Structural properties of urban street patterns, discovering the backbone of a city

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1 Introduction

Urban street patterns are a particular class of spatial networks. In order to compare different cities properties one has to find proper bounds to normalize the results. The main problem with spatial graphs is that, in most of the cases, the random graph and the complete graph are no more a good way to normalize the results. To overcome this problem, we consider both minimum spanning trees and greedy triangulations induced by the real distribution of nodes [1].

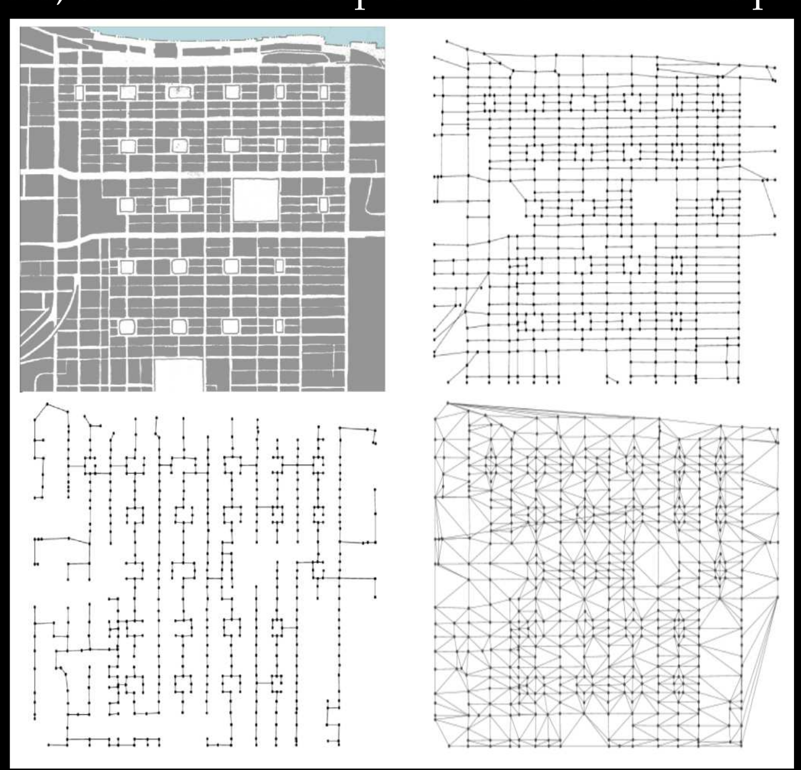
In urban design, a long-term effort has been spent in order to understand what streets and routes would constitute the so-called *"backbone of a city"* [2, 3]. Here we provide a tool to find out the backbone of networks of urban street patterns [4]. Such a tool is based on the concept of spanning trees, and on the capability of centrality measures to uncover the essential streets of a city. We compare the obtained trees with the standard spanning trees based on minimizing the total lengths.

2 Minimum Spanning Tree and Greedy Triangulation

In order to compare the different cities we use two particular graphs namely:

- **Minimum Spanning Tree** (MST) is the shortest tree which connects every node into a single connected component.
- **Greedy Triangulation** (GT) is the planar graph with the highest number of edges $K_{\max}$, and that minimize the total length.

For each of the twenty cities we have constructed the respective MST and GT. These two bounds make sense also as regards to the possible evolution of a city: the most primitive forms are close to trees, while more complex forms involve the presence of cycles.



The urban pattern of Savannah as it appears in the original map (top left), and reduced into a spatial graph (top right). We also report the corresponding MST (bottom left) and GT (bottom right).

3 Topological properties of urban street patterns networks

Networks considered here consist of twenty 1-square-mile samples of different world cities [1]. These cities differ in terms of cultural, social, economic, religious and geographic contexts. In particular, they can be roughly divided into two large classes: (1) patterns grown throughout a self-organized, fine-grained historical processe; (2) patterns realized over a short period of time as the result of a single plan (usually exhibiting a regular structure).

Those differences come out from the basic properties of the corresponding graphs. For example, the cost (defined as the sum of street lengths) assume widely different values, notwithstanding the fact that we have considered the same amount of land. The properties studied are listed in the table below.

City	N	K	M	W	$\langle l \rangle$	E	D_{box}
Ahmedabad	2870	4387	0.262	121037	27.59	0.818	1.92
Bologna	541	773	0.214	51219	66.26	0.799	1.95
Cairo	1496	2255	0.253	84395	37.47	0.809	1.82
London	488	730	0.249	52800	72.33	0.803	1.94
New Delhi	252	334	0.154	32281	96.56	0.766	1.85
Venice	1840	2407	0.152	75219	31.25	0.673	1.81
Vienna	467	692	0.242	49935	72.16	0.811	1.88
Washington	192	303	0.293	36342	119.94	0.837	1.93
Paris	335	494	0.241	44109	89.29	0.838	1.88
Seoul	869	1307	0.253	68121	52.12	0.814	1.87
Barcelona	210	323	0.275	36179	112.01	0.814	1.99
Brasilia	179	230	0.147	30910	134.39	0.695	1.83
Irvine 1	32	36	0.085	11234	312.07	0.755	–
Irvine 2	217	227	0.014	28473	128.26	0.374	1.81
Los Angeles	240	340	0.211	38716	113.87	0.782	1.90
New York	248	419	0.348	36172	86.33	0.835	1.72
Richmond	697	1086	0.279	62608	57.65	0.800	1.78
Savannah	584	958	0.322	62050	64.77	0.793	1.85
San Francisco	169	271	0.309	38187	140.91	0.792	1.90
Walnut Creek	169	197	0.084	25131	127.57	0.688	1.80

Topological properties of all the studied city networks. We find: number of nodes N, number of links K, the meshedness coefficient M, the total cost W, average edge length (in meters) $\langle l \rangle$, global efficiency E and fractal box-counting dimension D_{box}.

3.1 Local Properties

Buhl et.al. [5] have proposed a more general measure to characterize the local structure in planar graphs, the so called *meshedness coefficient* M defined as:

$$M = \frac{f}{f_{max}} \qquad (1)$$

where f is the number of faces (excluding the external ones) associated with a graph with N nodes and K edges, expressed by the Euler formula as: $f = K - N + 1$. f_{max} is the maximum possible number of faces obtained from the maximally connected planar graph, i.e. in a graph with N nodes and $K_{max} = 3N - 6$ edges. Thus $f_{max} = 2N - 5$. The meshedness coefficient can vary from zero for a tree, to one for the maximally connected planar graph, as in the GT (we will discuss below).

3.2 Global Properties

The global structural properties of the graphs have been evaluated analyzing the so called *global efficiency* E, a measure defined as:

$$E = \frac{1}{N(N-1)} \sum_{i,j,i \neq j} \frac{d_{ij}^{\text{Eucl}}}{d_{ij}}. \qquad (2)$$

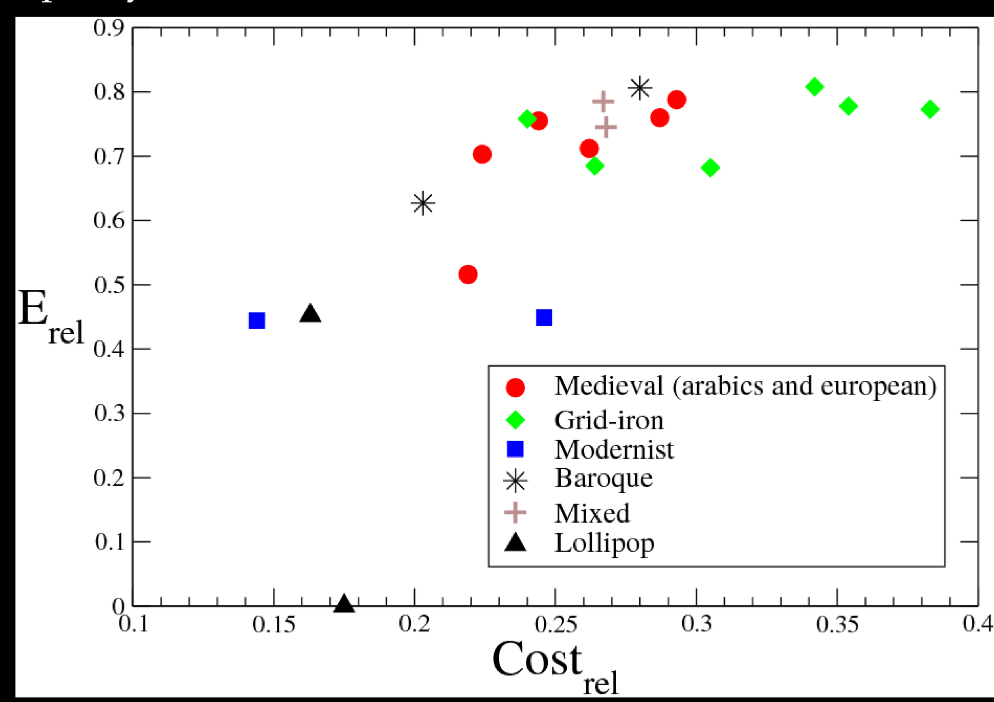
Where d_{ij} and d_{ij}^{Eucl} are the shortest path length and the Euclidean distance between node i and j respectively.

Of course, the counterpart of an increase in efficiency is an increase in the cost of construction i.e., an increase in the number and length of the edges. The cost of construction can be qualified by using the measure W (i.e. the total street length). To enable comparisons, we thus define a normalized efficiency and cost measures E_{rel} and W_{rel} as:

$$E_{rel} = \frac{E - E^{MST}}{E^{GT} - E^{MST}}; \qquad W_{rel} = \frac{W - W^{MST}}{W^{GT} - W^{MST}}; \qquad (3)$$

The plot E_{rel} vs W_{rel} has a certain capacity to characterize the different classes of cities.

Relative efficiency E_{rel} as a function of relative cost W_{rel} of cities divided into *classes* (medieval, grid-iron, etc.). The point of coordinate (0,0) would correspond to MST while the point (1,1) would correspond to the GT.

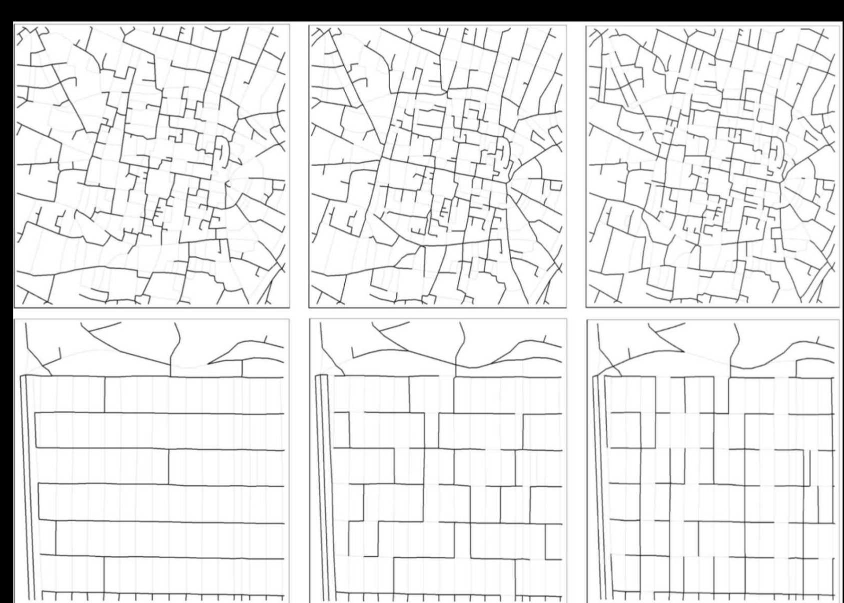


4 The Backbone of a City

Centrality is a fundamental concept in network analysis. Here we show how to construct spanning trees based on edge centrality to extract the skeleton of a city. For each city we have considered two centrality measures (Of course other definitions of centrality can be used as well) the *betweenness* and the *information*. The edge betweenness centrality, C^B, is based on the idea that an edge is central if it is included in many of the shortest paths connecting couples of nodes. Likewise, the edge information centrality, C^I, is a measure relating the edge importance to the ability of the network to respond to the deactivation of the edge itself. Betweennes and information of an edge α are defined as:

$$C^B_\alpha = \frac{1}{(N-1)(N-2)} \sum_{j,k \in G} \frac{n_{jk}(\alpha)}{n_{jk}}, \qquad (4) \qquad\qquad C^I_\alpha = \frac{\Delta E}{E} = \frac{E(G) - E(G')}{E(G)}; \qquad (5)$$

where G is the considered graph, n_{jk} is the number of shortest paths between nodes j and k, $n_{jk}(\alpha)$ is the number of shortest paths between nodes j and k that contain edge α, and G' is the graph obtained by removing edge α. We are now ready to build the **Maximum Centrality Spanning Trees** (MCSTs), i.e. maximum weight spanning trees where the edge weight is set equal to the centrality of that edge.



Spanning trees of Bologna (above) and San Francisco (below). From left to right, mLSTs (minimum spanning tree based on street length), betweenness-based and information-based MCSTs.

	BOL		SAF	
	Bet	Info	Bet	Info
% common links with mLST	82	75	70	76
% of total centrality in MCST	86	84	82	95

5 Conclusions

We have proposed a method to characterize both the local and the global properties of spatial graphs representing urban street patterns and to extract the skeleton of the network. The use of MST and GT gives the opportunity to compare different cities while the concept of MCST leads to a meaningful picture of the primary sub-system of a city network.

References

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