

Enhancement of cooperation in systems of moving agents playing public goods games

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Universidad
Zaragoza



Collaborators

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Sandro Meloni: BIFI, Zaragoza, Spain

Do the motion of agents helps the survival of cooperation?

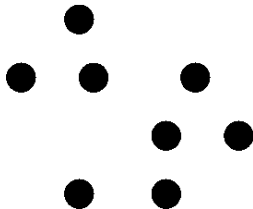
The screenshot shows the Physical Review E journal website. The header includes the journal title 'Physical Review E' with the subtitle 'statistical, nonlinear, and soft matter physics' and the American Physical Society logo. Below the header is a navigation bar with links for Home, Browse, Search, Subscriptions, and Help. A search bar is present with the text 'Citation Search: Phys. Rev. Lett.' and a search button. The main content area displays the article title 'Effects of mobility in a population of prisoner's dilemma players' and its citation 'Phys. Rev. E 79, 067101 (2009) [4 pages]'. There are tabs for 'Abstract', 'References', and 'Citing Articles (12)'. A download link for the PDF is provided. The authors are listed as S. Meloni¹, A. Bucarino^{2,3}, L. Fortunato^{2,3}, M. Prasca^{2,3}, J. Gómez-Gardeñes^{4,5,3}, V. Latora^{6,3}, and Y. Moreno^{5,7}. A sidebar on the right features the 'AMERICAN PHYSICAL SOCIETY'S NEW JOURNAL' logo and the 'PRX' logo with the text 'Physical Review X' and 'Committed to Excellence'.

S. Meloni *et al.* – Phys. Rev. E **79**, 067101 (2009).

C. P. Roca and D. Helbing – Proc. Natl. Acad. Sci. **108**(28) (2011).

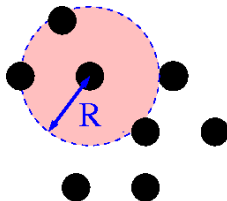
Random Geometric Graph (RGG)

- 1 Scatter N points (with some density ρ) at random on a surface;



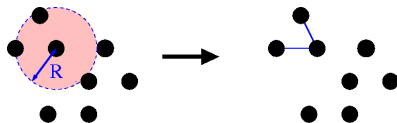
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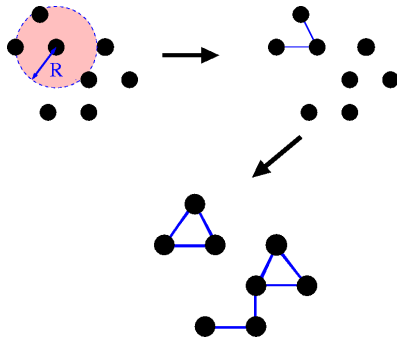
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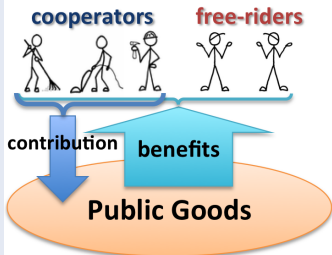
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- 4 Repeat operations 1 and 2 for all the nodes in the network.



Introduction to Public Goods Game (PGG)

The Public Goods Game



Synopsis

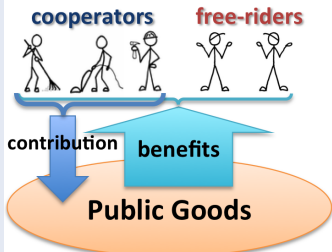
A **GROUP** of individuals have to contribute to the **public goods**:

- **Cooperators** ($x_i = 1$) contribute with an amount c (cost) into the public goods;
- **Free Riders** ($x_i = 0$) do not contribute;
- The total amount collected is amplified by a factor $r \geq 1$ and then equally distributed among all the players.



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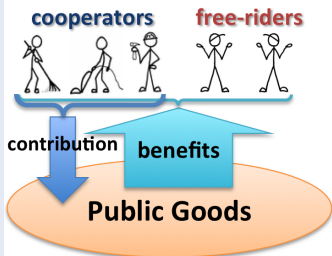
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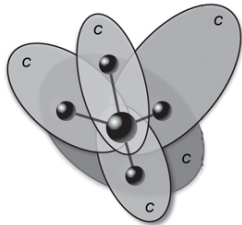
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The **payoff**, of the player i , p_i is equal to:

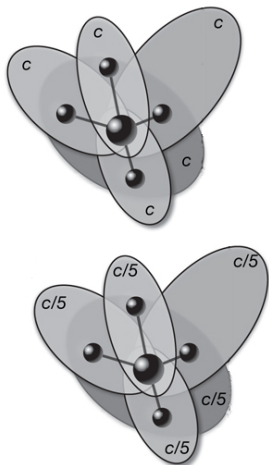
$$\text{ben}_i = \frac{r \left(\sum_{j \in N_i} x_j c + x_i c \right)}{k_i + 1},$$

$$p_i = \begin{cases} \text{ben}_i - c, & \text{if } i \text{ is a cooperator,} \\ \text{ben}_i, & \text{if } i \text{ is a free rider.} \end{cases}$$



- A player (if cooperates) contribute with the same amount c in each game she is involved with. **Fixed cost per interaction** (FCI).

PGG on complex networks \longrightarrow FCI & FCP



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- A player i , (if cooperates) has a finite amount of resources c and contribute with an amount $c_i = \frac{c}{k_i+1}$ in each game she is involved with. **Fixed cost per player** (FCP).

Normalized enhancement factor

Santos *et al.* have introduced a **normalized enhancement factor** η given by:

$$\eta = \frac{r}{\langle k \rangle + 1} \quad \rightarrow \quad \frac{r}{(\rho\pi R^2) + 1};$$

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Evolutionary rule

Players changes their strategies according to the **Fermi rule** given by:

$$P_{i \rightarrow j} = \frac{1}{1 + e^{-\beta(p_i - p_j)}};$$

Game: Public goods game played under FCI or FCP approach;

Update Rule: Fermi rule;

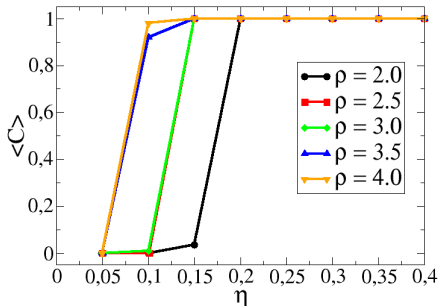
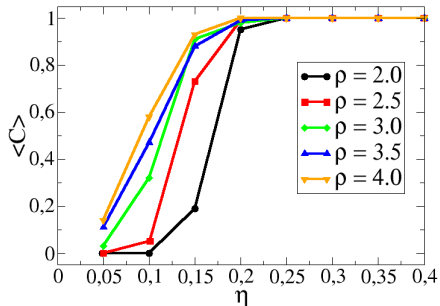
Topology: RGG built with density ρ and radius R and periodic boundaries;

Other information:

- Initial fraction of cooperators and defectors $C(0) = D(0) = 0.5$;
- Normalized enhancement factor $\eta \in [0.05, 1.2]$;
- Payoff does not accumulate through gaming and the update of the strategies is synchronous.
- Dynamic evolution of the system up to 50000 game rounds;
- All simulations averaged over 50 different realizations for each set of parameters;

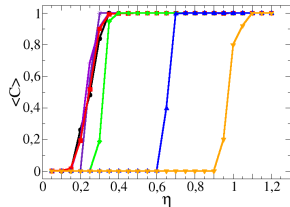


Results 1/3 – Static case

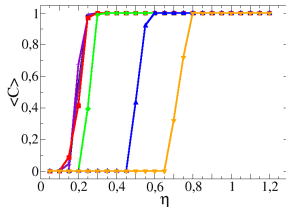


Results 2/3 – Dynamic scenario: effect of density

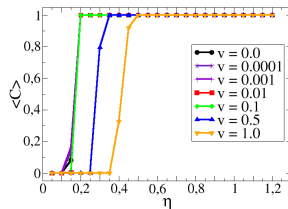
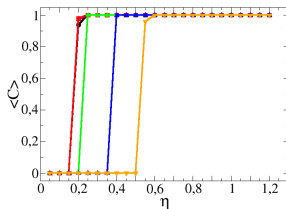
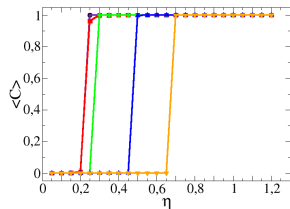
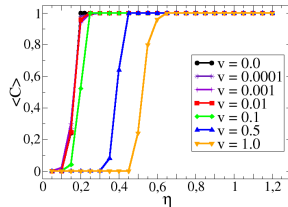
$\rho = 1.3$



$\rho = 1.6$

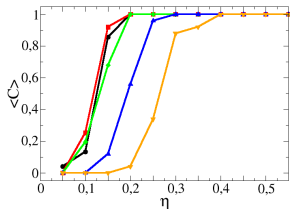


$\rho = 2.0$

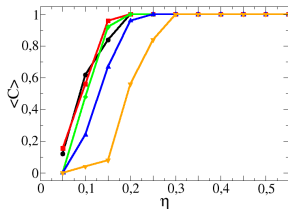


Results 3/3 – Dynamic scenario: effect of the radius

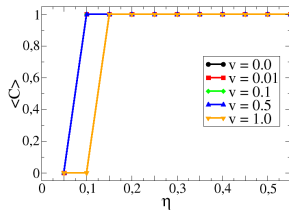
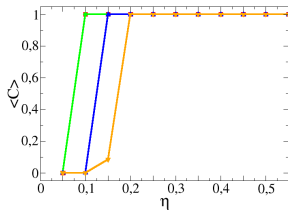
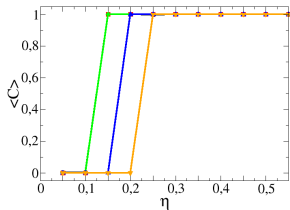
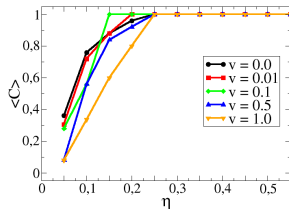
$R = 1.5$



$R = 1.75$



$R = 2.0$



Conclusions

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- Include some kind of adaptive mechanism in the motion of the agents to correlate it with their strategy/payoff.

