard, 2007).

Intro on evolutionary dynamics



Evolutionary theory

Replication The ability of an organism to reproduce.

Selection The ability of a species to replicate faster than another.

- M. A. Nowak, "*Evolutionary dynamics: exploring the equations of life*." (Belknap Press, Harvard, 2007).

Intro on evolutionary dynamics

Replicator equation

$$\rho_i^{T+1} = \rho_i^T \left[1 + \underbrace{\left(\Pi_i^T - \overline{\Pi}^T \right)}_{\text{selection}} \right]$$

T Time (discrete)

ρ_i Density of species i

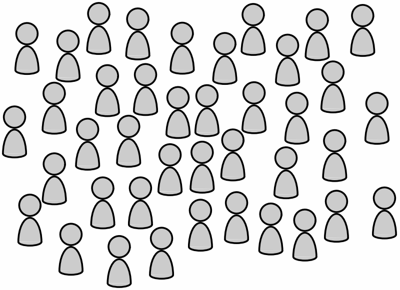
$\Pi_i(\rho)$ Fitness (payoff) of species i

$\overline{\Pi}$ Average fitness (whole population)

- M. A. Nowak, "*Evolutionary dynamics: exploring the equations of life*." (Belknap Press, Harvard, 2007).

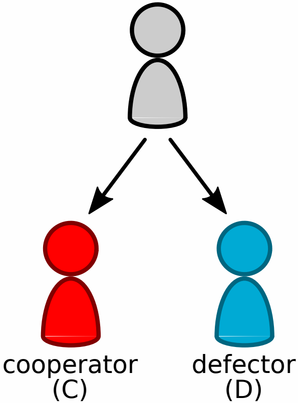
Intro on evolutionary dynamics

- Population of N agents



Intro on evolutionary dynamics

- Population of N agents
- Two strategies: **cooperation** (C) and **defection** (D)

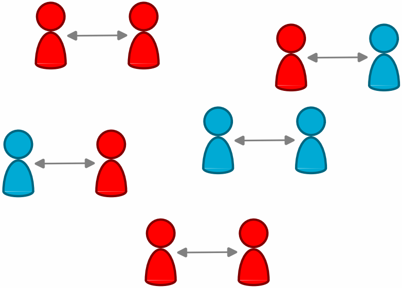


- H. Gintis, "*Game theory evolving (2nd ed.)*." (Princeton Univ. Press, 2009).

Intro on evolutionary dynamics

- Population of N agents
- Two strategies: **cooperation** (C) and **defection** (D)
- Pairwise game with **payoff matrix**

$$\begin{array}{cc} & \begin{array}{cc} \text{C} & \text{D} \end{array} \\ \begin{array}{c} \text{C} \\ \text{D} \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}$$



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Intro on evolutionary dynamics

- Population of N agents
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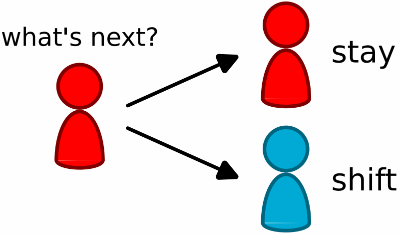
$$\begin{array}{c c} & \begin{array}{cc} \mathrm{C} & \mathrm{D} \end{array} \\ \begin{array}{c} \mathrm{C} \\ \mathrm{D} \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}$$

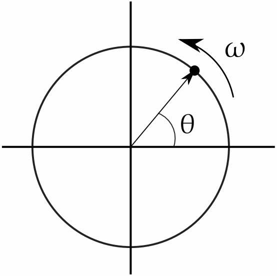
- Strategies evolve according to the **update rule** (*e.g.* Fermi rule)



• H. Gintis, “*Game theory evolving (2nd ed.)*.” (Princeton Univ. Press, 2009).

Introduction to synchronization

Introduction to synchronization



$\theta \in [0, 2\pi]$ Phase

$\omega \in [0, 2\pi]$ Natural frequency

$\lambda \geq 0$ Coupling

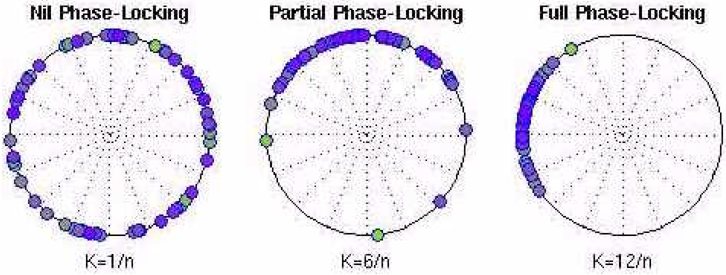
$$\dot{\theta}_l = \omega_l + \lambda \sum_{j=1}^{N} a_{lj} \sin(\theta_j - \theta_l)$$

- Y. Kuramoto, Progress of Theoretical Physics Supplement, **79**, 223–240, (1984).

Introduction to synchronization

Global order parameter

$$r_G\, e^{i\Psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j} \qquad r_G \in [0, 1]$$



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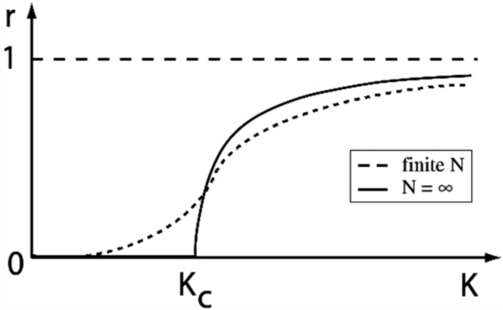
Introduction to synchronization



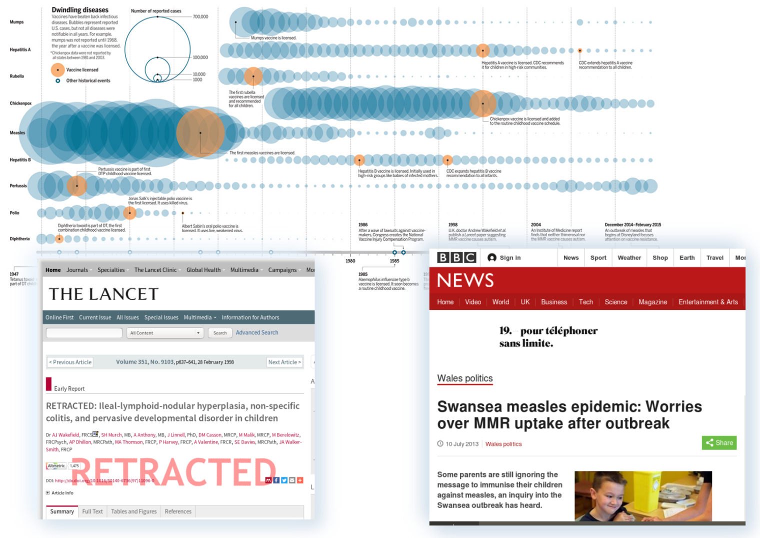
Critical coupling

$$\lambda_c = \lambda_c^{MF} \frac{\langle k \rangle}{\langle k^2 \rangle}$$

- S. H. Strogatz, "*From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators*." Physica D, **143**, 1–20, (2000). DOI: 10.1016/S0167-2789(00)00094-4
- A. Arenas, *et al.* Phys. Rep., **469**, 93–153, (2008). DOI: 10.1016/j.physrep.2008.09.002.

Examples

Spontaneous vaccination

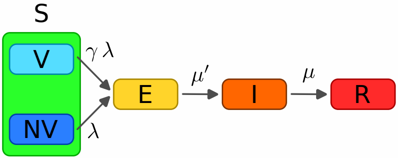


- M. Wadman, and J. You, Science, **356**(6336), 364–365, (2017). DOI: `10.1126/science.356.6336.364`
- C. Betsch, Nat. Microbio., **2**, 17106, (2017). DOI: `10.1038/nmicrobiol.2017.106`

Spontaneous vaccination

The model

Process 1: SEIR (*e.g.* influenza) with vaccination (effectiveness $\gamma \in [0, 1]$).

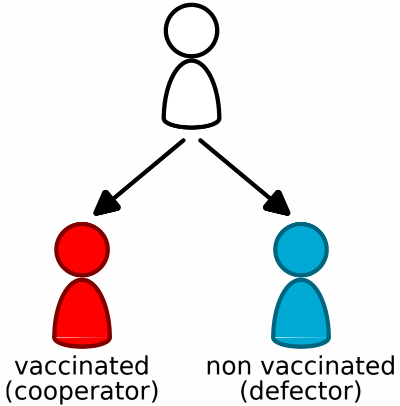


Spontaneous vaccination

The model

Process 1: SEIR (*e.g.* influenza) with vaccination (effectiveness $\gamma \in [0, 1]$).

Process 2: Vaccination game.



Spontaneous vaccination

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Process 1: SEIR (*e.g.* influenza) with vaccination (effectiveness $\gamma \in [0, 1]$).

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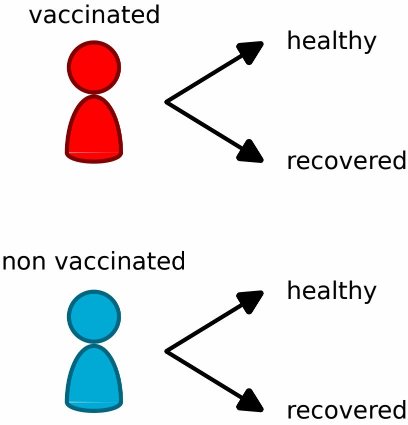
Payoffs:



Spontaneous vaccination

The model

Process 1: SEIR (*e.g.* influenza) with vaccination (effectiveness $\gamma \in [0, 1]$).

Process 2: Vaccination game.

Payoffs:

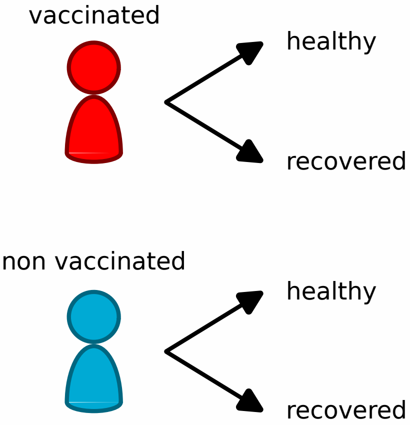
$$\pi_{V,H} = -c$$

$$\pi_{V,R} = -c - T_I$$

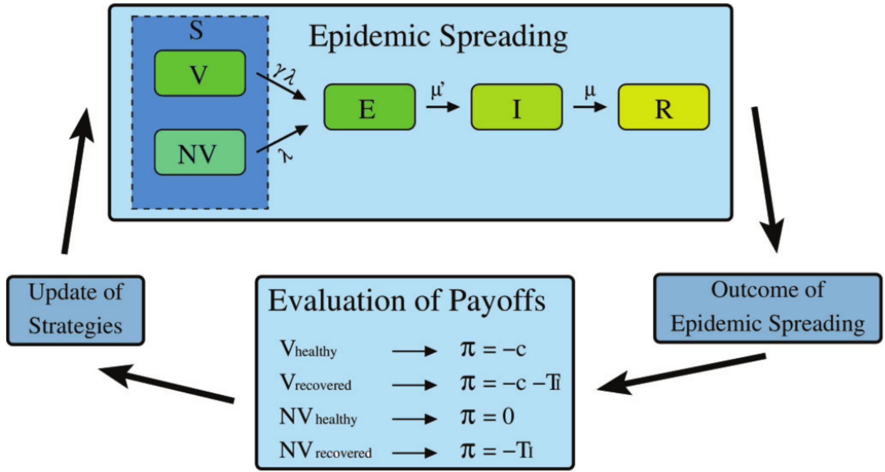
$$\pi_{NV,H} = 0$$

$$\pi_{NV,R} = -T_I$$

Update: Fermi (stochastic).



Spontaneous vaccination



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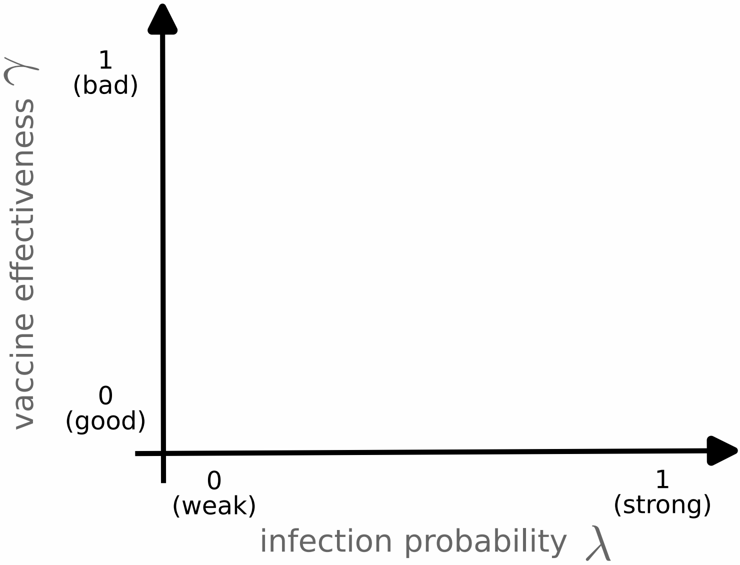

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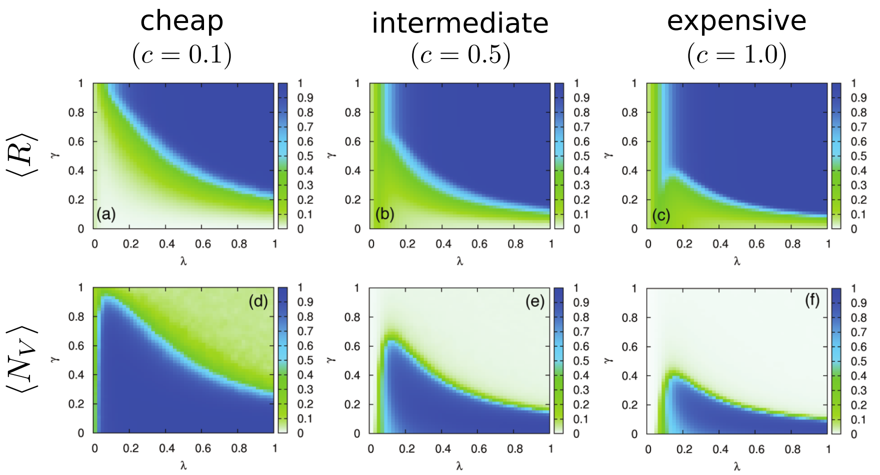

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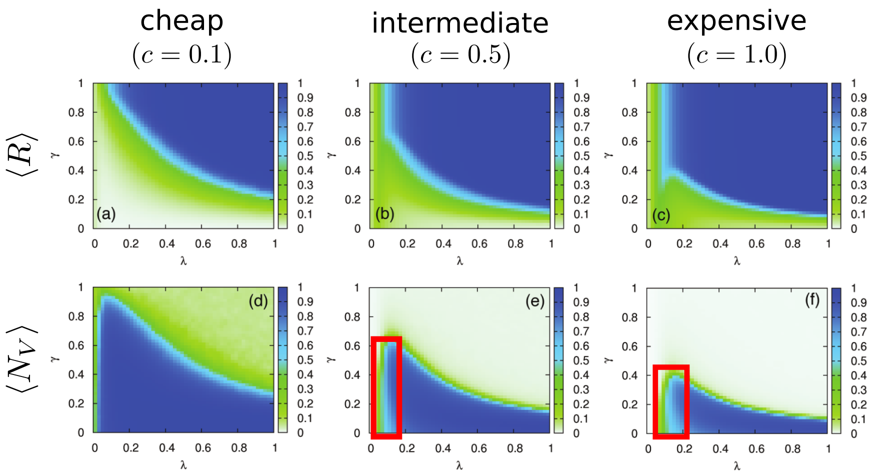
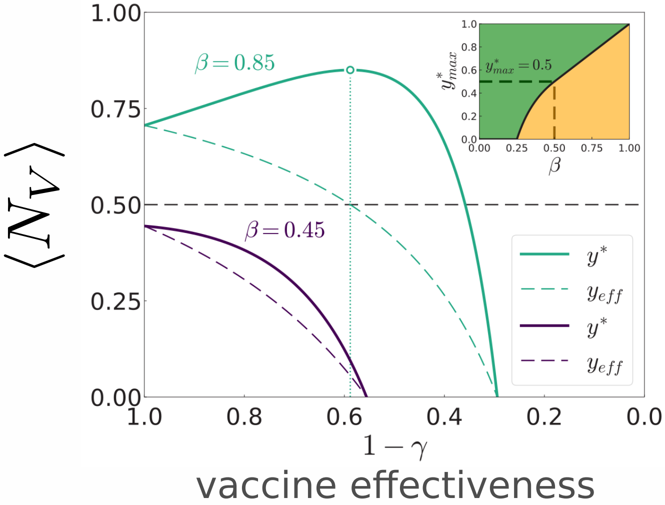


- B. Wu, *et al.* PLoS ONE, **6**, e20577, (2011). DOI: `10.1371/journal.pone.0020577`
- L. G. Alvarez-Zuzek, *et al.* PLoS ONE, **12**, e0186492, (2017). DOI: `10.1371/journal.pone.0186492`

Spontaneous vaccination



• B. Steinegger, *et al.* "*Interplay between cost and benefits triggers nontrivial vaccination uptake*." Phys. Rev. E, **97**, 032308, (2018). DOI: `10.1103/PhysRevE.97.032308`

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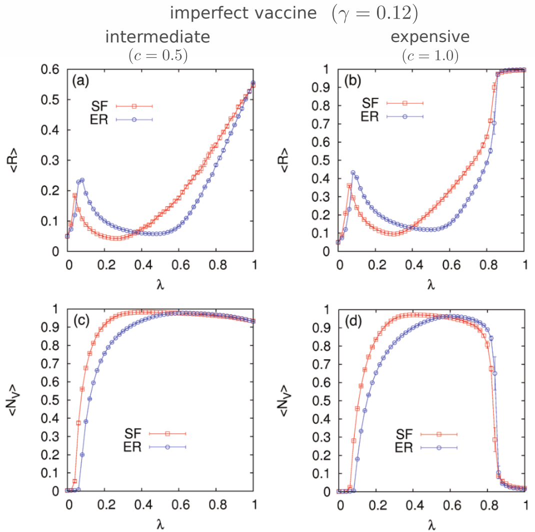

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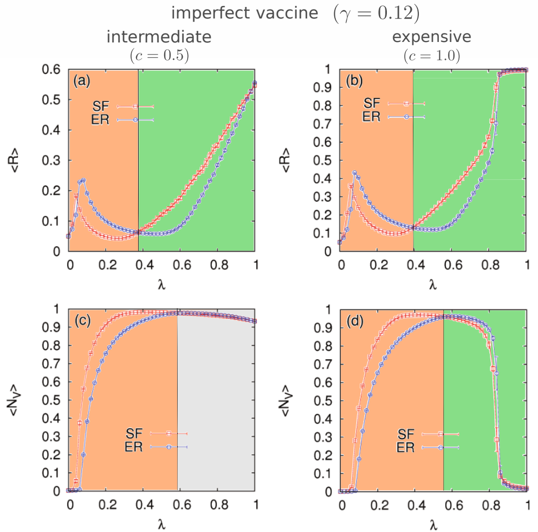

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Evolutionary synchronization

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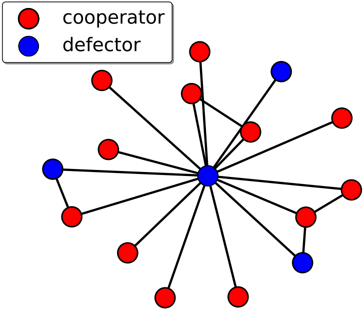

Strategy

$$s_l = \begin{cases} 1 & \text{if } l \text{ is cooperator} \\ 0 & \text{if } l \text{ is defector} \end{cases}$$

Phase

$$\theta_l \in [0, 2\pi]$$

Evolutionary synchronization

$$\dot{\theta}_l = \omega_l + \overbrace{s_l \lambda}^{\text{interaction}} \sum_{j=1}^{N} a_{lj} \sin(\theta_l - \theta_j)$$

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Evolutionary synchronization

Payoff

$$p_l = \underbrace{r_{L_l}}_{\text{benefit}} - \underbrace{\alpha \frac{c_l}{2\pi}}_{\text{cost}}$$

$$\alpha \in \,]0, \infty[\, .$$

Evolutionary synchronization

Payoff

$$p_l = \underbrace{r_{L_l}}_{\text{benefit}} - \underbrace{\alpha \frac{c_l}{2\pi}}_{\text{cost}}$$

$$\alpha \in \,]0, \infty[\,.$$

Benefit

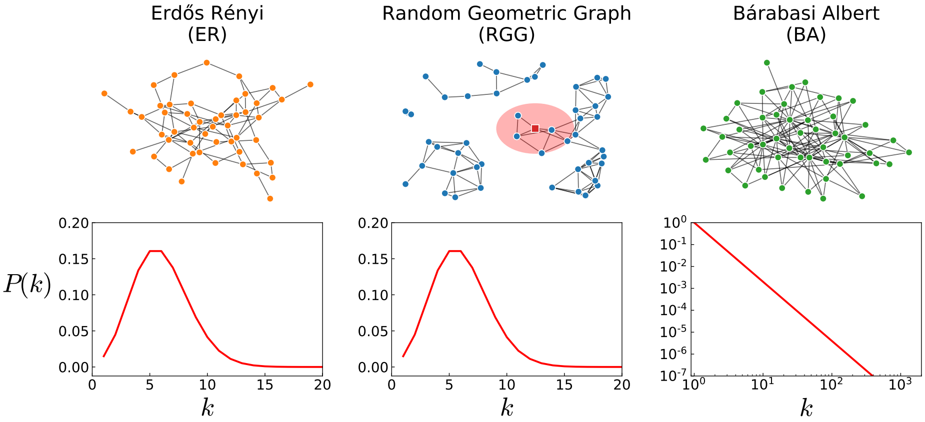
$$r_{L_l} = \frac{1}{k_l} \sum_{j=1}^{N} a_{lj} \frac{|e^{i\theta_l} + e^{i\theta_j}|}{2}$$

$$r_L \in [0, 1]\,,$$

Cost

$$c_l = \Delta\dot{\theta}_l = \left|\dot{\theta}_l(t) - \dot{\theta}_l(t-1)\right|$$

Evolutionary synchronization

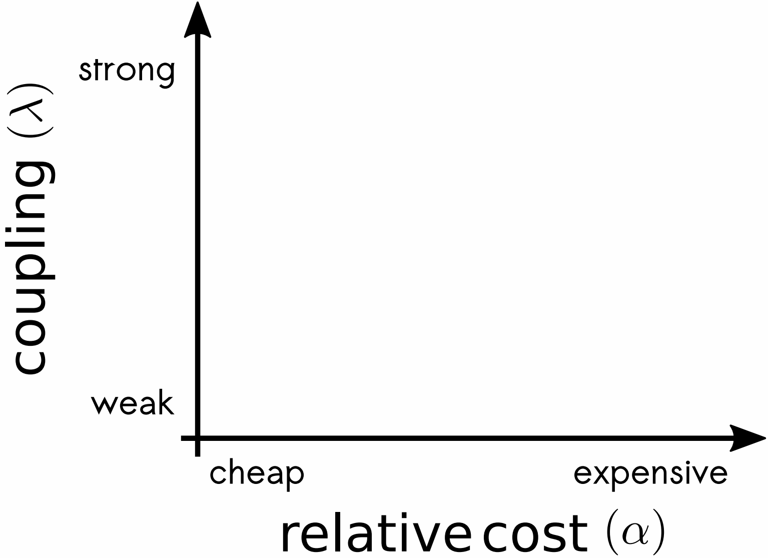


Note:

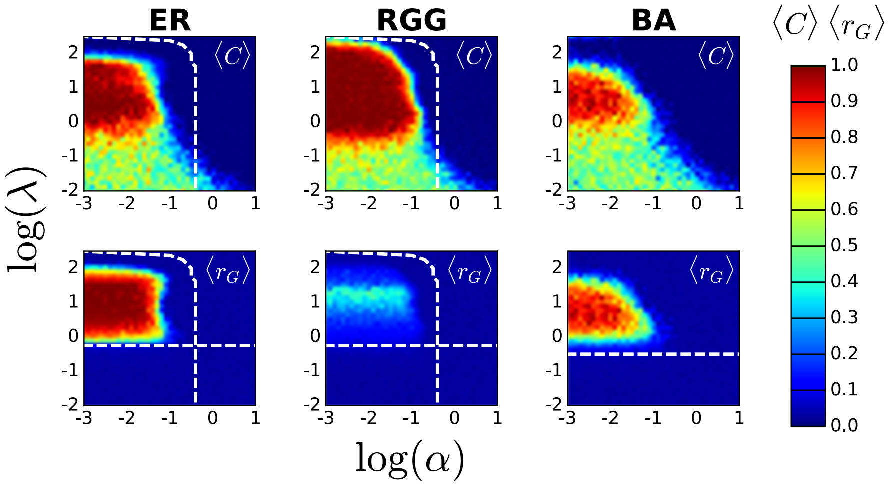
All nets have $N = 1000$ and $\langle k \rangle = 8$

Evolutionary synchronization

Evolutionary synchronization

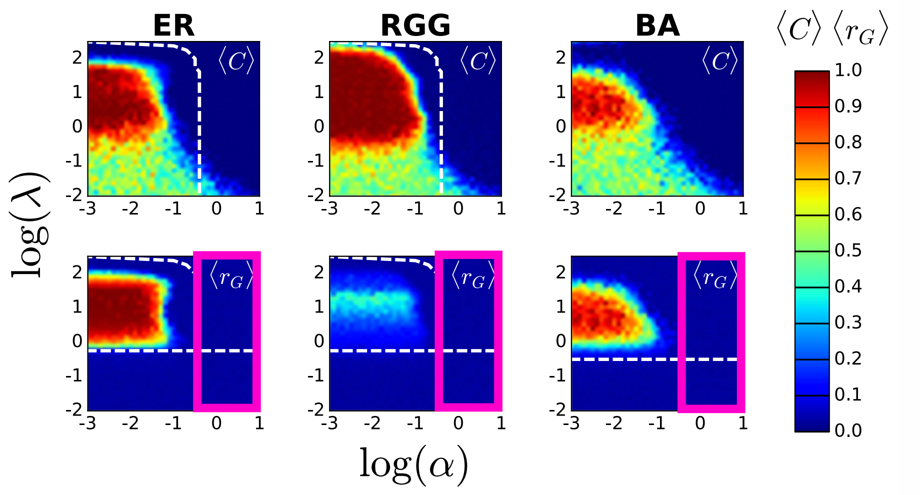


Evolutionary synchronization



- A. Antonioni, and A. Cardillo, "*Coevolution of Synchronization and Cooperation in Costly Networked Interactions*." Phys. Rev. Lett., **118**, 238301, (2017). DOI: `10.1103/PhysRevLett.118.238301`

Evolutionary synchronization



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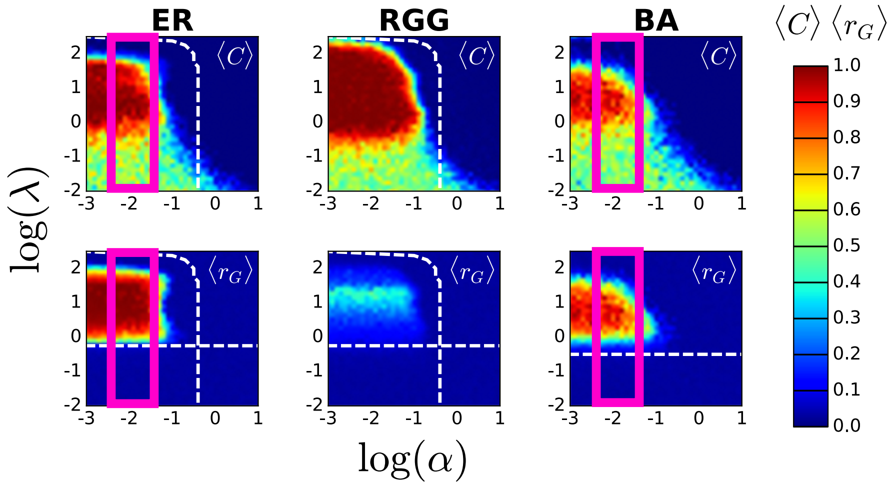

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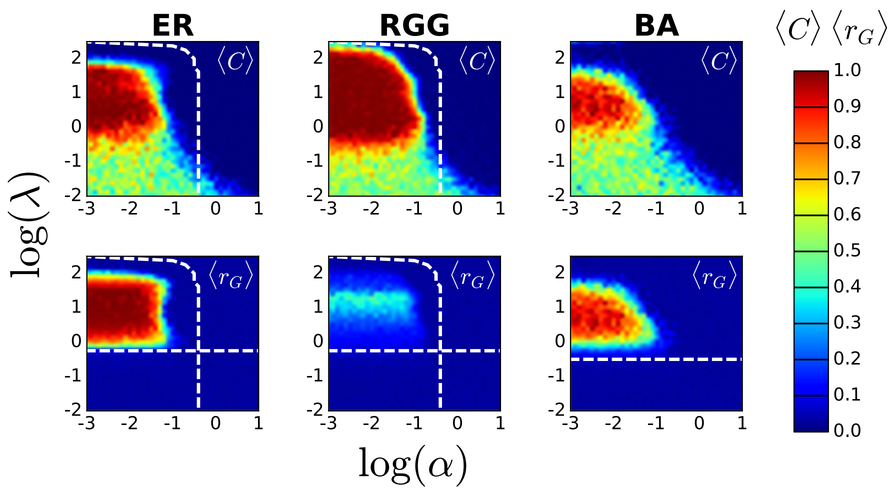

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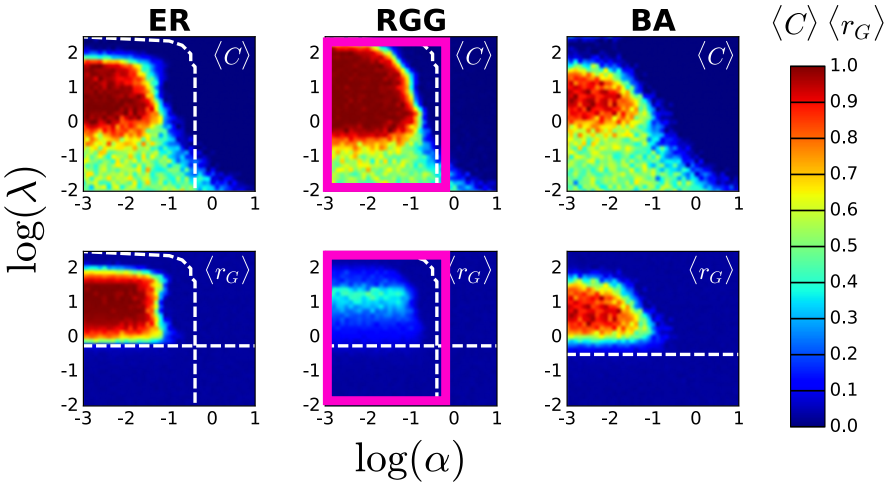

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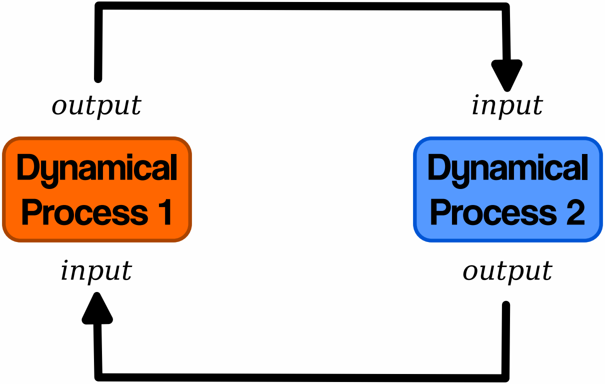
Evolutionary synchronization

Evolutionary synchronization

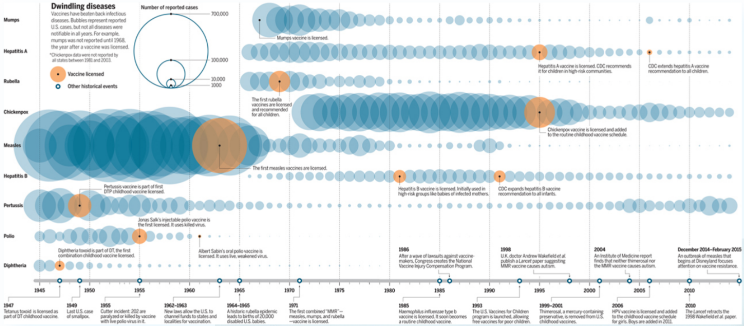
Summing up . . .

Take home messages

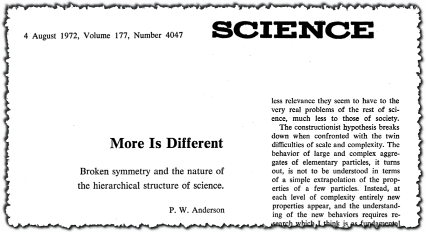


Coevolutionary dynamics as a way to model complex phenomena

Take home messages



Take home messages


4 August 1972, Volume 177, Number 4047

SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental

The interplay between dynamical processes (and the topology) allow to observe/explain new phenomena. However, there are some caveats . . .

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Benjamin Steinegger

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Alex Arenas

Unil
UNIL | Université de Lausanne

Alberto Antonioni

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Biocomputación y Física de Sistemas Complejos
Universidad Zaragoza

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Alessio Cardillo, Catalina Reyes-Suárez, Fernando Naranjo, and Jesús Gómez-Gardeñes
Phys. Rev. E **88**, 032803 – Published 5 September 2013

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Emergence of synergistic and competitive pathogens in a coevolutionary spreading model

Fakhteh Ghanbarnejad, Kai Seegers, Alessio Cardillo, and Philipp Hövel
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10.1103/PhysRevE.88.032803

10.1103/PhysRevE.97.032308

10.1103/PhysRevE.105.034308

10.1103/PhysRevLett.118.238301

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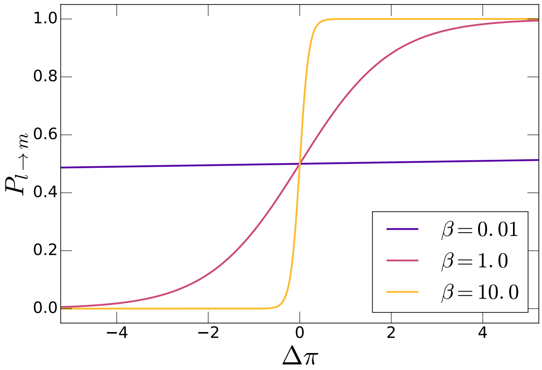
Extra contents

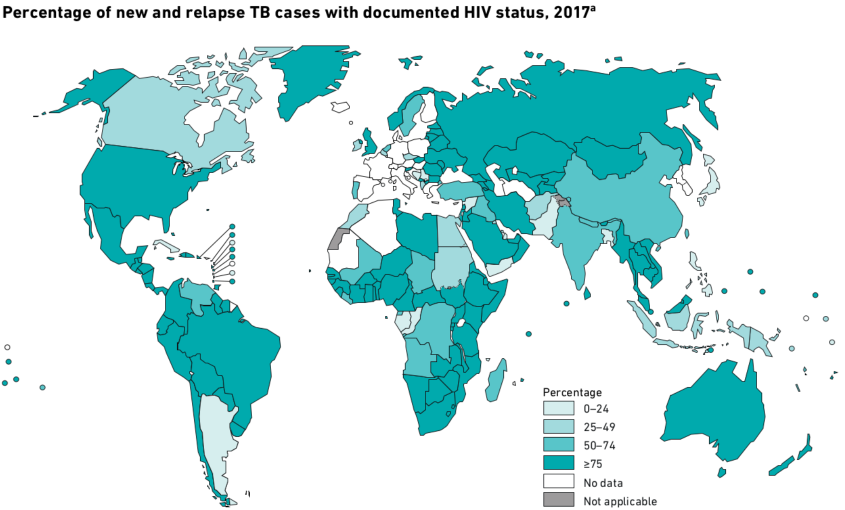
Fermi's Rule

$$P_{l \to m} = \frac{1}{1 + e^{-\beta(p_m - p_l)}}.$$



• G. Szabó, and C. Tőke, "*Evolutionary prisoner's dilemma game on a square lattice*." Phys. Rev. E, **58**, 69–73, (1998). DOI: `10.1103/PhysRevE.58.69`

Spreading of multiple pathogens



- WHO Global Tuberculosis report 2017. Available at:

`https://www.who.int/teams/global-tuberculosis-programme/tb-reports`

Spreading of multiple pathogens

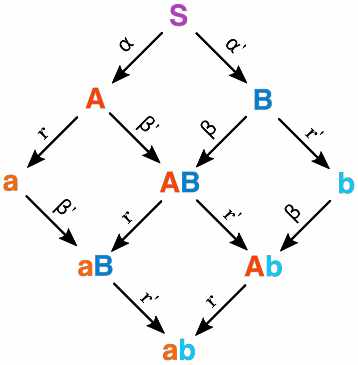
Spreading of multiple pathogens

$$\begin{tikzcd}
\mathbf{S} \arrow[r, "\alpha"] \arrow[d] & \mathbf{I} \arrow[r, "r"] \arrow[d] & \mathbf{R} \arrow[d] \\
\mathbf{S} & \mathbf{A} & \mathbf{a}
\end{tikzcd}$$

	Single SIR	Double SIR	Double SIR & games
Nr. pathogens	1	2	2
Nr. strategies	–	–	2
Nr. species	1	2	4
Nr. states	3	9	25
Nr. parameters	2	6	>7

• Anderson, R. M., & May, R. M. "*Infectious diseases of humans: Dynamics and control*". (Oxford University Press, Oxford, 1991).

Spreading of multiple pathogens



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- Chen, L., *et al.* Europhys. Lett., **104**, 50001 (2013). DOI: `10.1209/0295-5075/104/50001`

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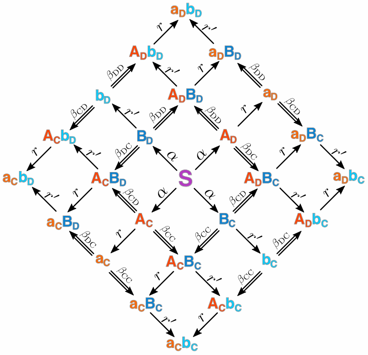

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Note

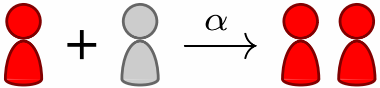
- α Single pathogen infection rate
- β Multiple pathogen infection rate
- c Parameter $\in]0, +\infty[$

Spreading of multiple pathogens

Let's simplify a bit . . .

- **Single pathogen infection** independent of pathogen's strategy (*i.e.* $\alpha_C = \alpha_D = \alpha$).

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

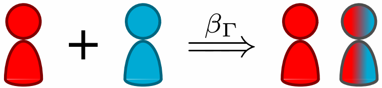


Spreading of multiple pathogens

Let's simplify a bit . . .

- **Single pathogen infection** independent of pathogen's strategy (*i.e.* $\alpha_C = \alpha_D = \alpha$).
- **Multiple pathogens infection** dependent only on host's strategy (*i.e.* $\beta_{CC} = \beta_{DC} = \beta_C$ and $\beta_{CD} = \beta_{DD} = \beta_D$).

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xrightarrow{\beta_\Gamma} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda\mathbf{Y}_\Gamma$$



Spreading of multiple pathogens

Let's simplify a bit . . .

- **Single pathogen infection** independent of pathogen's strategy (*i.e.* $\alpha_C = \alpha_D = \alpha$).
- **Multiple pathogens infection** dependent only on host's strategy (*i.e.* $\beta_{CC} = \beta_{DC} = \beta_C$ and $\beta_{CD} = \beta_{DD} = \beta_D$).
- Easier to infect a host occupied by a cooperator pathogen than a defector one (*i.e.* $\beta_C > \beta_D$).

Spreading of multiple pathogens

Spreading of multiple pathogens

Phase 1 (within season T)

Coupled SIR (25 states) dynamics

Single pathogen infection

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

Multiple pathogens infection

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_{\Lambda\Gamma}} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda \mathbf{Y}_\Gamma$$

Accumulation of Payoff

Single pathogen infection

$$\pi_{\mathbf{X}_\Lambda} = 1$$

Multiple path. infection (HD game)

Infecting $\pi_{\mathbf{X}_\Lambda}$

$$\begin{array}{c|cc} & C & D \\ \hline C & \frac{1}{2} & \gamma \\ D & 1-\gamma & -\frac{1}{2} \end{array}$$

Infected $\pi_{\mathbf{Y}_\Gamma}$

$$\begin{array}{c|cc} & C & D \\ \hline C & \frac{1}{2} & 1-\gamma \\ D & \gamma & -\frac{1}{2} \end{array}$$

Evaluation of fitness

$$\rho_i^{T+1}\Big|_{t_0} = \rho_i^{T}\Big|_{t_0}\left[1 + \Pi_i^{T}\Big|_{t_\infty} - \overline{\Pi}^{T}\Big|_{t_\infty}\right]$$

Phase 2: Evolution of concentrations/strategies (between season T and $T+1$)

New epidemic season

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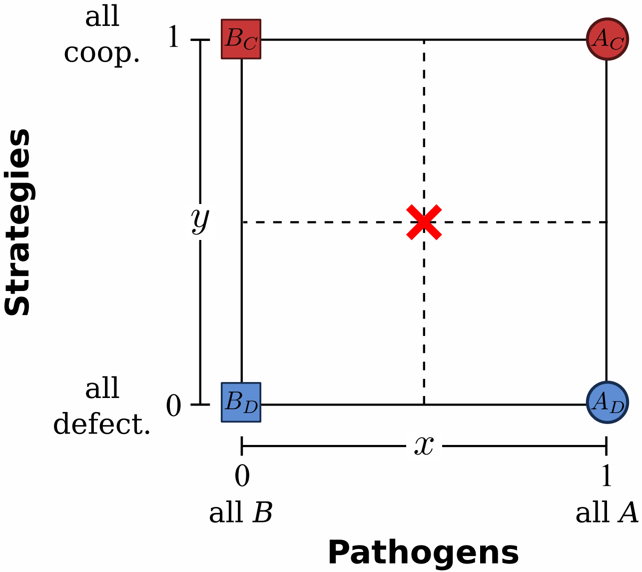

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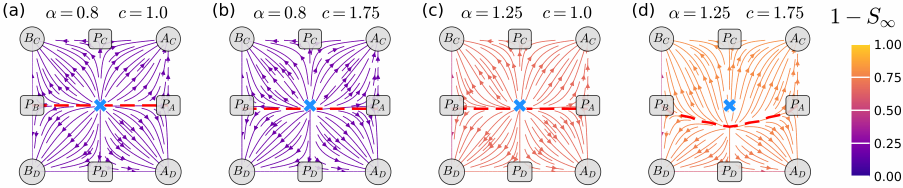

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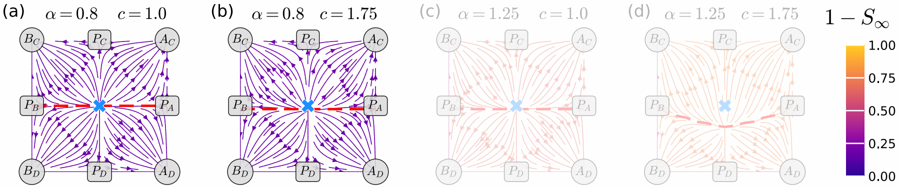

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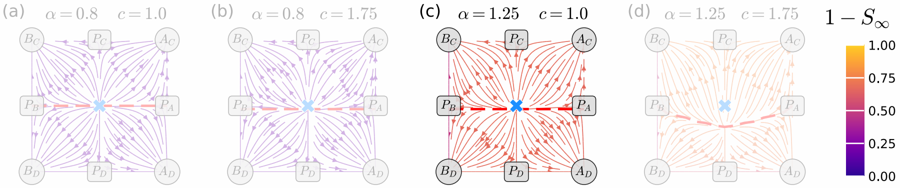

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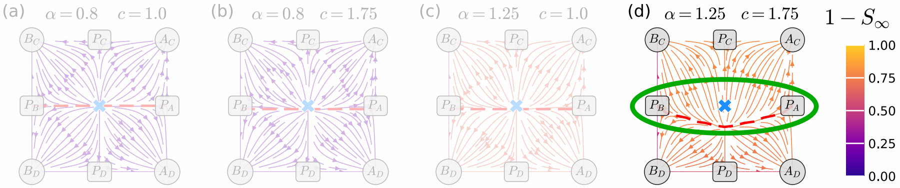

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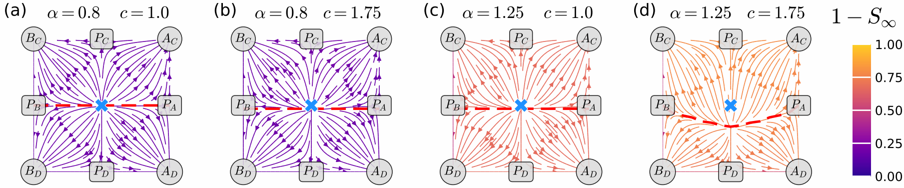

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Evolutionary Kuramoto – macroscopic behavior

Lower bound

$$\lambda_c = \lambda_c^{MF} \frac{\langle k \rangle}{\langle k^2 \rangle}$$

- A. Arenas, *et al.* Phys. Rep., **469**, 93–153, (2008). DOI: `10.1016/j.physrep.2008.09.002`
- H. Ohtsuki, *et al.* Nature, **441**, 502–505, (2006). DOI: `10.1038/nature04605`

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Upper bound

$$\frac{\Delta b}{\Delta c} = \frac{b_{Coop} - b_{Def}}{c} > \langle k \rangle$$

$$\frac{\sqrt{2\left[1 + \sin(\varepsilon\lambda)\right]} - \sqrt{2}}{\varepsilon\lambda\langle k \rangle}\pi > \alpha \, .$$

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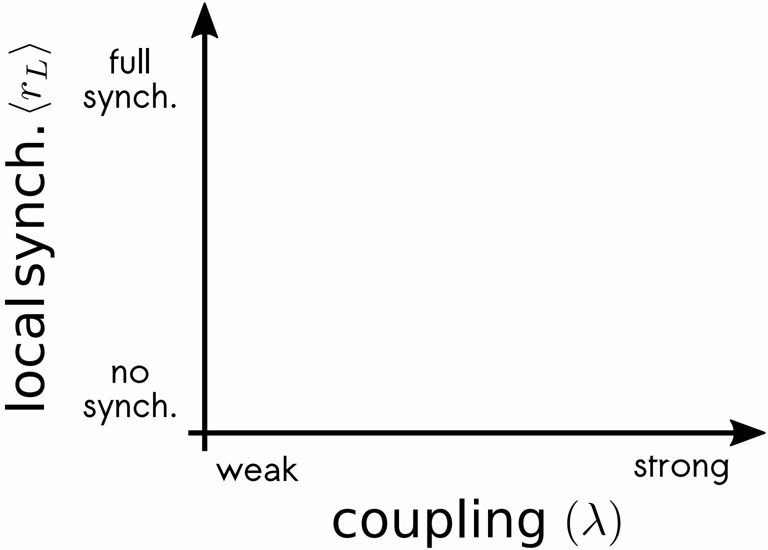

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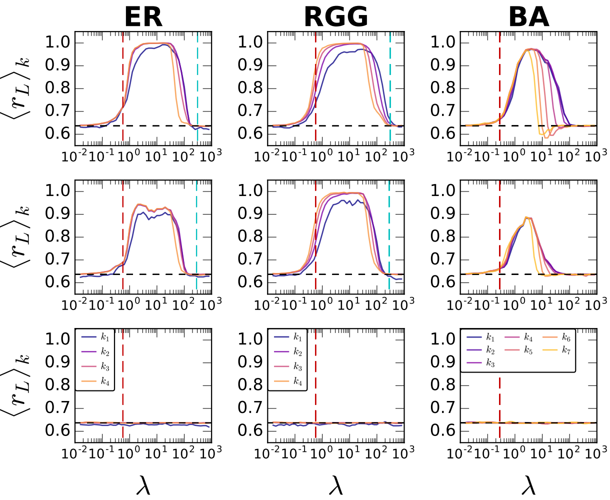

Three regimes of relative cost:

$\alpha = 10^{-3}$ Cheap

$\alpha = 10^{-1.4}$ Medium

$\alpha = 10^{0}$ Expensive

Evolutionary Kuramoto – microscopic behavior

average pairwise order parameter

$$\overline{r_{lm}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\left\|1+e^{i\theta}\right\|}{2}\,d\theta =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\|1+\cos\theta + i\sin\theta\|}{2}\,d\theta =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\sqrt{[1+\cos\theta]^2+\sin^2\theta}}{2}\,d\theta = \frac{4}{2\pi} = \frac{2}{\pi} \sim 0.6366\,.$$