Comparing spatial networks: A 'one size fits all' efficiency-driven approach

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NetSci-X 2020 – Tokyo
Monday, January 20, 2020

UNIVERSITAT ROVIRA i VIRGILI

Systems embedded in (metric) spaces

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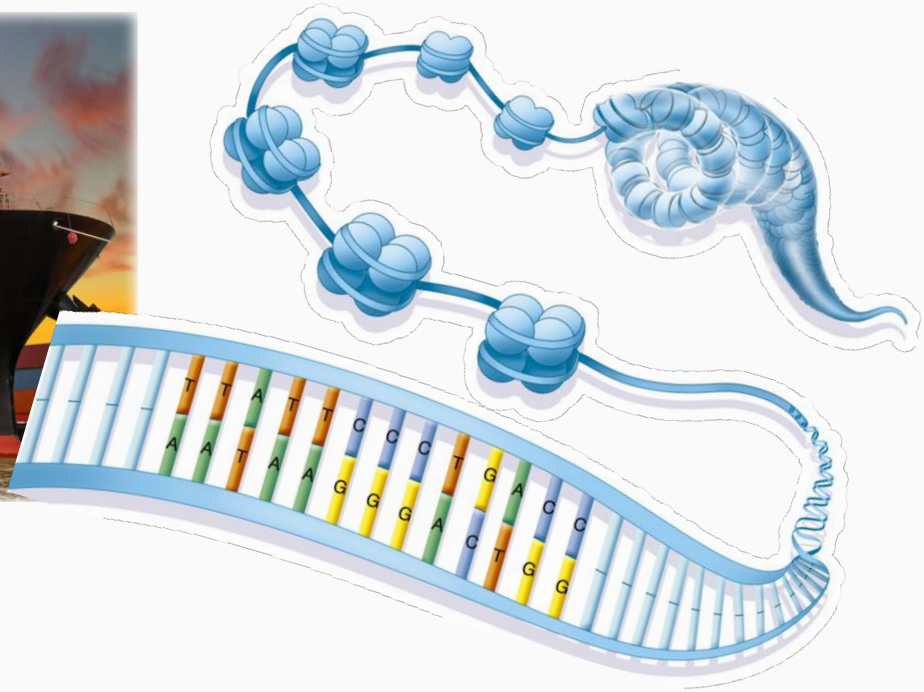

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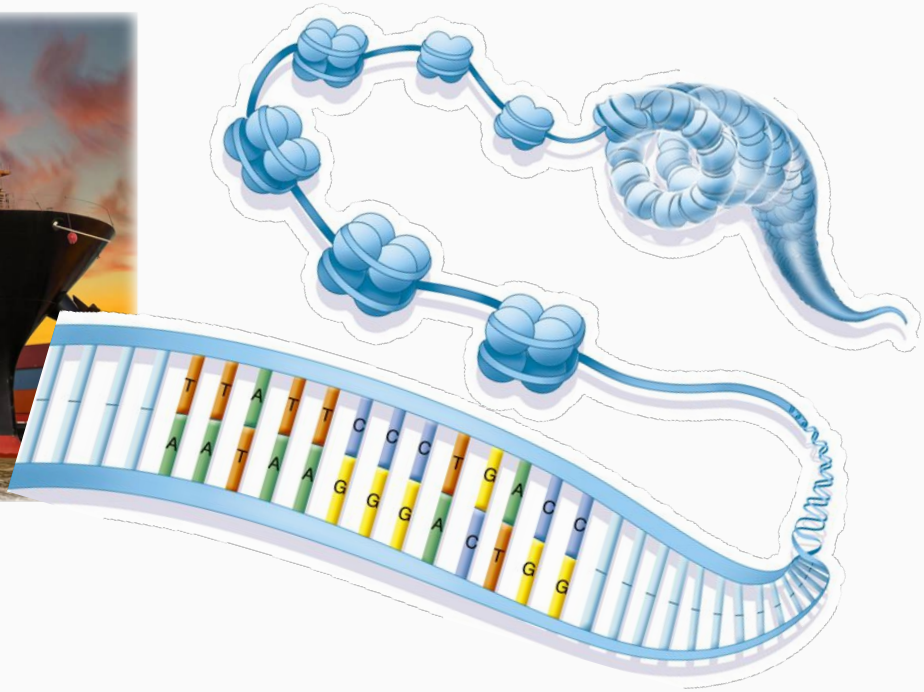

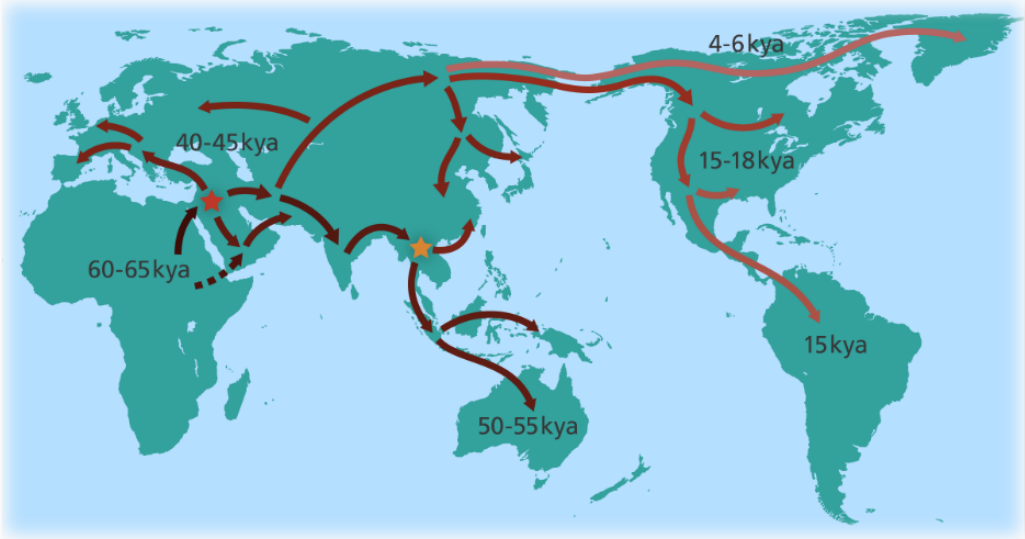

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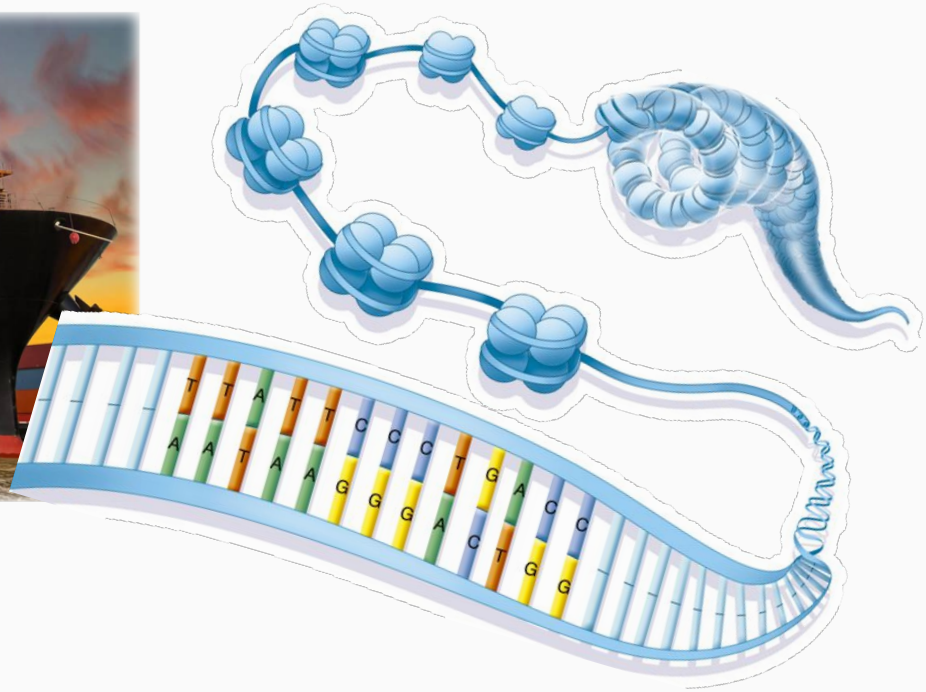
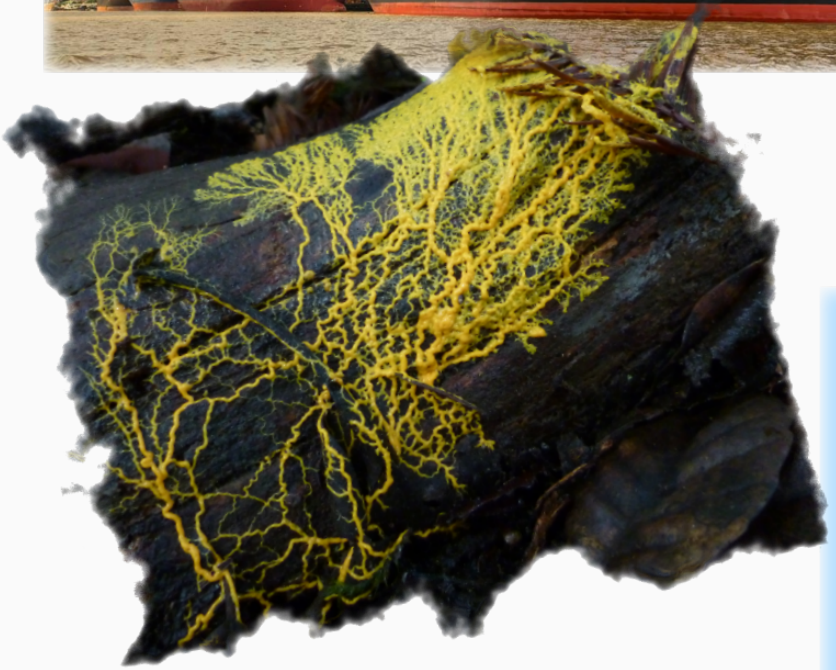
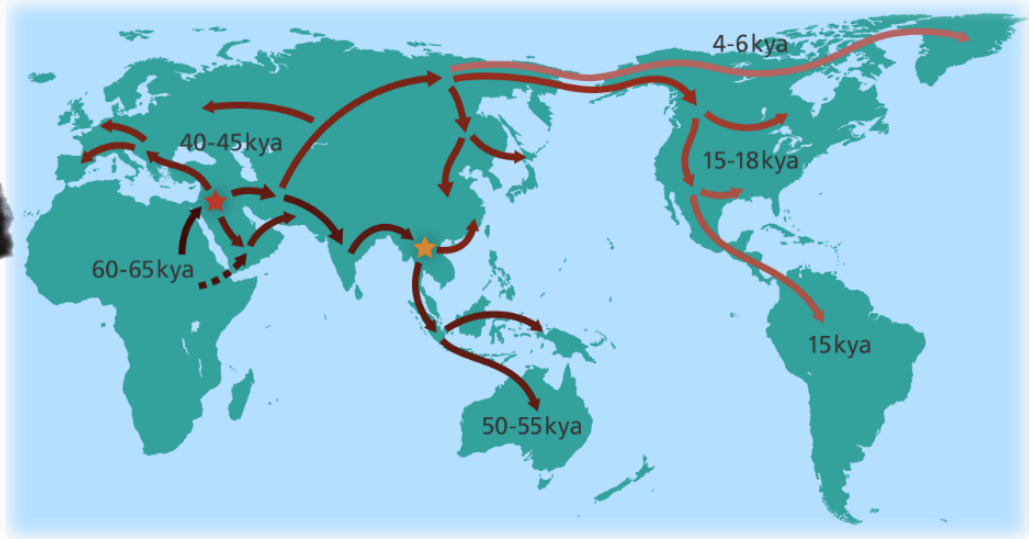
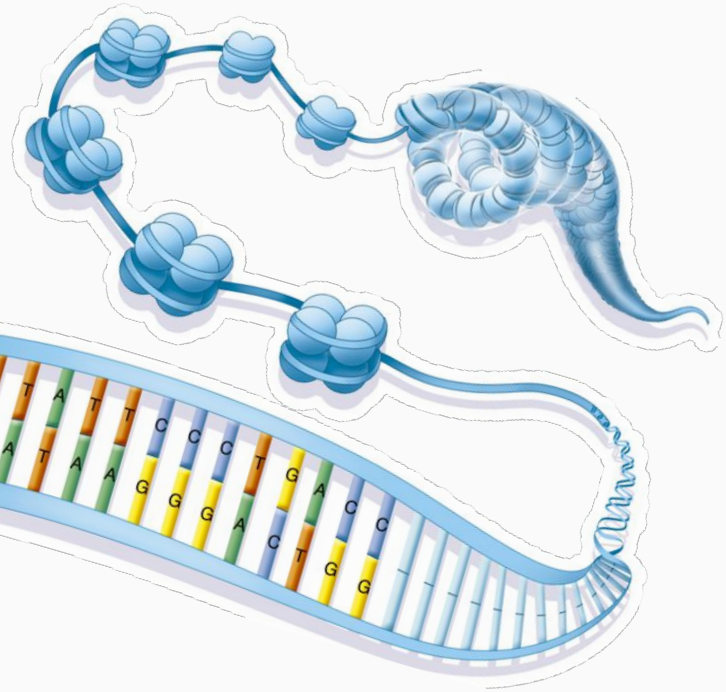
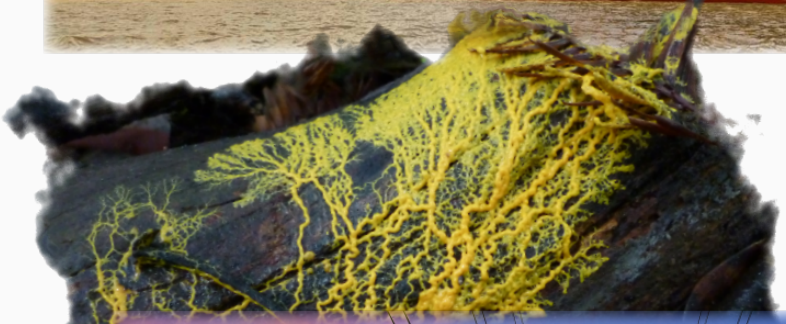


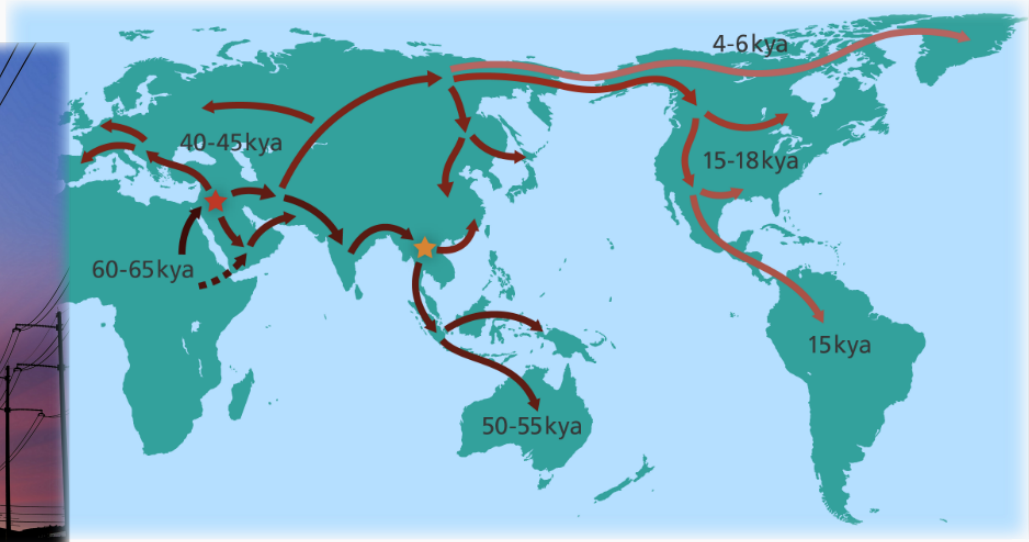




Systems embedded in (metric) spaces







Systems embedded in (metric) spaces

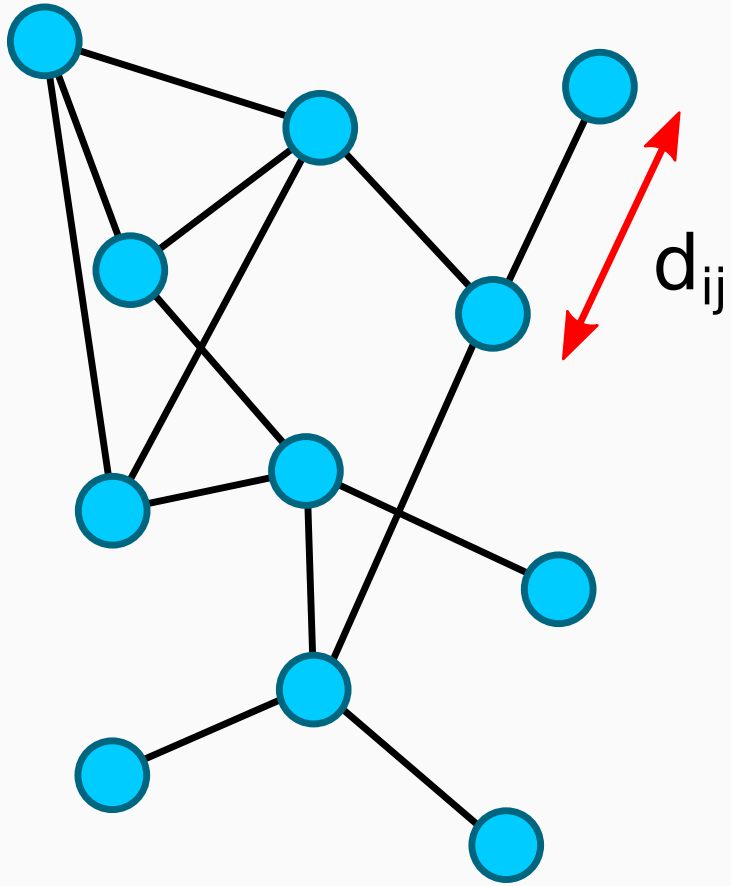

How can we measure (and compare)
the performances of a spatial network?


SAMSUNG
SAMSUNG
SAMSUNG

Cost

Total cost

$$L_{\text{tot}} = \sum_{ij} a_{ij}\, d_{ij}$$

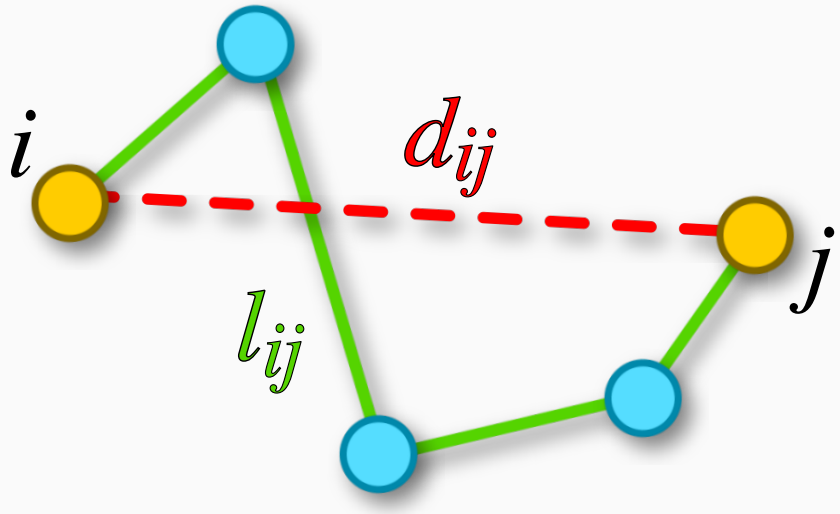


- Barthélemy M 2011 *Phys. Rep.* **499** 1–101

Cost

The amount of resources available reverberates on the structure of the system

Efficiency



Detour index/distance ratio

$$e_{ij} = \frac{d_{ij}}{l_{ij}} \in [0, 1]$$

Efficiency

Global Efficiency

$$E_{\text{glob}} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{d_{ij}}{l_{ij}} \qquad E_{\text{glob}} \in [0, 1] \,.$$

- Latora V and Marchiori M *Phys. Rev. Lett.* **87** 198701 (2001).

Efficiency

Global Efficiency

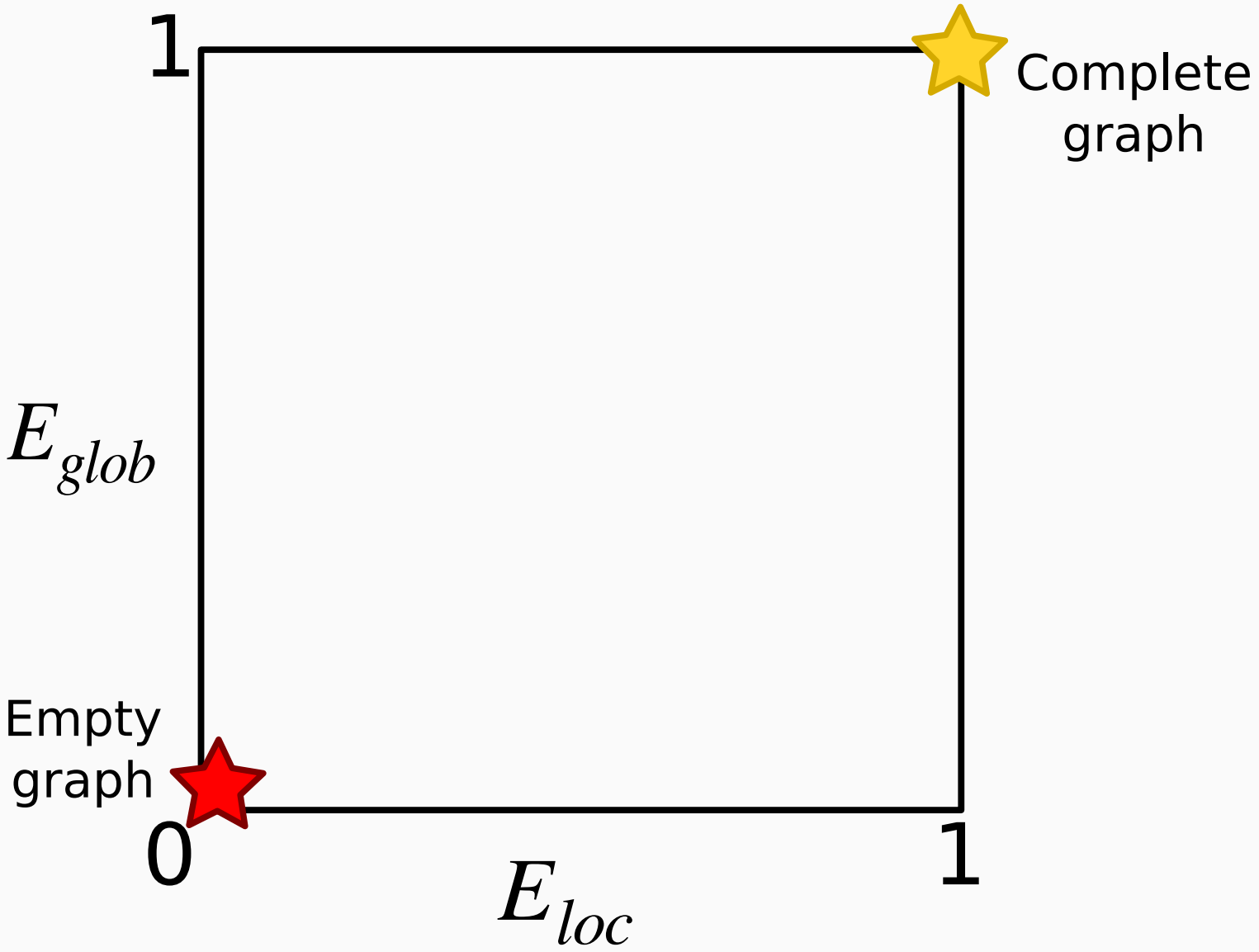
$$E_{\text{glob}} = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{d_{ij}}{l_{ij}} \qquad E_{\text{glob}} \in [0,1] .$$

Local Efficiency

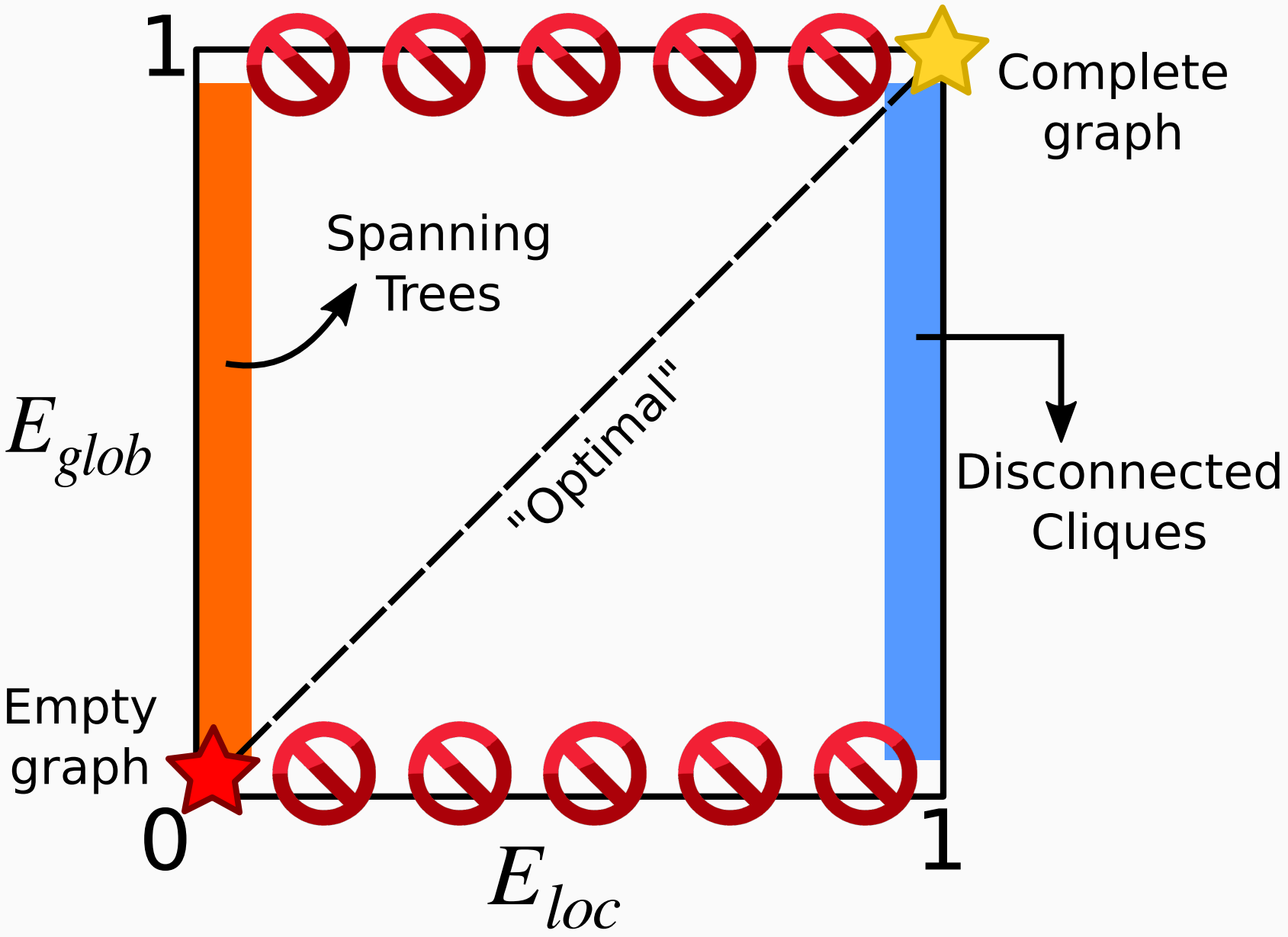
$$E_{\text{loc}} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{k_i (k_i - 1)} \sum_{j \neq m \in \Gamma_i} \frac{d_{jm}}{l_{jm/i}} \qquad E_{\text{loc}} \in [0,1] .$$

- Latora V and Marchiori M *Phys. Rev. Lett.* **87** 198701 (2001).
- Vragović I, L E and Díaz-Guilera A *Phys. Rev. E* **71** 036122 (2005).

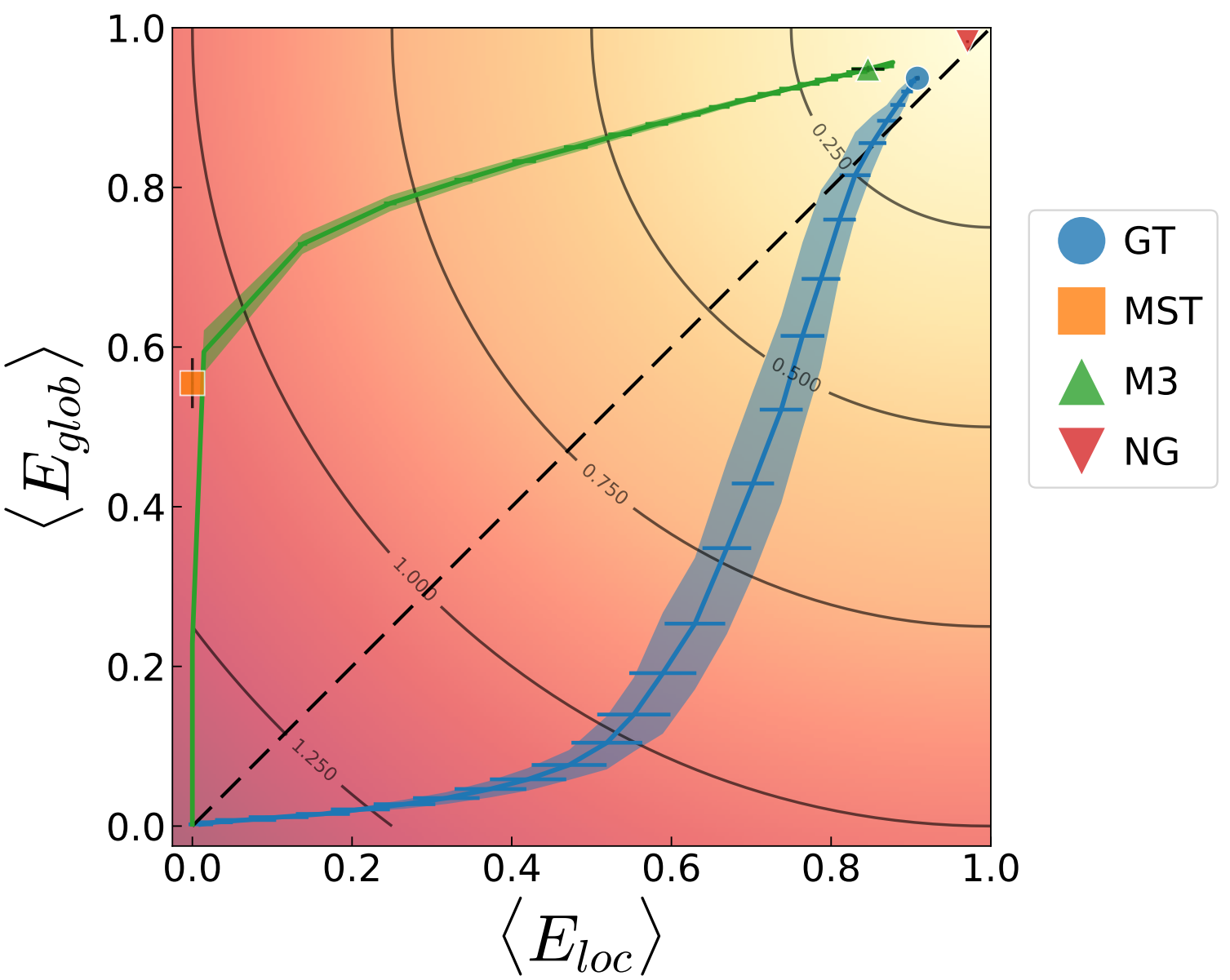
Efficiency



Efficiency


1
Complete graph
Spanning Trees
E_{glob}
"Optimal"
Disconnected Cliques
Empty graph
0
1
E_{loc}

Efficiency


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$\langle E_{glob} \rangle$
$\langle E_{loc} \rangle$
0.250
0.500
0.750
1.000
1.250
GT
MST
M3
NG


YES WE CAN

Another measure of Efficiency

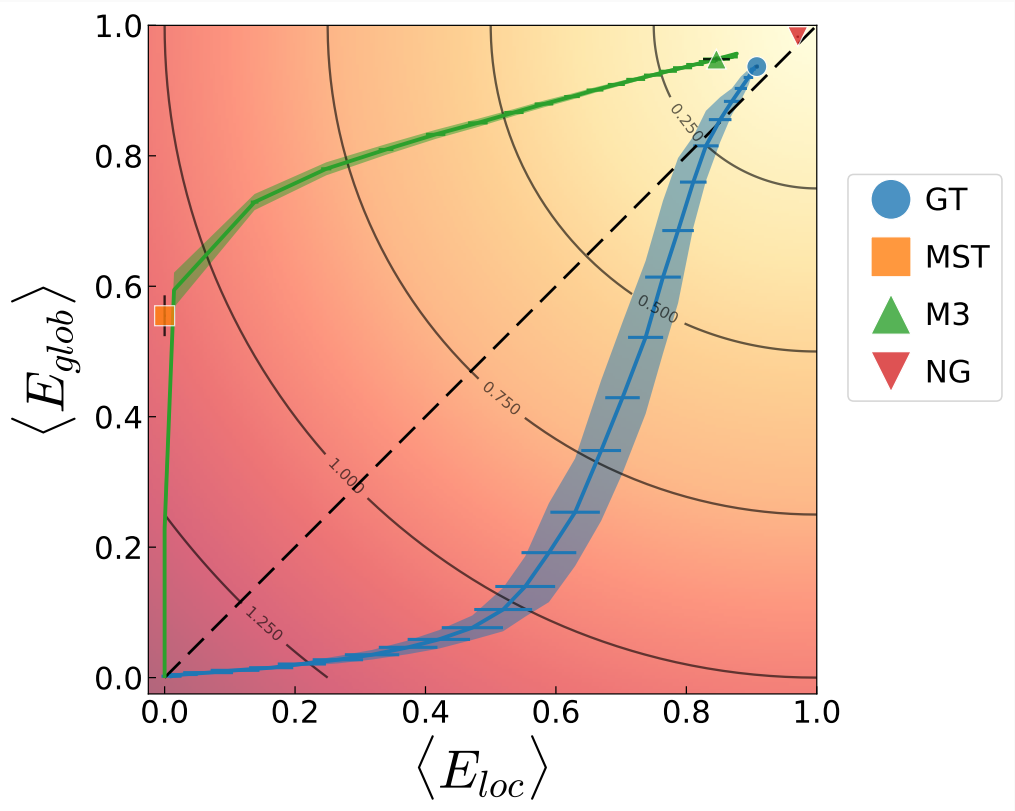
Integrated Efficiency

$$E_{\text{int}} = 1 - \sqrt{\frac{(1 - E_{\text{glob}})^2 + (1 - E_{\text{loc}})^2}{2}}\,.$$

Another measure of Efficiency

Integrated Efficiency

$$E_{\text{int}} = 1 - \sqrt{\frac{(1 - E_{\text{glob}})^2 + (1 - E_{\text{loc}})^2}{2}}\,.$$



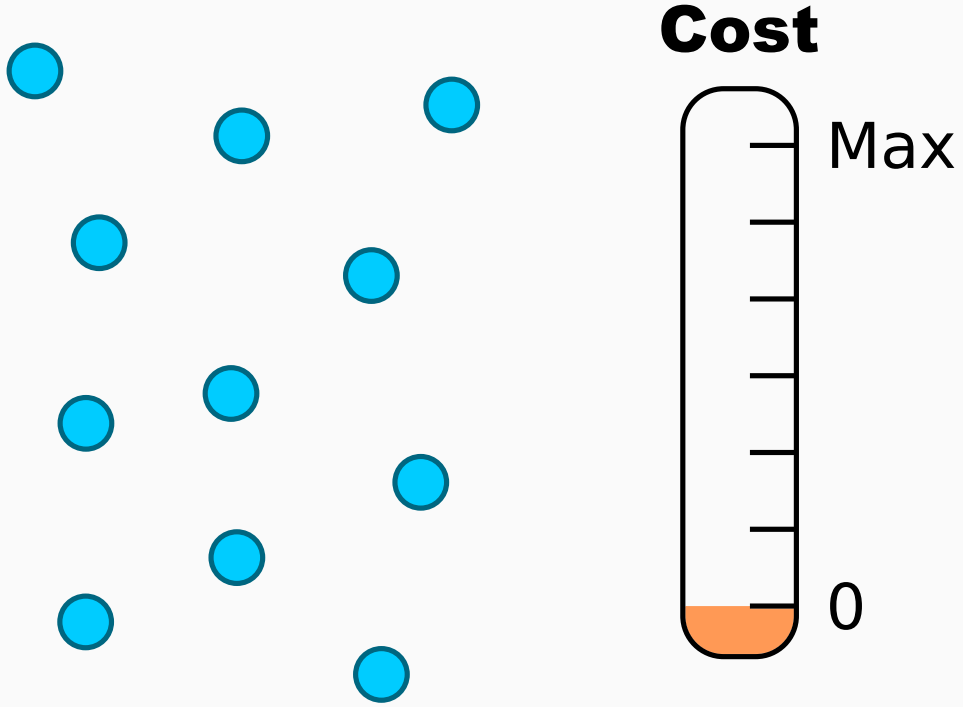
Efficiency optimal networks

Given a certain **layout** of nodes, $\tilde{\mathcal{L}}$, and a fixed **amount of resources**, $\tilde{L_{\text{tot}}}$, can we find the set of connections that **maximizes** E_{int}?

Efficiency optimal networks

Network growth algorithm

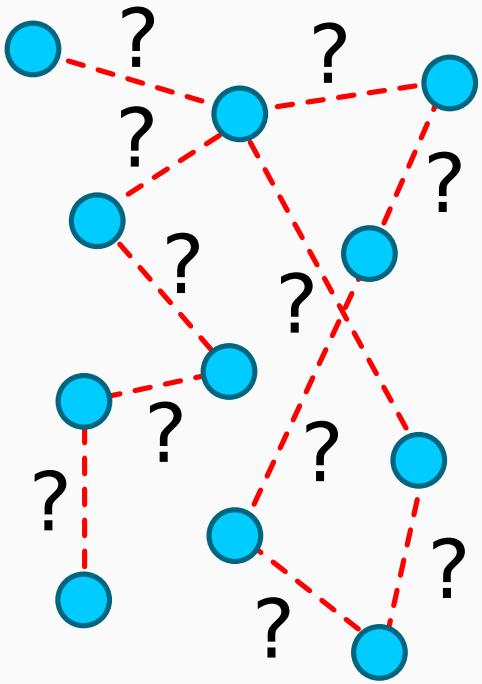
1. Start at $t = 0$ from an empty graph, G, with N nodes placed according to layout $\tilde{\mathcal{L}}$.

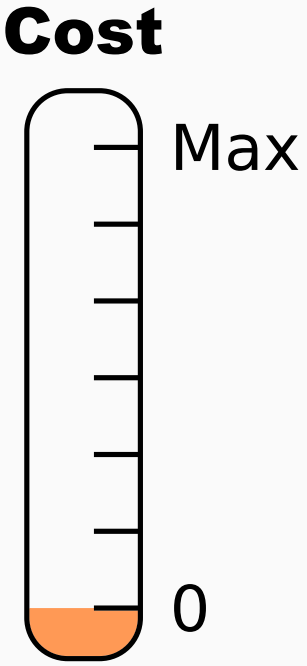


Efficiency optimal networks

Network growth algorithm

1. Start at $t = 0$ from an empty graph, G, with N nodes placed according to layout $\tilde{\mathcal{L}}$.
2. For $t > 0$, for each link $(i, j) \notin G$:

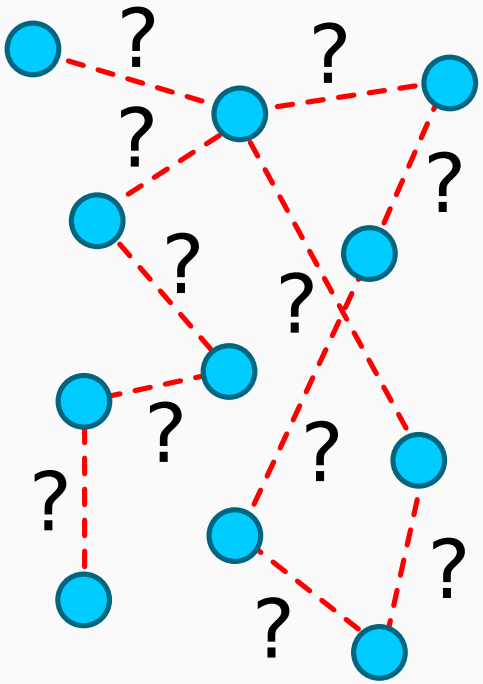


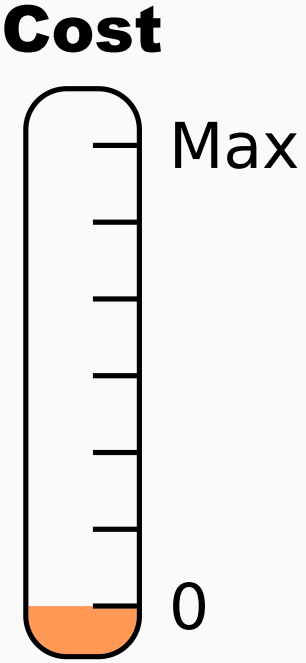


Efficiency optimal networks

Network growth algorithm

1. Start at $t = 0$ from an empty graph, G, with N nodes placed according to layout $\tilde{\mathcal{L}}$.
2. For $t > 0$, for each link $(i, j) \notin G$:
   2.1 Compute
   $$\alpha_{ij} = \left\{ \frac{\Delta E_{\text{int}}(i,j)}{d_{ij}} \right\}$$





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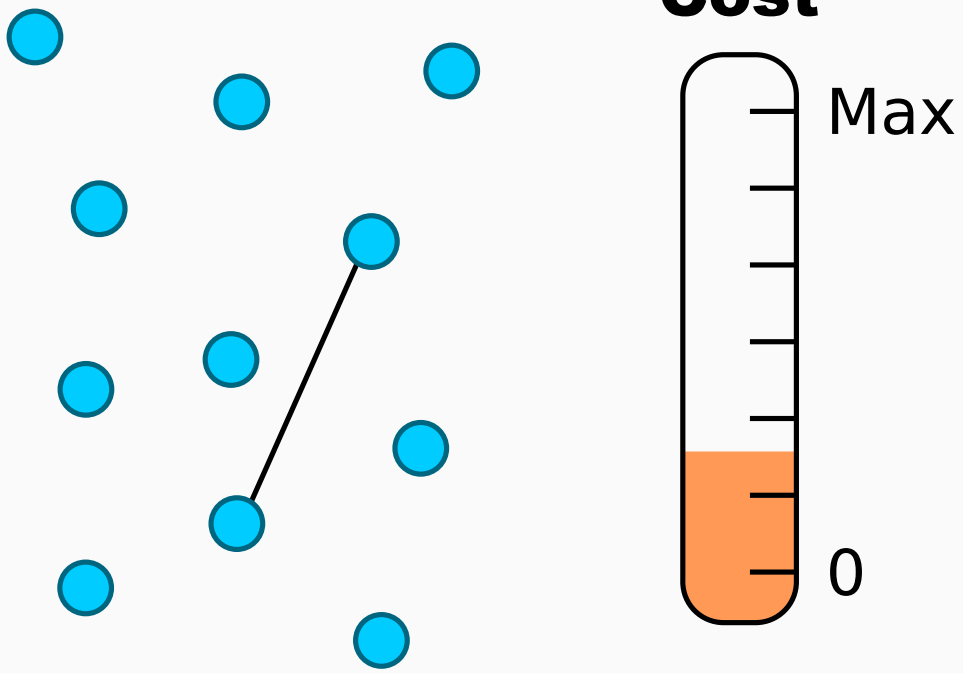
   - 2.1 Compute
     $$\alpha_{ij} = \left\{ \frac{\Delta E_{\text{int}}(i,j)}{d_{ij}} \right\}$$
   - 2.2 Add edge
     $$(i^\star, j^\star) : \alpha_{i^\star j} = \max_{i,j} (\alpha_{ij})$$



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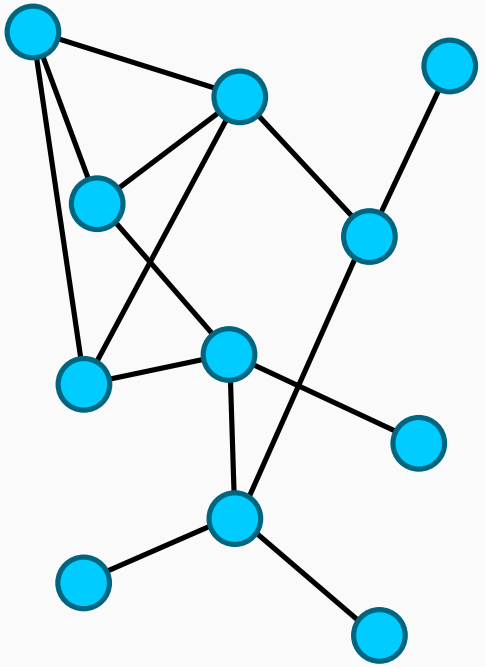
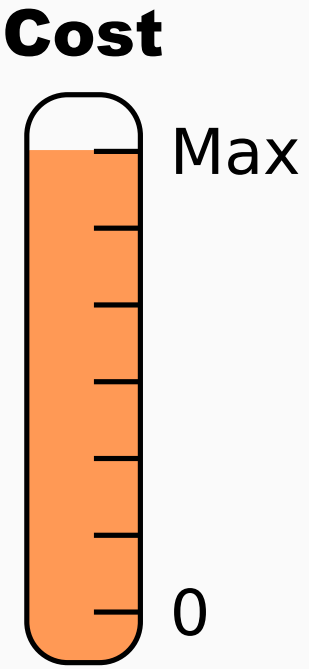
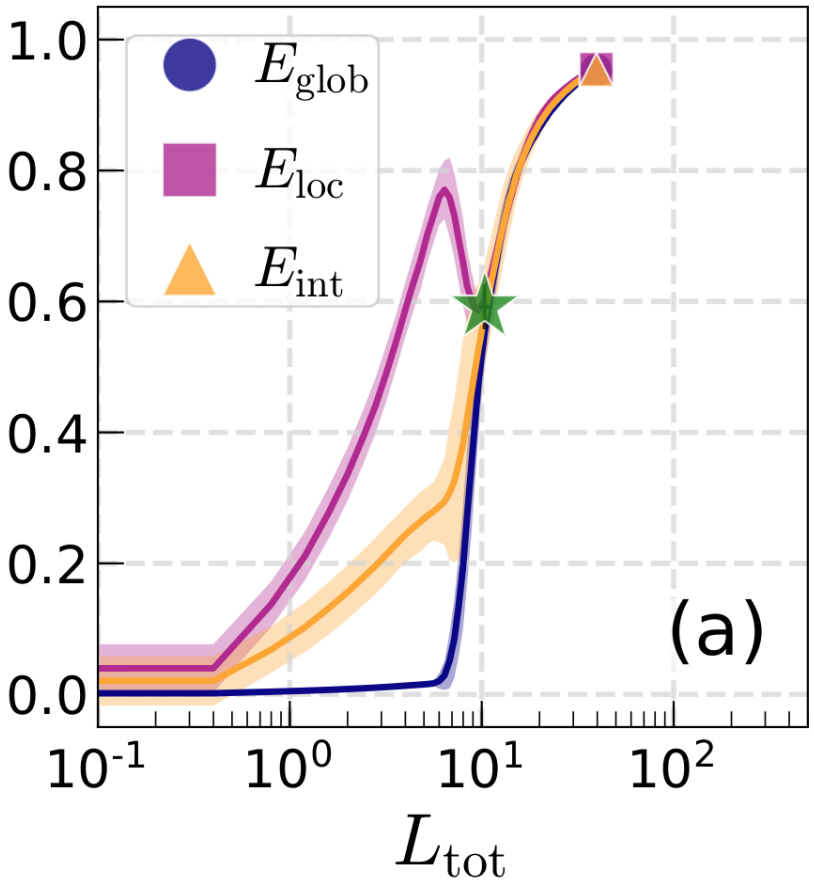
   2.2 Add edge
   $$(i^\star, j^\star) : \alpha_{i^\star j} = \max_{i,j} (\alpha_{ij})$$
3. Repeat step 2 until $L_{\text{tot}}(G) \simeq \tilde{L_{\text{tot}}}$.





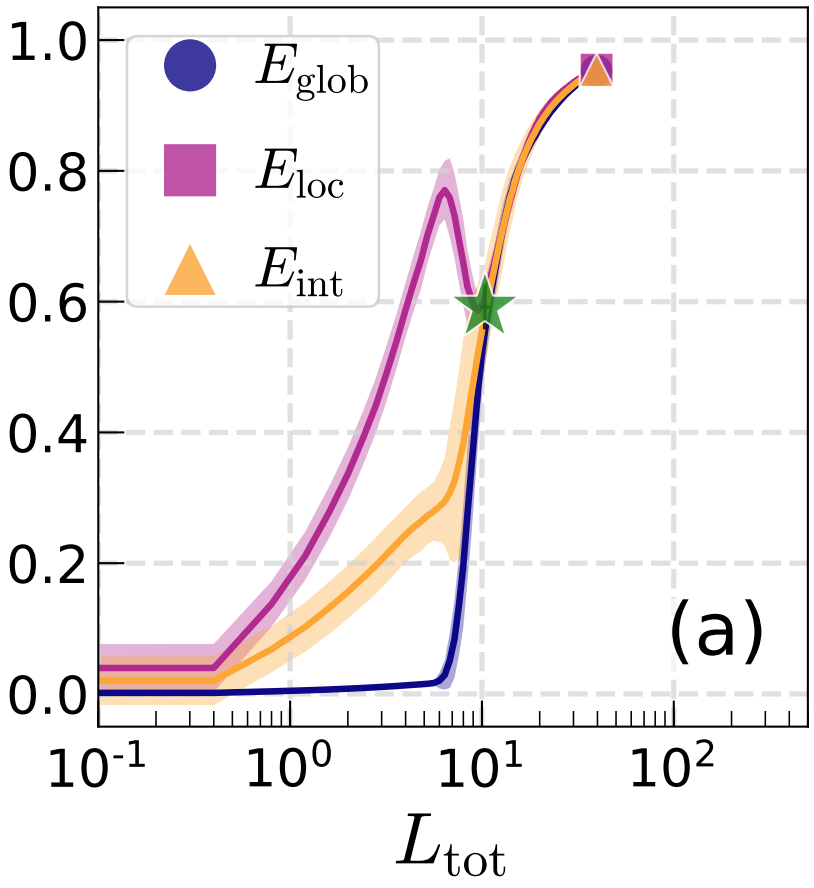
Efficiency optimal networks

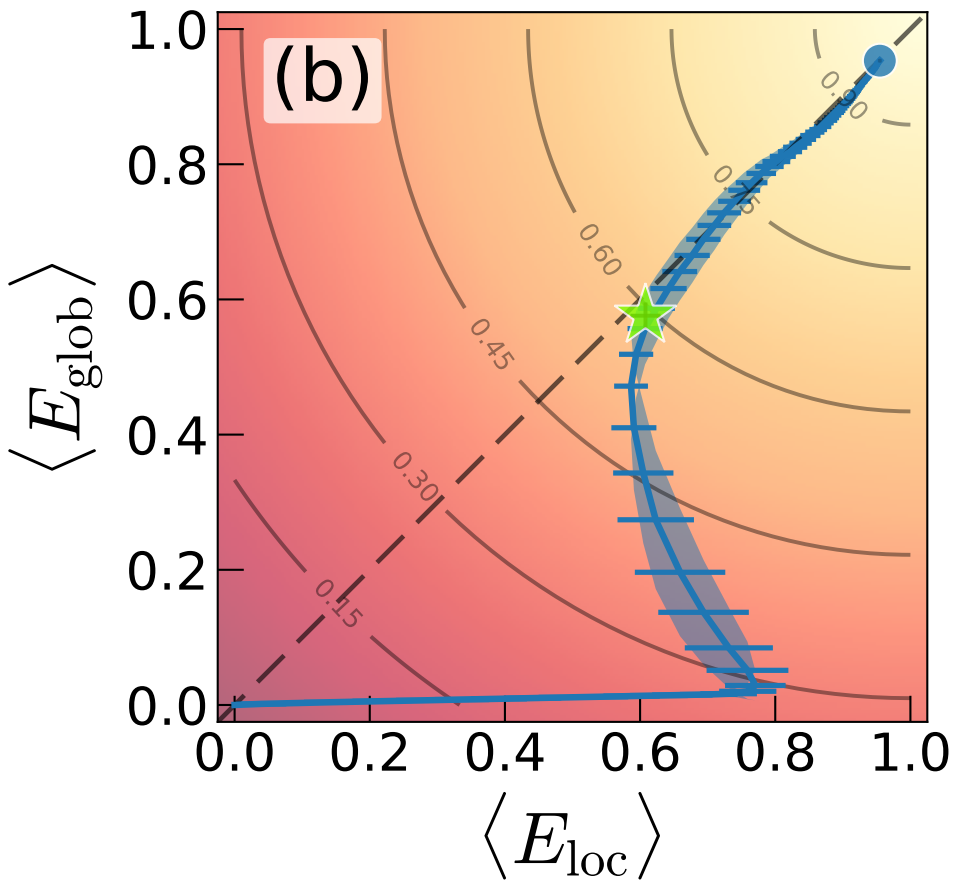


Note:

$N_{rel} = 100$ realizations of networks with $N = 100$ nodes uniformly distributed at random on $[0, 1] \times [0, 1] \in \mathbb{R}^2$.

Efficiency optimal networks





Note:

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Efficiency optimal networks

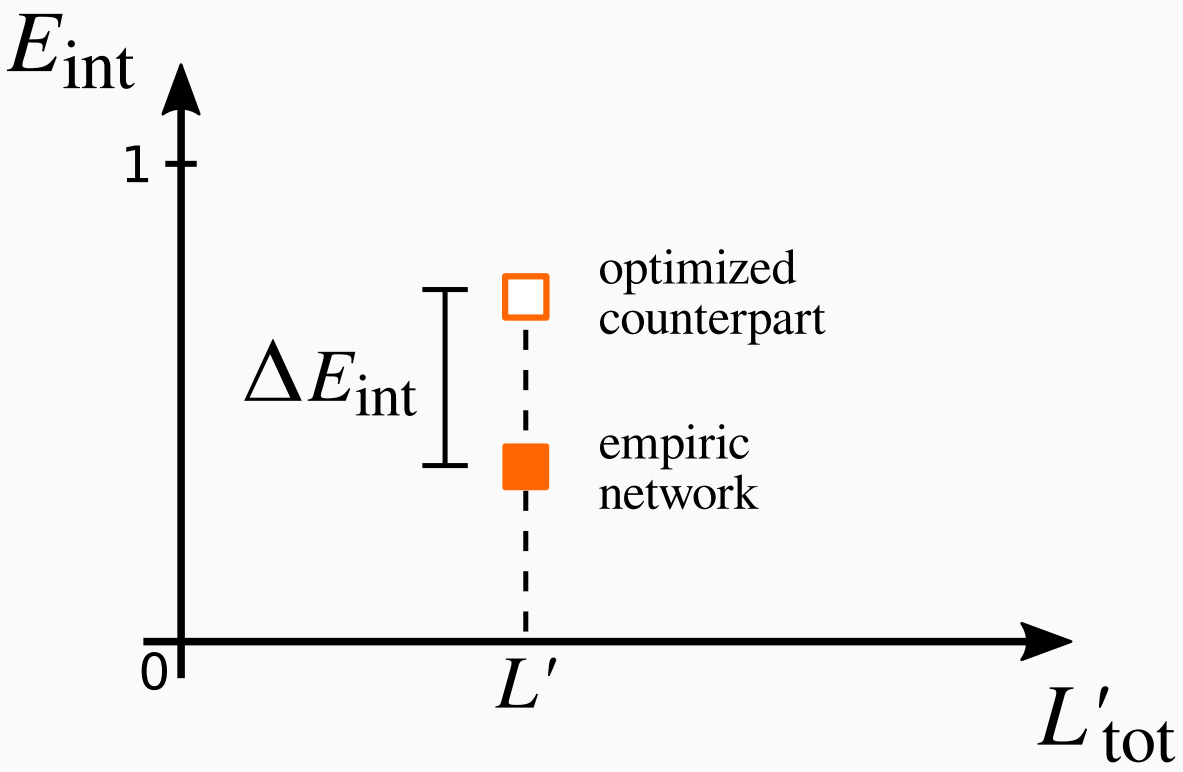
Let's compare networks!

Comparison between networks

Data

	description	time var. (# snap.)
UK Flights	Domestic flights in UK	✓ (4)
Cities	Road patterns in Northern Italy	✓ (4)
Latium Vetus & South Etruria	Trails among villages during the Iron Age (950 – 509 BC)	✓ (5 + 5)
Hispania	Main road network in the Iberian peninsula during the Roman Empire	✗
Catalonia Railway	—	✗
Rome Railway	—	✗
Power Grid	Italian power grid	✗

Comparison between networks

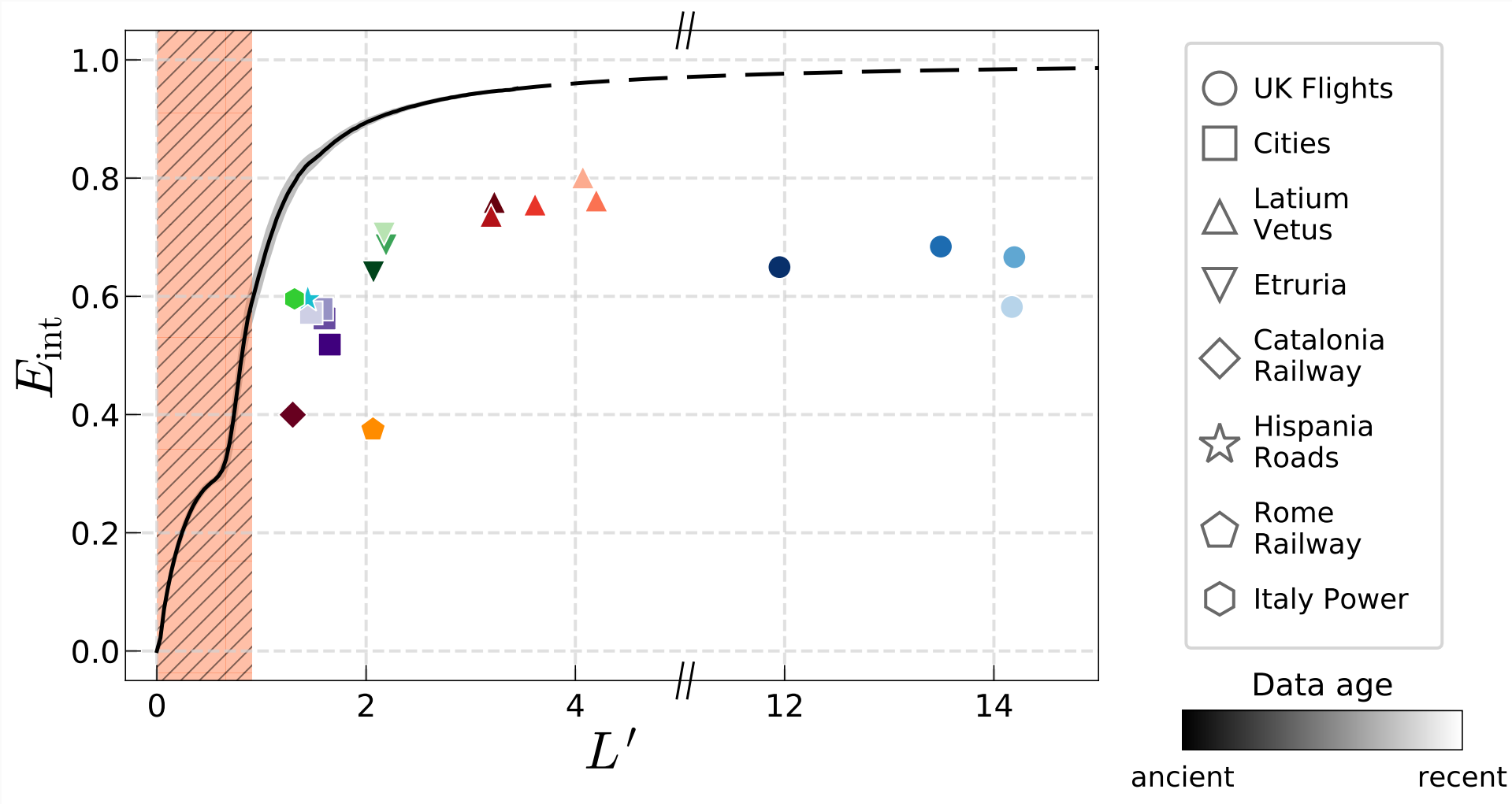


Note 1

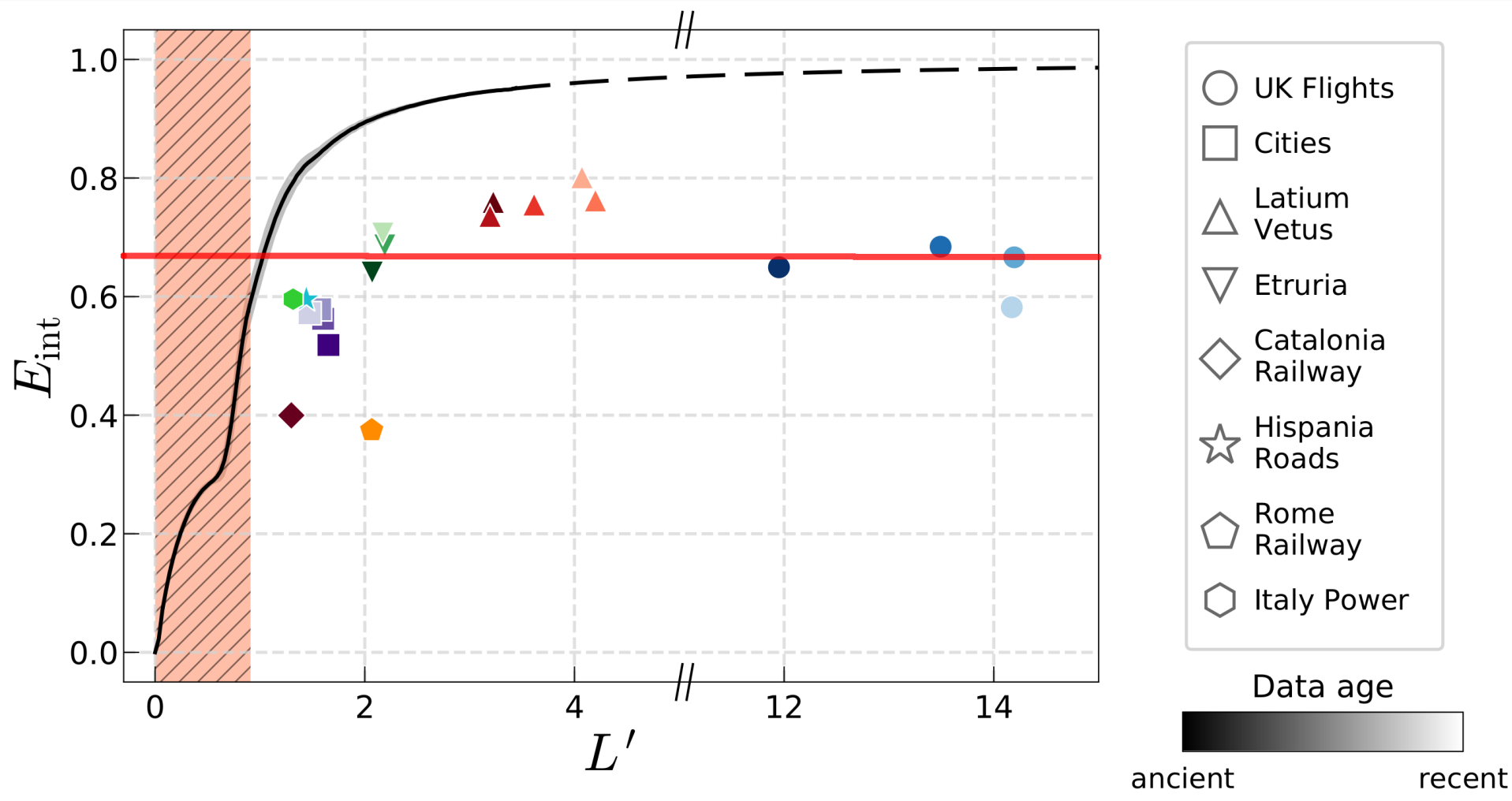
To ensure consistency between units, we must rescale the lengths!

$$L' \simeq \frac{L_{\text{tot}}}{\langle d \rangle \cdot N^{2\alpha}}$$

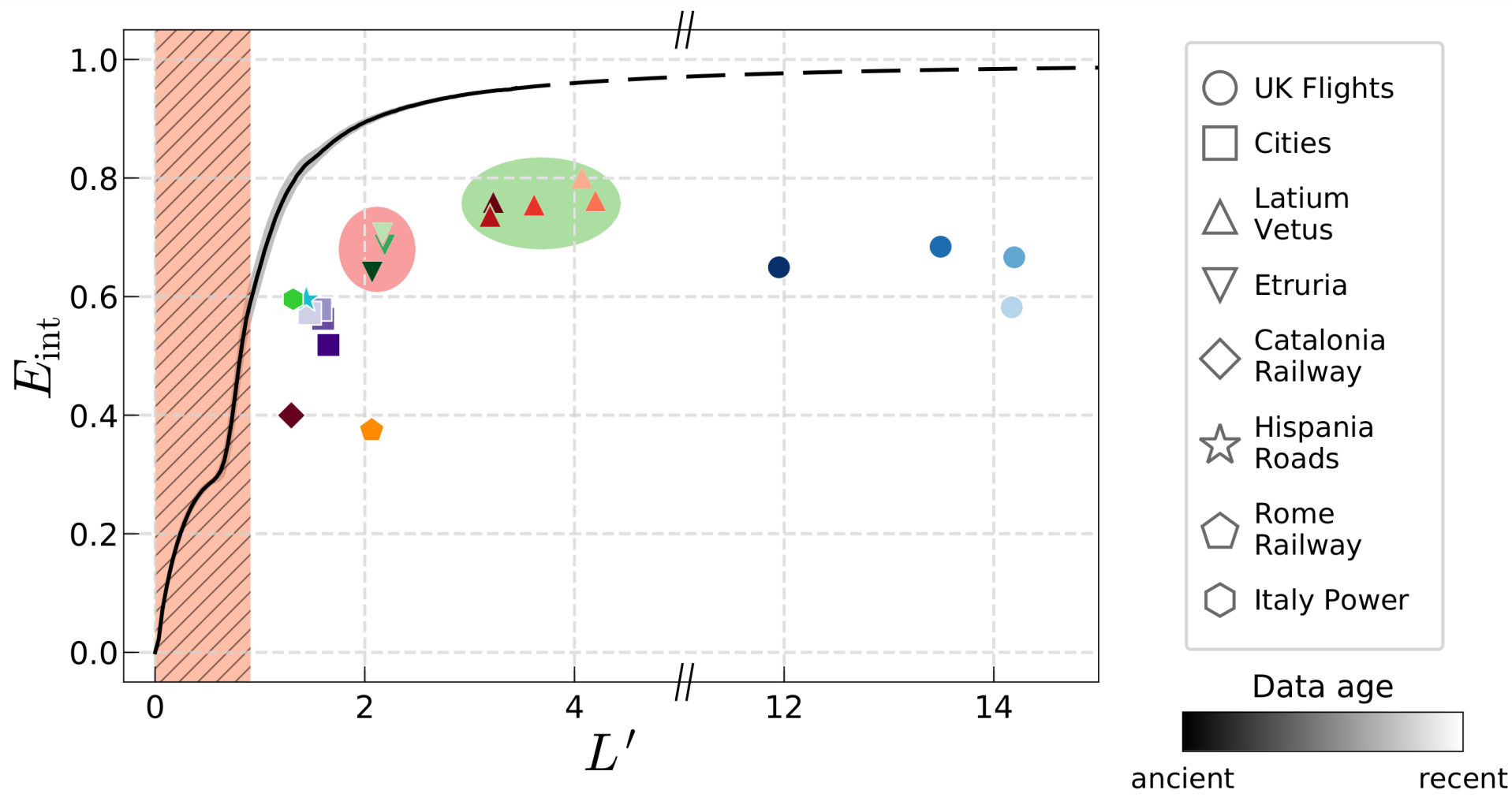
Comparison between networks



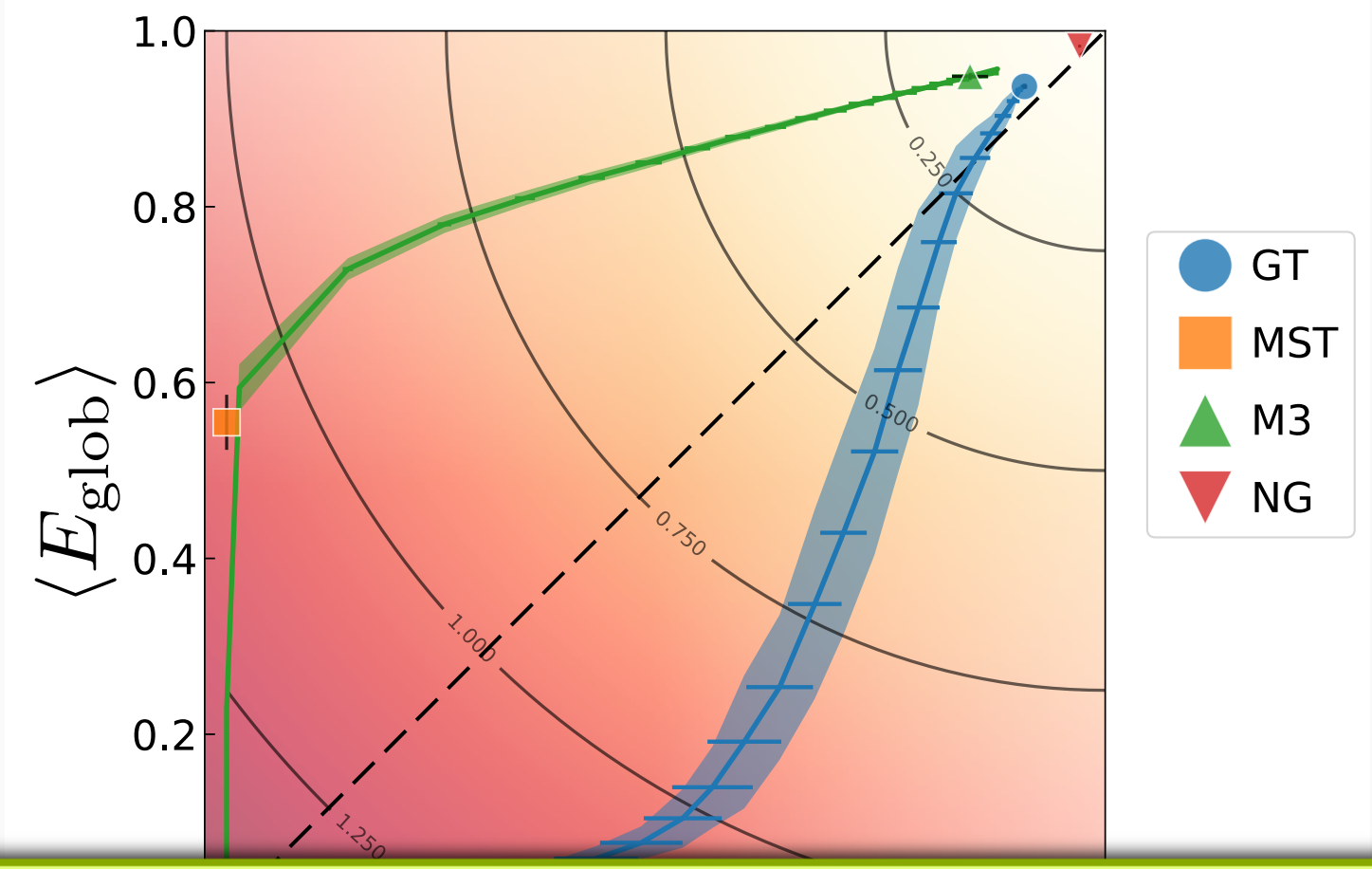
Comparison between networks



Comparison between networks

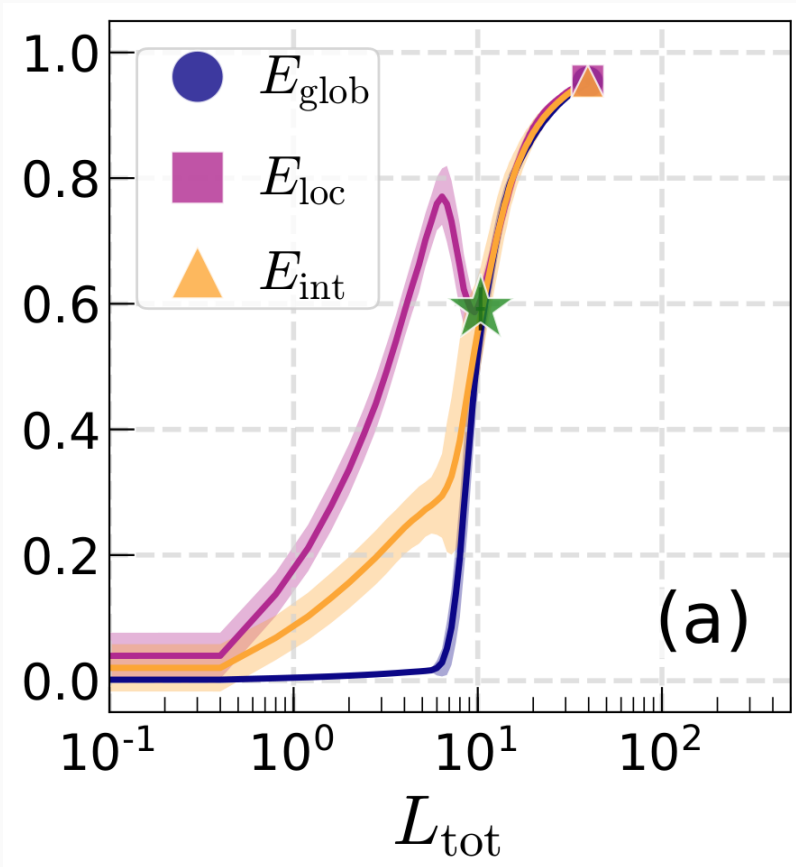


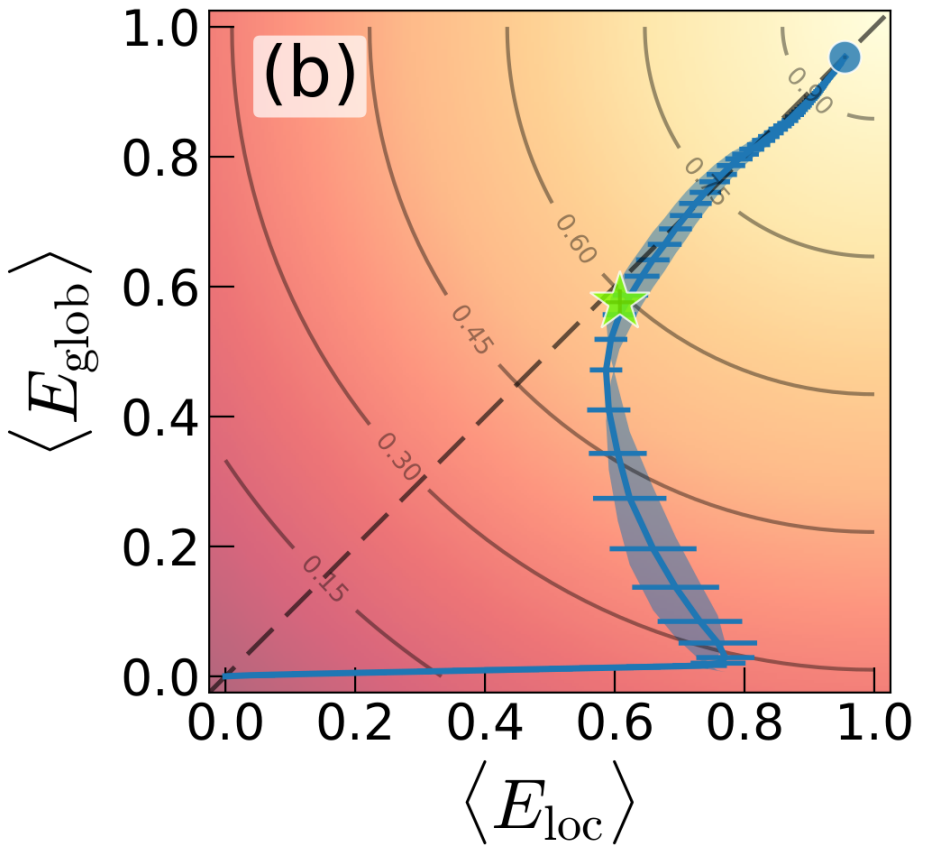
Summing up ...

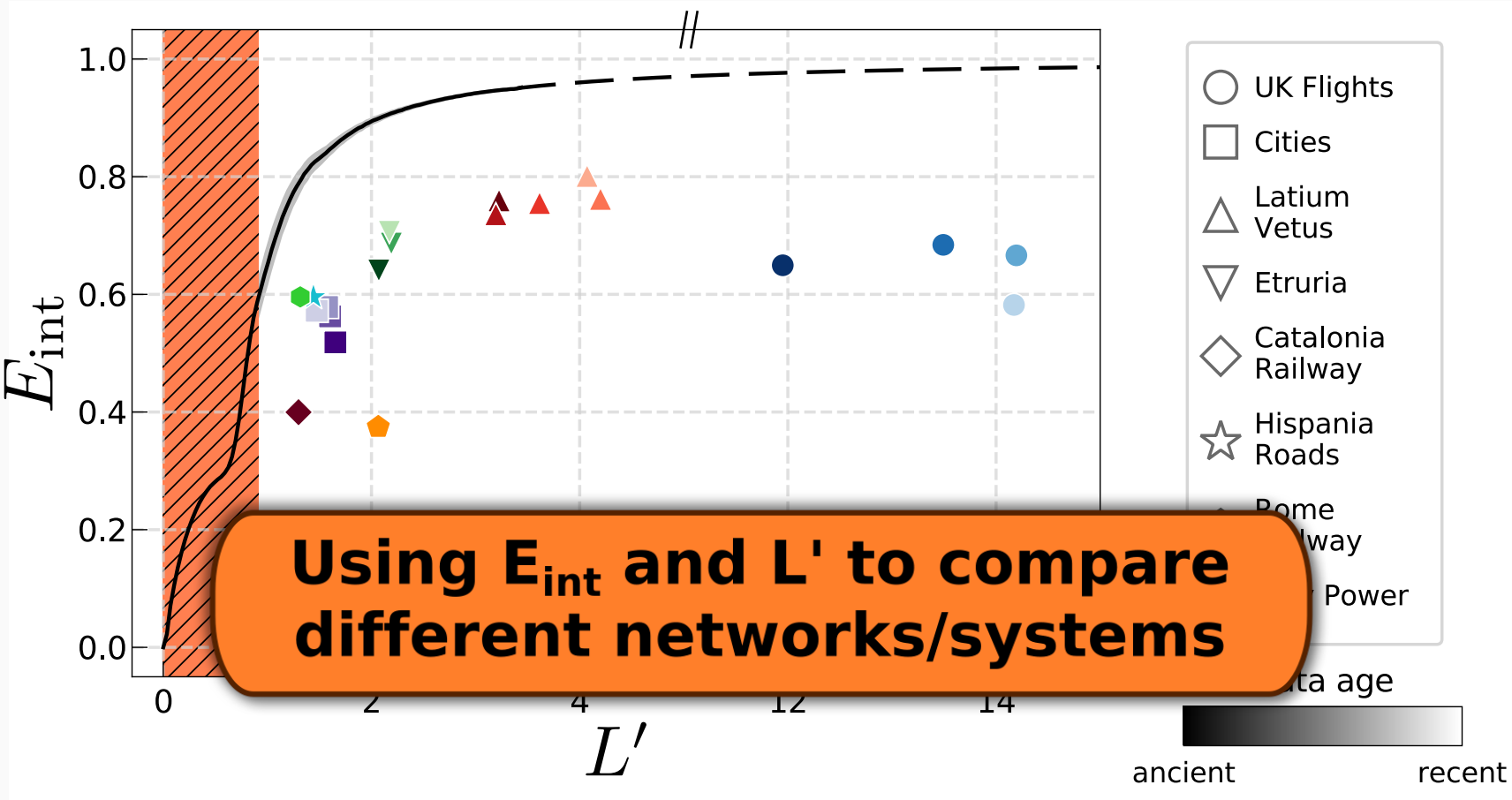


Integrated efficiency as a metric for performances at a global and local scale

A growth model maximizing integrated efficiency to gauge the performance of a network



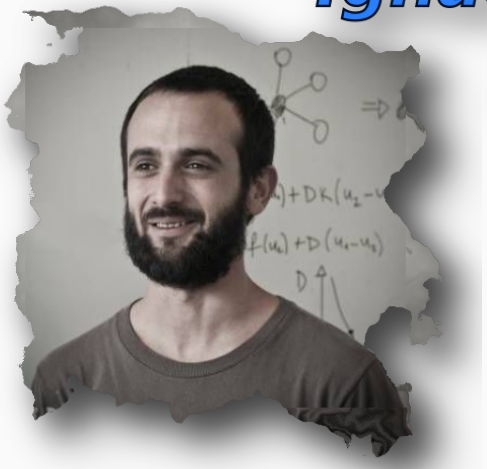




Using E_{int} and L' to compare different networks/systems

Acknowledgements

Ignacio Morer (UB)



Albert Diaz-Guilera (UB)

Luce Prignano (UB)

Sergi Lozano (IPHES - UB)

Acknowledgements

Ignacio Morer et al.

Comparing spatial networks:
A 'one size fits all' efficiency-driven approach

https://arxiv.org/abs/1807.00565

UK Air Data Available!

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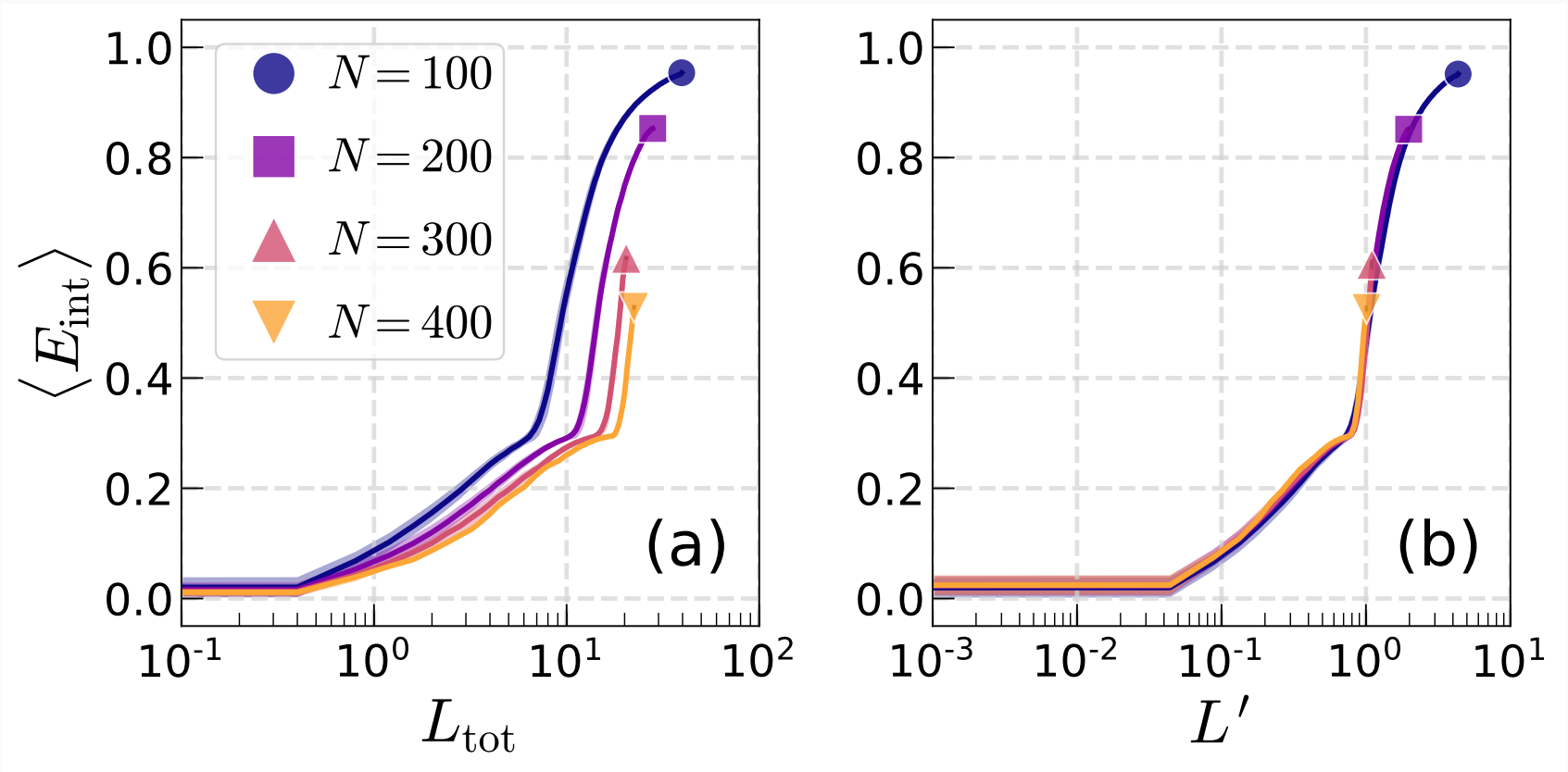
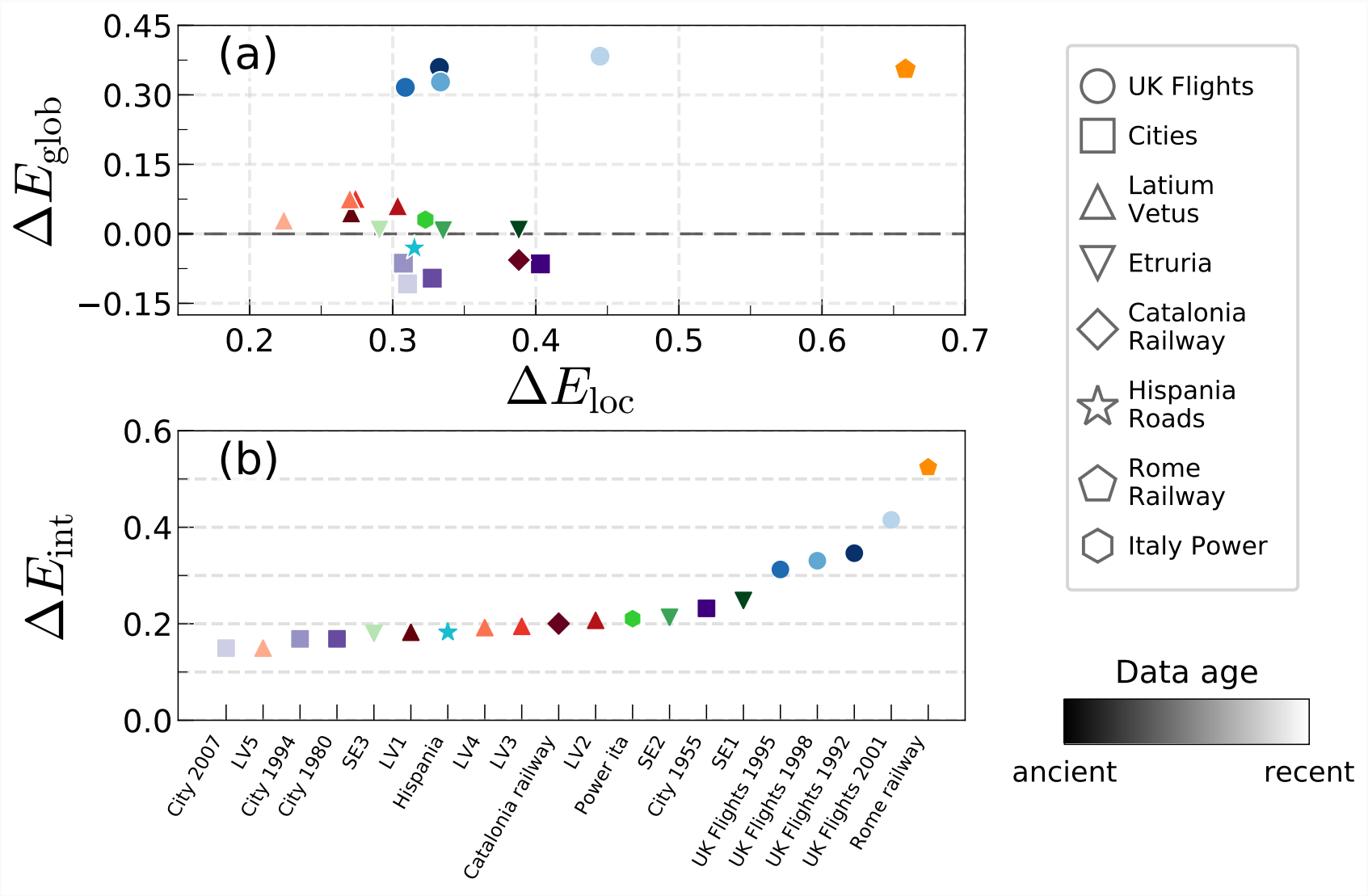
R. Kujala, C. Weckström, R. K. Darst, M. N. Mladenović, and J. Saramäki, A collection of public transport network data sets for 25 cities. Scientific Data, **5**, 180089 (2018). https://doi.org/10.1038/sdata.2018.89

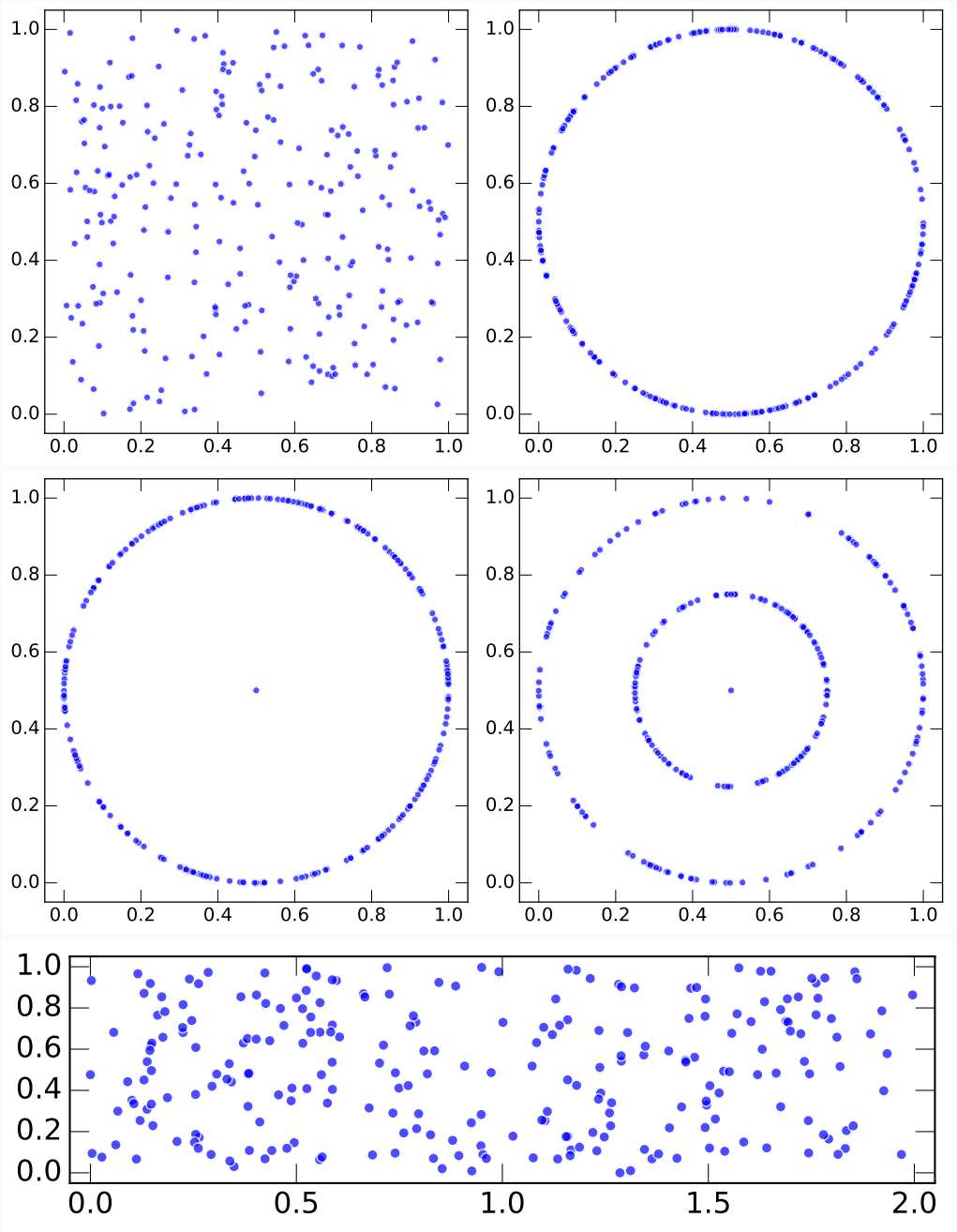

Fulminante F, Prignano L, Morer I and Lozano S 2017 *Front. Dig. Hum.* **4** 1–12 https://doi.org/10.3389/fdigh.20

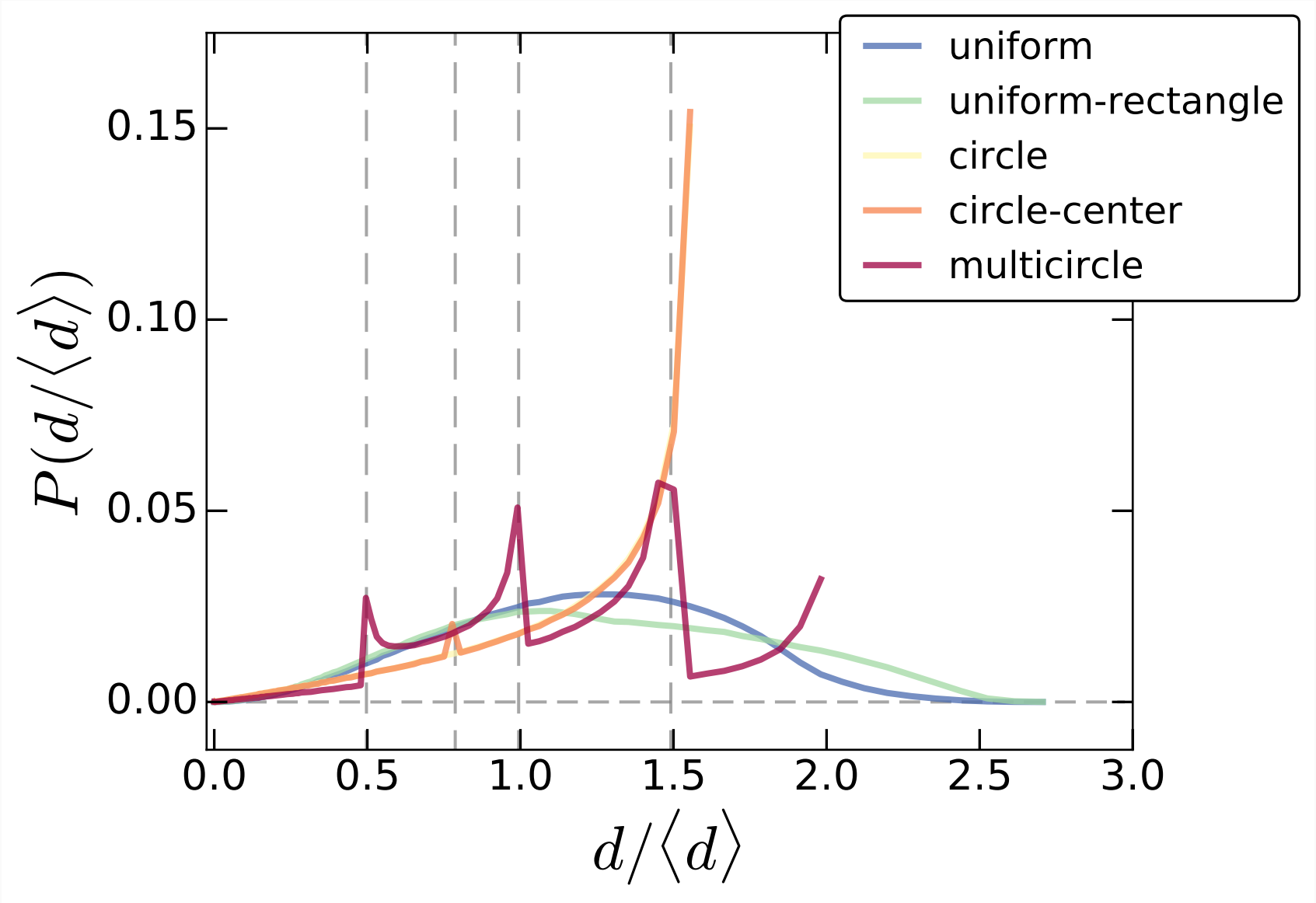

Strano E, Nicosia V, Latora V, Porta S and Barthélemy M 2012 *Sci. Rep.* **2** 1–8 https://doi.org/10.1038/srep00296

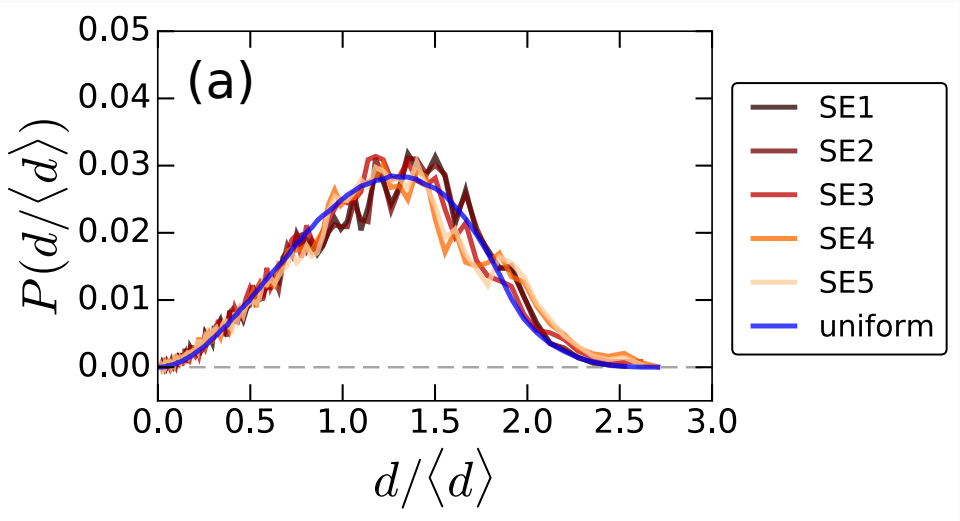
Extra contents

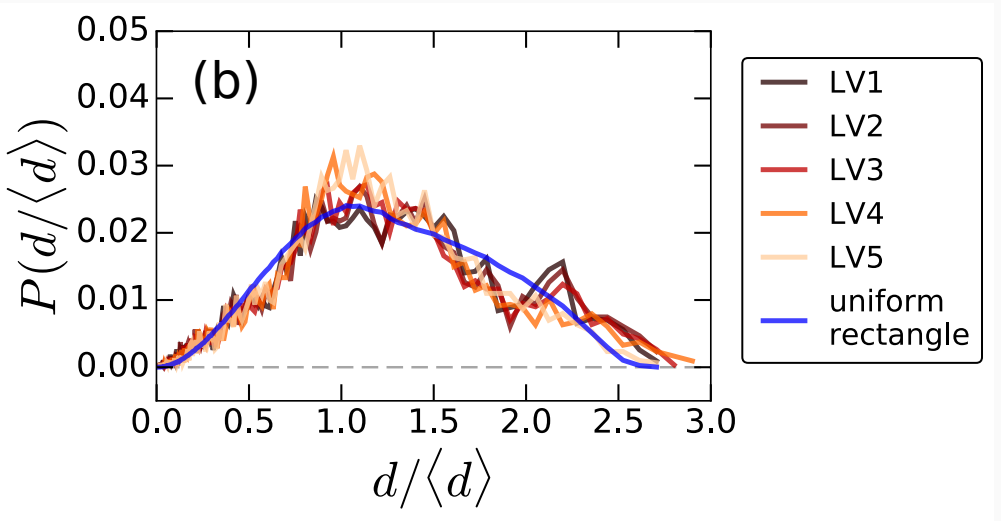

1.0
0.8
0.6
0.4
0.2
0.0
$\langle E_{\rm int} \rangle$
$N = 100$
$N = 200$
$N = 300$
$N = 400$
(a)
10^{-1}
10^{0}
10^{1}
10^{2}
$L_{\rm tot}$
(b)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
L'


(a)
0.45
0.30
0.15
0.00
−0.15
ΔE_{glob}
0.2
0.3
0.4
0.5
0.6
0.7
ΔE_{loc}
(b)
0.6
0.4
0.2
0.0
ΔE_{int}
City 2007
LV5
City 1994
City 1980
SE3
LV1
Hispania
LV4
LV3
Catalonia railway
LV2
Power ita
SE2
City 1955
SE1
UK Flights 1995
UK Flights 1998
UK Flights 1992
UK Flights 2001
Rome railway
UK Flights
Cities
Latium Vetus
Etruria
Catalonia Railway
Hispania Roads
Rome Railway
Italy Power
Data age
ancient
recent


1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
0.5
1.5
2.0


uniform
uniform-rectangle
circle
circle-center
multicircle
0.15
0.10
0.05
0.00
$P(d/\langle d \rangle)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$d/\langle d \rangle$


(a)
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$P(d/\langle d \rangle)$
$d/\langle d \rangle$
SE1
SE2
SE3
SE4
SE5
uniform


(b)
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$P(d/\langle d \rangle)$
$d/\langle d \rangle$
LV1
LV2
LV3
LV4
LV5
uniform
rectangle