d-Covering on Weighted Complex Networks

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1 Introduction

The search for an efficient allocation of resources in complex networks is a topical issue concerning communication, trasportation and social systems. Most of the proposed protocols are based on the degree distribution of the graph. However, the complex networks under study are better represented as weighted graphs. We report here on a heuristic method to find near-optimal solutions to the covering problem in real communication networks, taking into account links weights. We analyse the distribution of covers over the network when different protocols are used. Besides, a SIR dynamics is studied in order to highlight the performance of the proposed covering algorithm when covers are seen as immune nodes to the disease.

2 Topological properties of networks

To evaluate the effectiveness of our method we tested it on three different weighted networks. These networks are:

- a Peer to Peer connections network ;
- the US Airports passengers network ;
- the Los Alamos `hep-lat` Scientific Collaboration Network ;

From now on we will refers at them as *Net1*, *Net2* and *Net3*.

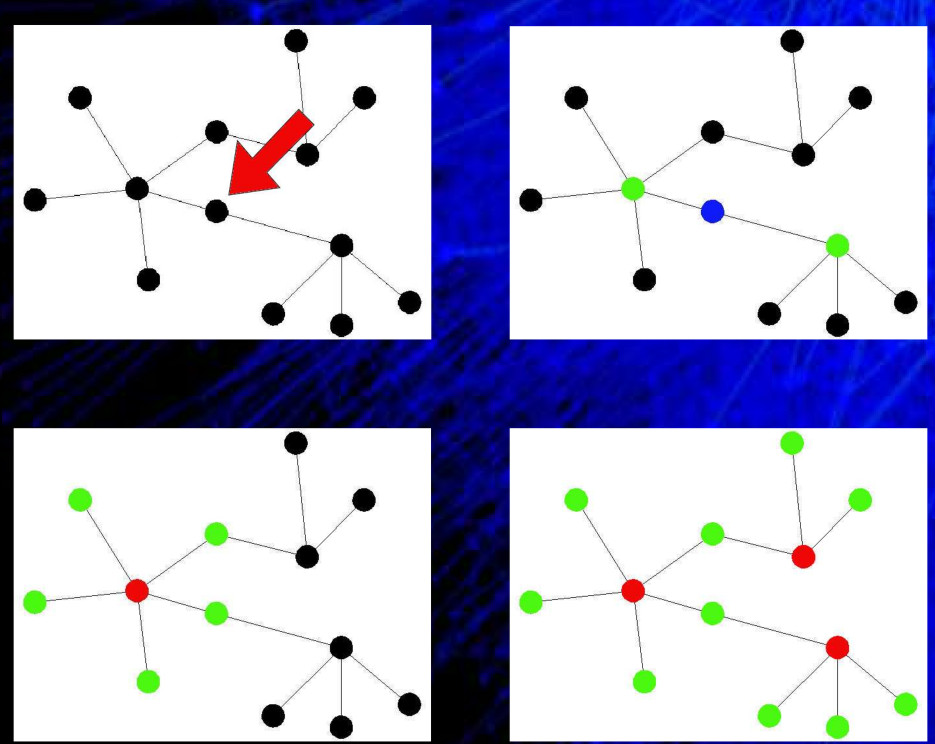
	Net 1	Net 2	Net 3
N	280	500	589
K	922	2980	2718
$\langle k \rangle$	6.586	11.920	9.229
k_{max}	60	145	54
w_{max}	157	2253992	15
s_{max}	617	49316361	130
$\overline{w}_{max}$	19.00	440473.75	7.00
$\langle L \rangle$	4.6963	48673.4012	6.7514
D	39	332141	21

Topological properties of all the studied networks. In order we find: number of nodes N, number of links K, mean degree $\langle k \rangle$, maximum: degree, weight and strenght k_{max}, w_{max}, s_{max}, maximum mean weight per node $\overline{w}_{max}$, average path length $\langle L \rangle$ and diameter D.

3 d-Covering

To optimize the resources needed to give a minimum set of immune nodes following the idea of Echenique et.al. [1] we propose an implementation of the *d-covering* algorithm [1]. The algorithm acts as follows:

1. We select at random a node i and we consider all its d-neighbors (neighbors at distance d);
2. we select the one with the highest value of a certain quantity (i.e. degree, strenght, betweenness) and *cover* it ;
3. now we consider the d-neighbors of the cover node and consider them as *covered* ;
4. repeat all the above operations for all the uncovered nodes until all the nodes in the network are covered ;



We used two different quantities to determine the covering status of the nodes: degree and strenght. This is to emphasize the relevance of link weights in the infection spreading and in the immunization policies.

3.1 Unweighted Covering

If we decide to ignore the "interaction" between nodes considering only the topology of the network, a good measure to perform covering is the degree.

$$k_i = \sum_{j=1}^{\mathcal{N}} a_{ij} \; ; \qquad (1)$$

Where a_{ij} is the adjacency matrix. In this way we decide to immunize those nodes which are connected with the highest number of nodes possible (according to the algorithm procedures).

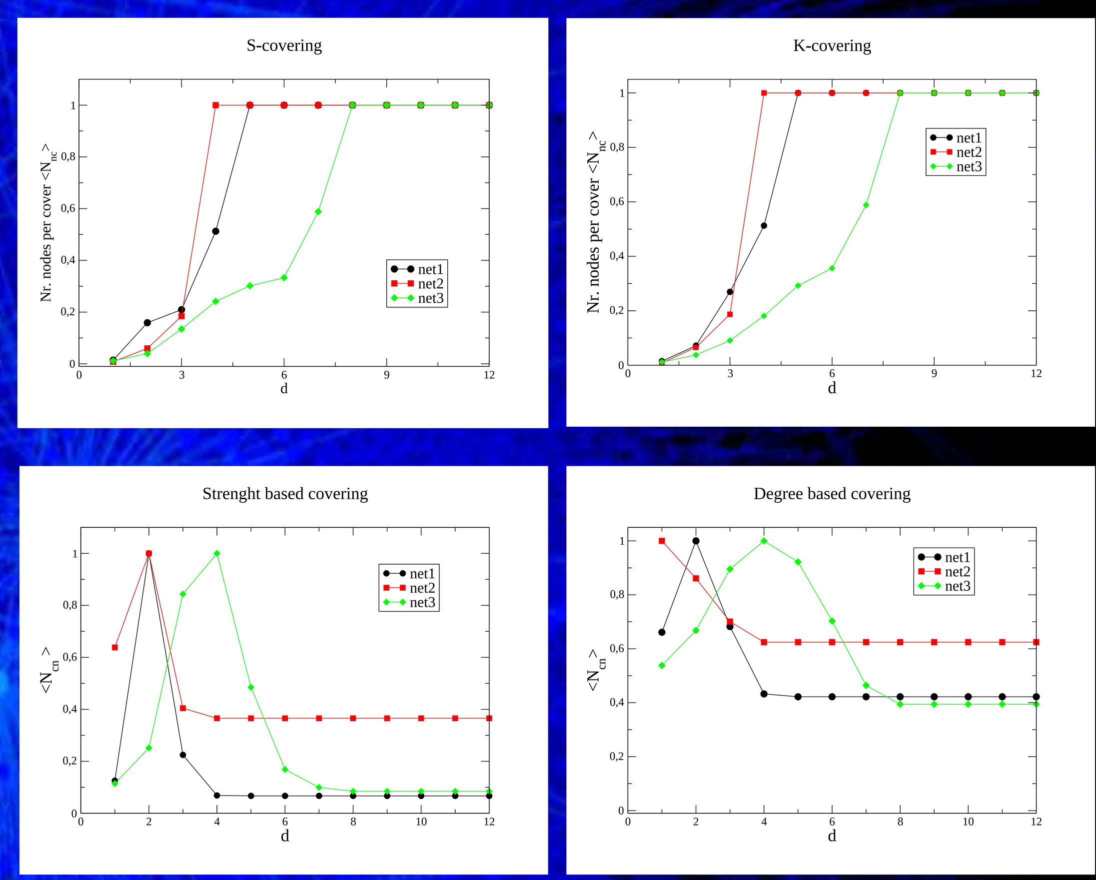
3.2 Weighted Covering

Since almost all real networks are weighted we decided to test the method on a weighted network. These weights must be taken into account when the covering algorithm is performed on such networks. This lead to the choice of strenght as parameter to be used during covering. The strenght of node i is defined as:

$$s_i = \sum_{j=1}^{\mathcal{N}} a_{ij}\, w_{ij} \; ; \qquad (2)$$

Where a_{ij} is the adjacency matrix and w_{ij} is the weight of the link $i - j$.

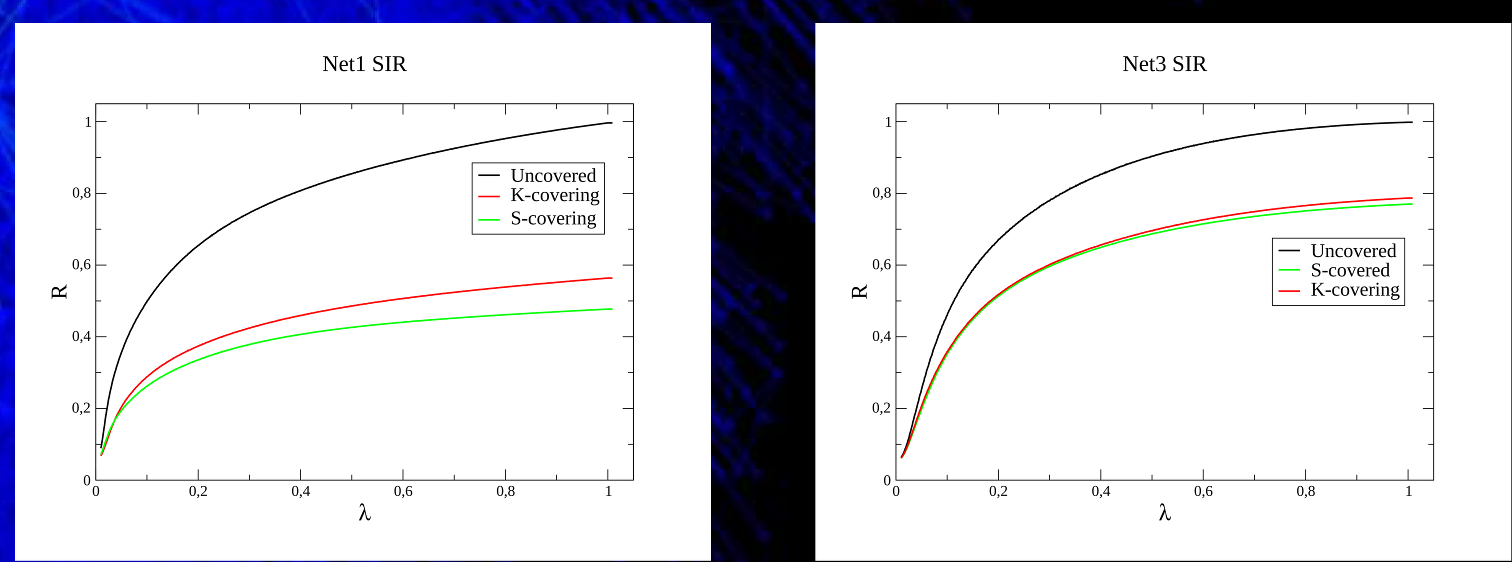
Fraction of nodes covered by a cover as a function of d shows a smooth transition from 0 to 1. Fraction of covers covering a node as a function of d shows an optimal cover set for d 1.



4 SIR model

In order to compare the efficiency of the different immunization strategies, we first perform extensive numerical simulations of an epidemic spreading process on top of Net1 and Net3 [2]. We consider the $\mathcal{SIR}$ *model* as a plausible model for epidemic spreading. In this model, nodes can be in three different states. Susceptible nodes, have not been infected and are healthy. They catch the disease via direct contact with Infected nodes at a rate λ. Finally, those nodes that have caught the disease are Removed with probability β. The relevant order parameter of the disease dynamics is the fraction of nodes R that got infected once the epidemic process dies out, i.e., when no infected nodes are left in the system. In the general case of weighted networks, the number of contacts per unit time between two neighbours is proportional to the weight of the link between them.

We compare the weighted and unweighted d-covering. The figure below show the fraction of removed nodes in the cases of unimmunized, and d = 1 weighted and unweighted immunization. We obtain that in both cases weighted covering outperforms the unweighted one. This confirms our hypotesis: weights must be taken into account during immunization.



5 Conclusion

In Conclusion the weighted d-covering ensures a better performance in terms of immunization when tested using SIR model on weighted nets.

References

[1] P. Echenique , J. Gómez-Gardeñes , Y. Moreno and A. Vàsquez , Phys. Rev. E **71** 035102(R) (2005).

[2] J. Gómez-Gardeñes , P. Echenique and Y. Moreno, Eur. Phys. J. B **49** 259-264 (2006).