Evolutionary Game Theory on networks

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Outline

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1. Introduction
2. Basics of game theory
3. Basics of evolutionary dynamics
    - Constant selection
    - Frequency dependent selection

Outline

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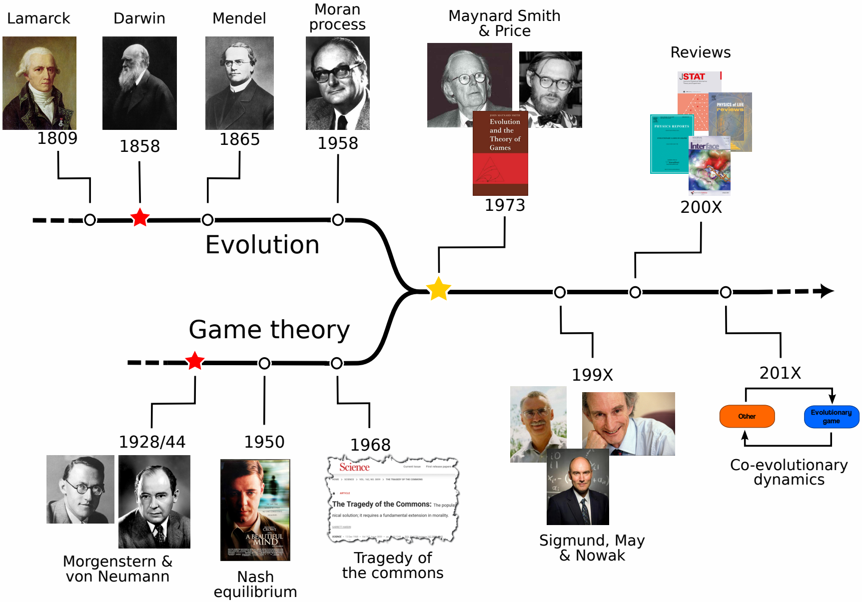
Introduction

A bit of history ...



A bit of history ...

- 11 Nobel prizes
- 1 Blockbuster movie
- 1 Viral YouTube video (at least)

- https://www.nobelprize.org/prizes/lists/all-prizes-in-economic-sciences
- https://www.imdb.com/title/tt0268978
- https://www.youtube.com/watch?v=S0qjK3TWZE8

A bit of history ...

Why “individuals” are willing to pay some **cost** to provide **benefits** for themselves and **others**?

Game Theory

Basics of game theory

Definition (pairwise-game):

1. The state of a player corresponds to its **strategy**

- Gintis, H. (2009). Princeton University Press.

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Payoff matrix

$$
\begin{array}{c} \\ S_1 \\ S_2 \\ \vdots \\ S_m \end{array}
\begin{array}{c}
\begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\
\begin{pmatrix}
\pi_{S_1,S_1} & \pi_{S_1,S_2} & \cdots & \pi_{S_1,S_m} \\
\pi_{S_2,S_1} & \pi_{S_2,S_2} & \cdots & \pi_{S_2,S_m} \\
\vdots & \vdots & \ddots & \vdots \\
\pi_{S_m,S_1} & \pi_{S_m,S_2} & \cdots & \pi_{S_m,S_m}
\end{pmatrix}
\end{array}
$$

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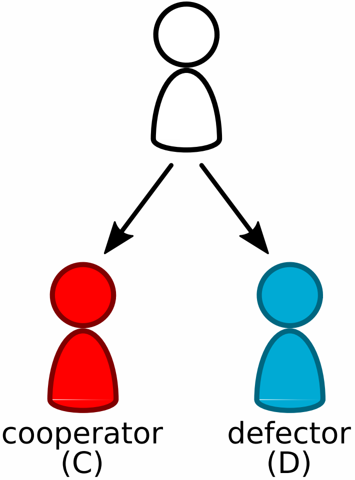
Basics of game theory

An example:

- Two strategies: cooperation (C) and defection (D).



Basics of game theory

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- Four possible strategies' combinations: (C, C), (C, D), (D, C), and (D, D).

Basics of game theory

An example:

- Two strategies: cooperation (C) and defection (D).
- Four possible strategies' combinations: (C, C), (C, D), (D, C), and (D, D).
- We get a 2×2 payoff matrix:

  Reward $R \longrightarrow (C, C)$.
  Sucker $S \longrightarrow (C, D)$.
  Temptation $T \longrightarrow (D, C)$.
  Punishment $P \longrightarrow (D, D)$.

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}$$

Basics of game theory

Basics of game theory

Definition of Nash equilibrium

Given a game played by N players, a **set of strategies** $\mathcal{S}^\star \equiv \{S_1, S_2, \dots, S_N\}$ is a Nash equilibrium if no player, $i \in \{1, \dots, N\}$, can do unilaterally better by changing its strategy, S_i.

- Nash, J. F. Proc. Natl. Aca. Sci. USA, **36**, 48–49 (1950).
- Gintis, H. (2009). Princeton University Press.

Basics of game theory

An example: The Prisoner's dilemma

- Two robbers are arrested after a bank robbery and held separately by the police. However, the police **does not have enough evidences** to have them convicted.

- Szabó, G., and Fáth, G. Phys. Rep., **446**, 97 (2007).

Basics of game theory

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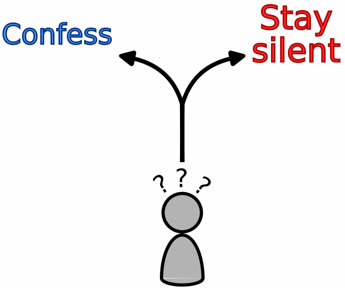
- The prosecutor **offers to each robber the same deal**: he can confess (*i.e.*, defect) and get a discount on the sentence, or remain silent (*i.e.*, cooperate with the other prisoner) and get no discount (but a shorter jail time).



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- The prosecutor **offers to each robber the same deal**: he can confess (*i.e.*, defect) and get a discount on the sentence, or remain silent (*i.e.*, cooperate with the other prisoner) and get no discount (but a shorter jail time).
- The payoff matrix (jail's years) is:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}$$

For example:

$$\begin{pmatrix} -1 & -10 \\ 0 & -7 \end{pmatrix}$$

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Basics of game theory

$$
\begin{array}{c c}
 & \begin{array}{cc} C & D \end{array} \\
\begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \color{blue}{-1} & -10 \\ 0 & \color{red}{-7} \end{pmatrix}
\end{array}
$$

the dilemma

Although the optimal choice would be for both players to **cooperate**, assuming that both players will try to maximize their own payoff, the Nash equilibrium tells us that it best to **defect** regardless of what the other player will do.

Basics of game theory

Question:

Given the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 10 & 0 \\ 7 & 5 \end{pmatrix} \end{array}$$

What is the Nash equilibrium?

Basics of game theory

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What is the Nash equilibrium?

Answer

Both (C, C) and (D, D) are Nash equilibria, albeit the latter is a strict one.

Evolutionary Dynamics

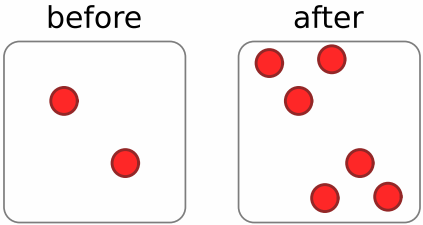
Basics of evolutionary dynamics

Foreword:

Evolutionary theory stands on three pillars:

- Nowak, M. A. (2007). *Evolutionary dynamics: exploring the equations of life*. Belknap Press.

Basics of evolutionary dynamics



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Evolutionary theory stands on three pillars:

Replication The ability of an organism to reproduce.

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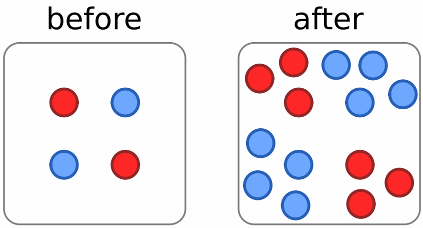

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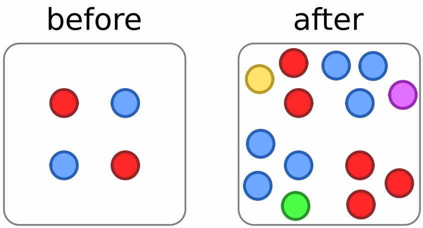

Foreword:

Evolutionary theory stands on three pillars:

Replication The ability of an organism to reproduce.

Selection The ability of a species to replicate faster than another.

Mutation The ability of creating new species from the existing ones.

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Basics of evolutionary dynamics

Replication

Suppose to have a fraction x_0 of individuals of species X reproducing with **rate** r and study the evolution of the fraction of agents of species X over (continuous) time.

Basics of evolutionary dynamics

Logistic equation

$$\frac{dx}{dt} = \dot{x} = rx\left(1 - \frac{x}{\kappa}\right)$$

- x density of individuals of species X, $x \in [0, 1]$.
- r reproduction rate $r \in [0, \infty[$.
- κ Carrying capacity $\kappa \in [0, 1]$.

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Note

In discrete time the logistic equation is equivalent to the so-called **logistic map** (May, 1976):

$$x_{t+1} = rx_t\left(1 - x_t\right)$$

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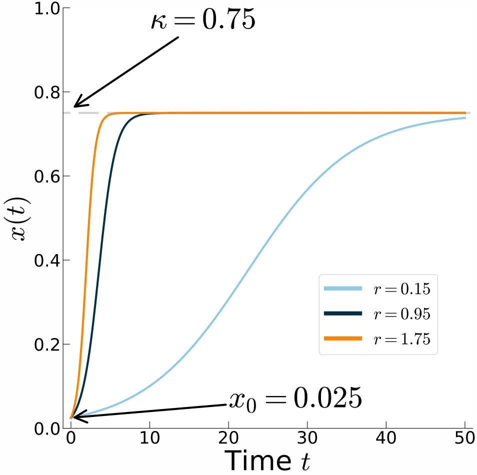
Basics of evolutionary dynamics

$$x(t) = \frac{\kappa x_0 \, e^{rt}}{\kappa + x_0 \, \left(e^{rt} - 1\right)}$$



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Basics of evolutionary dynamics

Selection

Suppose to have an infinite population of individuals of **two species**: A and B. Each species reproduces with rate r_A and r_B, respectively. The fractions (*i.e.*, relative abundances or densities) of individuals of species A is x and of species B, y, instead. The sum of densities is constant (*i.e.*, $x + y = 1$).

Basics of evolutionary dynamics

$$\begin{cases} \dot{x} &= x\,(r_A - \varphi) \\ \dot{y} &= y\,(r_B - \varphi) \end{cases}$$

where

$$\varphi = r_A x + r_B y$$

$\{x, y\}$ Species' densities
$x, y \in [0, 1]$
$x(t) + y(t) = 1 \ \forall t.$

$\{r_A, r_B\}$ Species' reproduction rates .

φ Average fitness of the whole population.

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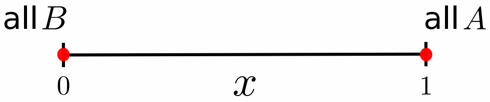
Then

$$\begin{aligned} \dot{x} &= x\left[r_A - r_A x - r_B\left(1 - x\right)\right] \\ &= x\left[r_A\left(1 - x\right) - r_B\left(1 - x\right)\right] \\ &= x\left(1 - x\right)\left(r_A - r_B\right) \ . \end{aligned}$$

Basics of evolutionary dynamics

Fixed points

$$\dot{x} = 0 \Leftrightarrow \begin{cases} x^\star = 0 \text{ (all } B) \\ x^\star = 1 \text{ (all } A) \end{cases}$$



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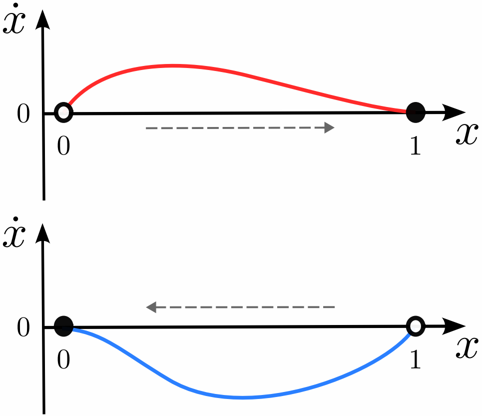
Basics of evolutionary dynamics

Solutions' stability

$$\dot{x} = \underbrace{x}_{\geq 0}\underbrace{(1-x)}_{\geq 0}\underbrace{(r_A - r_B)}_{\lessgtr 0\ ?}$$

$$(r_A - r_B)\begin{cases} > 0 \Leftrightarrow r_A > r_B \\ < 0 \Leftrightarrow r_A < r_B \end{cases}$$



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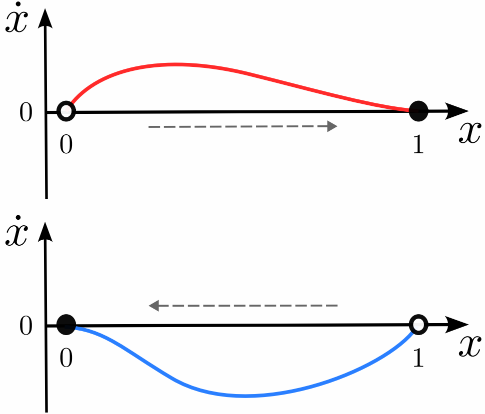

Note

The stability of the fixed points **does not depend** on the composition of the population (*i.e.*, x and y).

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Evolutionary game theory

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Game theory is not enough because

- Players do not play **only once**.

Evolutionary game theory

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- Players are neither **smart** (*i.e.*, they do not know how to compute the Nash equilibrium) nor **fully rational** (*i.e.*, they do not act always to maximize their payoff).

Evolutionary game theory

Game theory is not enough because

- Players do not play **only once.**
- Players are neither **smart** (*i.e.*, they do not know how to compute the Nash equilibrium) nor **fully rational** (*i.e.*, they do not act always to maximize their payoff).
- Players do not have **full knowledge** (*i.e.*, they know all the entries of the payoff matrix) and tend to **learn** by adopting a strategy ensuring them the best success in the next round.

Evolutionary game theory

Solution

- Players **interact via a game** and play it **multiple times.**
- **Payoff translates into fitness** and **success in the game translates into reproductive success.**
- The reproduction rate depends on the density of agents (*i.e.*, **frequency dependent selection**).


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Published: 02 November 1973

The Logic of Animal Conflict

J. MAYNARD SMITH & G. R. PRICE

Nature **246**, 15–18 (1973) | Cite this article

14k Accesses | **3637** Citations | **82** Altmetric | Metrics

- Maynard Smith, J., Price, G. Nature **246**, 15–18 (1973).

Evolutionary game theory

Preamble

Let us consider an infinite population of individuals of species A and B, whose relative abundances are x and y.

Moreover $x + y = 1$.

Let us denote the **fitness** of species A with $f_A(x, y)$ and of species B as $f_B(x, y)$, respectively.

Evolutionary game theory

Preamble

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Let us denote the **fitness** of species A with $f_A(x, y)$ and of species B as $f_B(x, y)$, respectively.

$$\begin{cases} \dot{x} &= x\left(f_A(x,y) - \varphi\right) \\ \dot{y} &= y\left(f_B(x,y) - \varphi\right) \end{cases}$$

where

$$\varphi = x\, f_A(x,y) + y\, f_B(x,y)$$

Note

The above equation is known as the **replicator equation**.

Evolutionary game theory

$$\begin{cases} \dot{x} & = x\big(f_A(x,y) - \varphi\big) \\ \dot{y} & = y\big(f_B(x,y) - \varphi\big) \end{cases}$$

Evolutionary game theory

$$\begin{cases} \dot{x} & = x\big(f_A(x,y) - \varphi\big) \\ \dot{y} & = y\big(f_B(x,y) - \varphi\big) \end{cases}$$

As $x + y = 1$, we can write

$$\varphi = x\, f_A(x,y) + y\, f_B(x,y) = x\, f_A(x) + (1-x)\, f_B(x)$$

Evolutionary game theory

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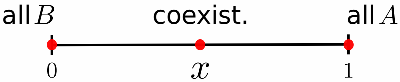
Then

$$\begin{aligned} \dot{x} &= x\Big[f_A(x) - x\, f_A(x) - (1-x)\, f_B(x)\Big] \\ &= x\,\Big[\Big(1-x\Big)\; f_A(x) - (1-x)\; f_B(x)\Big] \\ &= x\,(1-x)\,\Big(f_A(x) - f_B(x)\Big)\,. \end{aligned}$$

Evolutionary game theory

Fixed points

$$\dot{x} = 0 \Leftrightarrow \begin{cases} x^\star = 0 & \text{(all } B) \\ x^\star = 1 & \text{(all } A) \\ 0 < x^\star < 1 & \text{(coexist.)} \end{cases}$$



Solutions' stability

$$\dot{x} = \underbrace{x}_{\geq 0} \underbrace{(1-x)}_{\geq 0} \underbrace{\left(f_A(x) - f_B(x)\right)}_{\lesseqgtr 0\ ?}$$

Evolutionary game theory

Case study: 2 strategy pairwise games

Let us consider a population of players with two strategies: cooperation (C) and defection (D). The payoff matrix is:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}$$

Evolutionary game theory

Intermediate fixed point

$$\dot{x} = x\,(1-x)\,\big(f_C(x) - f_D(x)\big)$$

where

$$f_C(x) = xR + (1-x)\,S$$
$$f_D(x) = xT + (1-x)\,P$$

$$\begin{aligned}
\dot{x} = 0 \Leftrightarrow f_C(x) - f_D(x) &= 0 \\
f_C(x) - f_D(x) &= xR + (1-x)\,S - xT - (1-x)\,P \\
&= x\,(R-T) + (1-x)\,(S-P) \\
&= x\,(R-T) + (S-P) - x\,(S-P) \\
&= x\,(R-T-S-P) + (S-P)\ .
\end{aligned}$$

Thus

$$f_C(x) - f_D(x) = 0 \Leftrightarrow \boxed{\boxed{x^\star = \frac{P-S}{R-T-S+P}}}$$

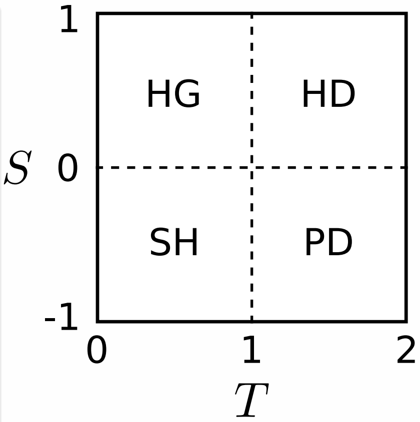
Evolutionary game theory

Simplified payoff matrix

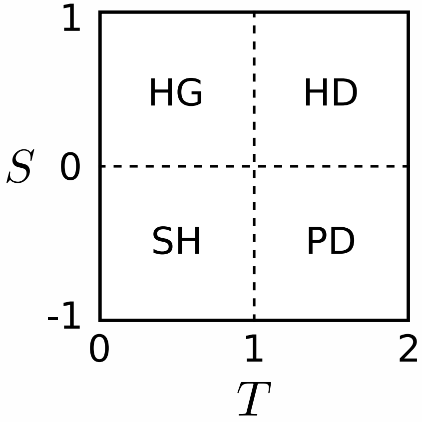
One way to reduce the complexity of the problem is to use a **simplified payoff matrix**

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} = \begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & S \\ T & 0 \end{pmatrix} \end{array}$$

With $S \in [-1, 1]$ and $T \in [0, 2]$.



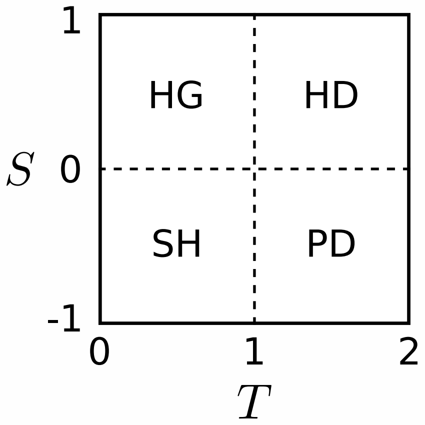
Evolutionary game theory



Harmony game (HG)

($S \geq 0$ and $T \leq 1$).

Evolutionary game theory



Harmony game (HG)

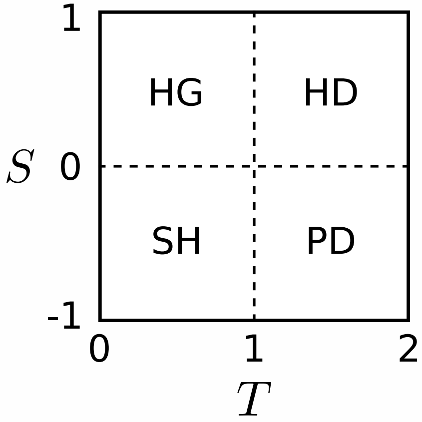
($S \geq 0$ and $T \leq 1$).

Hawk and Dove (HD)

($S \geq 0$ and $T \geq 1$).

Note: Known also as **Snowdrift Game** or **Chicken Game**.

Evolutionary game theory



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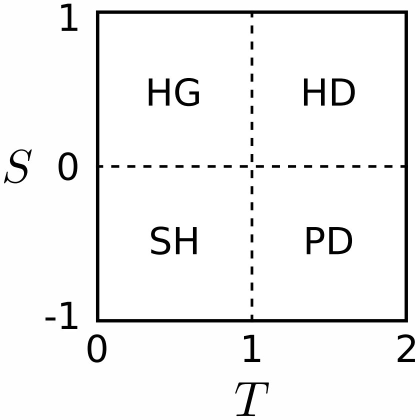
Prisoner's Dilemma (PD)
($S \leq 0$ and $T \geq 1$).

Evolutionary game theory



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Hawk and Dove (HD)
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Prisoner's Dilemma (PD)
($S \leq 0$ and $T \geq 1$).

Stag Hunt (SH)
($S \leq 0$ and $T \leq 1$).

Evolutionary game theory

$$x^\star = \frac{S}{T + S - 1} \qquad f_C(x) - f_D(x) = (S - P) + \\ + x\,(R - T - S - P)$$

Evolutionary game theory

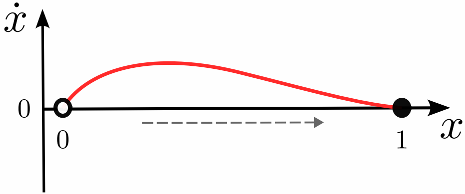
$$x^\star = \frac{S}{T+S-1}$$

$$f_C(x) - f_D(x) = (S-P) + \\ + x\,(R-T-S-P)$$

Harmony game

Only two fixed points

$$x^\star = \begin{cases} 0 & \longrightarrow \text{unstable} \\ 1 & \longrightarrow \text{stable} \end{cases}$$



Evolutionary game theory

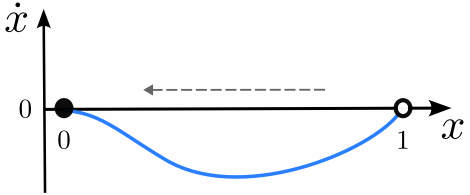
$$x^\star = \frac{S}{T + S - 1}$$

$$f_C(x) - f_D(x) = (S - P) + \\ + x\,(R - T - S - P)$$

Prisoner's Dilemma

Only two fixed points

$$x^\star = \begin{cases} 0 & \longrightarrow \text{stable} \\ 1 & \longrightarrow \text{unstable} \end{cases}$$



Evolutionary game theory

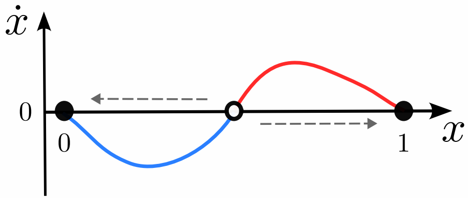
$$x^\star = \frac{S}{T+S-1}$$

$$f_C(x) - f_D(x) = (S-P) + \\ + x\,(R-T-S-P)$$

Stag Hunt

Three fixed points

$$x^\star = \begin{cases} 0 & \longrightarrow \text{stable} \\ 0 < x^\star < 1 & \longrightarrow \text{unstable} \\ 1 & \longrightarrow \text{stable} \end{cases}$$



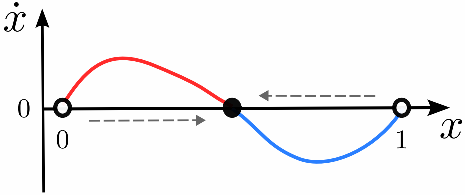
Evolutionary game theory

$$x^\star = \frac{S}{T+S-1} \qquad f_C(x) - f_D(x) = (S-P) + \\ + x\,(R-T-S-P)$$

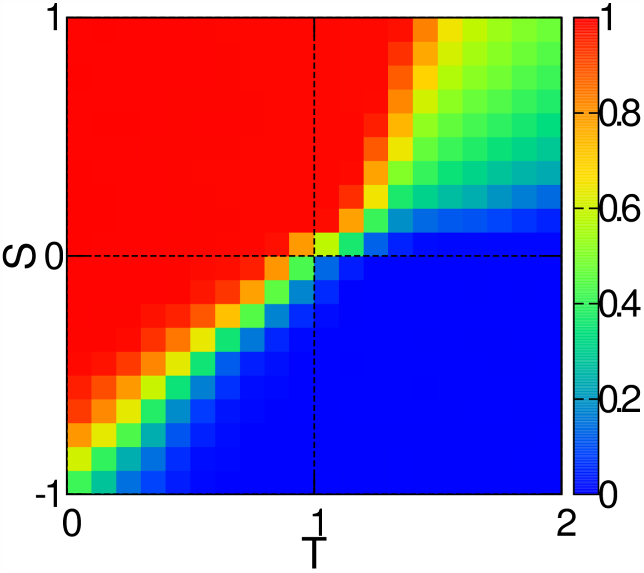
Hawk and Dove

Three fixed points

$$x^\star = \begin{cases} 0 & \longrightarrow \text{unstable} \\ 0 < x^\star < 1 & \longrightarrow \text{stable} \\ 1 & \longrightarrow \text{unstable} \end{cases}$$



Evolutionary game theory



Evolutionary game theory

Note

The stability of the fixed points (especially of those corresponding to pure strategies) is intimately related with the concept of **evolutionary stable strategy** (ESS) which is the evolutionary counterpart of the **Nash equilibrium**.

Evolutionary Game Theory on Graphs

Pairwise games on networks

- Nowak, M. A. Science, **314**, 1560, (2006).

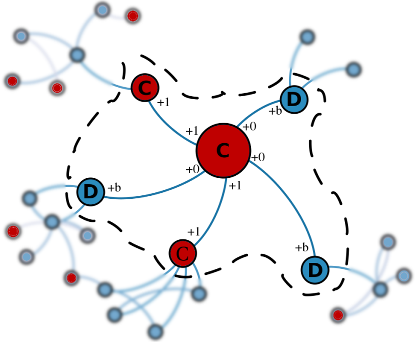
Pairwise games on networks

Network reciprocity

Pairwise games on networks

Network reciprocity

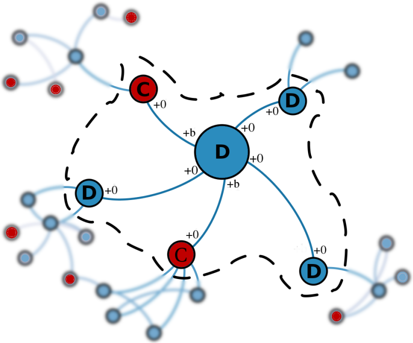
- Each player corresponds to **a vertex of the network** and interacts **ONLY with her neighbors**.



Pairwise games on networks

Network reciprocity

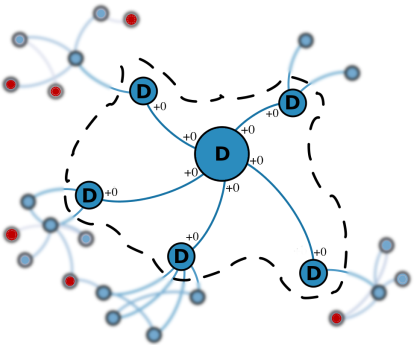
- Each player corresponds to **a vertex of the network** and interacts **ONLY with her neighbors**.
- Players play a game and **accumulate payoff** according to its payoff matrix, and then **update their strategies** according to some update rule.



Pairwise games on networks

Network reciprocity

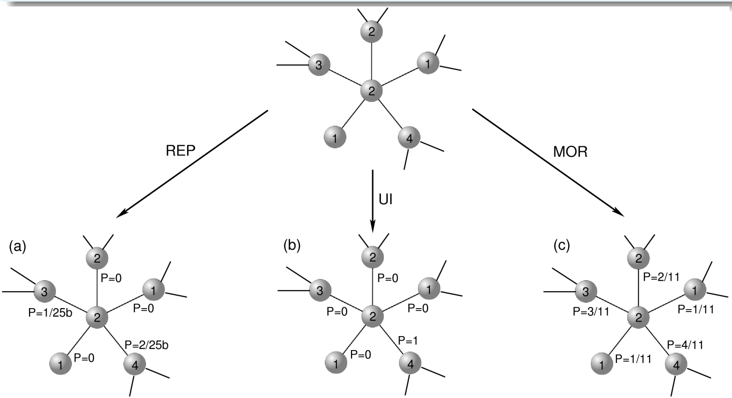
- Each player corresponds to **a vertex of the network** and interacts **ONLY with her neighbors**.
- Players play a game and **accumulate payoff** according to its payoff matrix, and then **update their strategies** according to some **update rule**.
- The dynamics takes place until the system ends up in one of the so-called **absorbing states** (*i.e.*, pure strategy equilibria).



Pairwise games on networks

Update Rules

Replicator (REP): Player i chooses one of her neighbors at random and compares their payoffs. If $f_j > f_i$ player i copies j's strategy with probability $\Pi \propto f_j - f_i$.

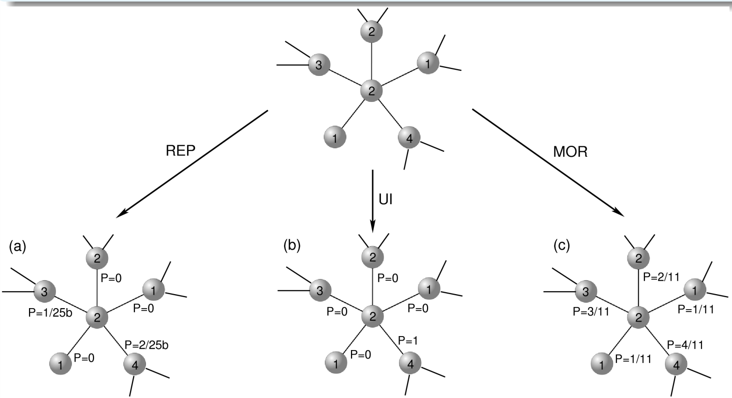


- Schlag, K. H. Jour. Econ. Theo., **78**, 130, (1998).

Pairwise games on networks

Update Rules

Unconditional Imitator (UI): Player i looks at all her neighbors and chooses the one with the highest payoff, j, and copy her strategy if $f_j > f_i$.

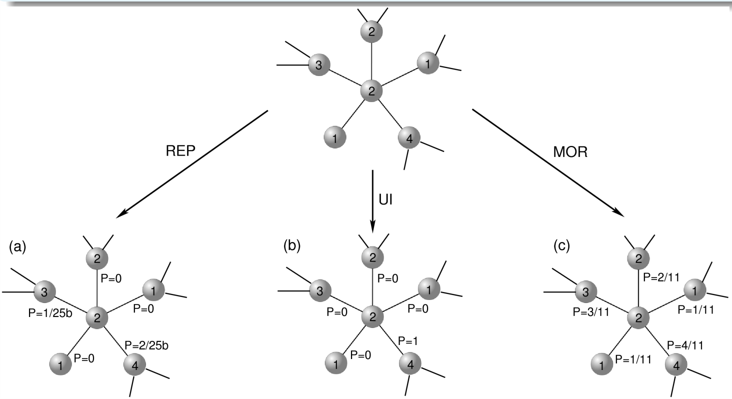


- Nowak, M. A., and May, R. Nature **359**, 826 (1992).

Pairwise games on networks

Update Rules

Moran Rule (MOR): Player *i* chooses one of her neighbors proportionally to her payoff, and changes her strategy to that of the chosen one.



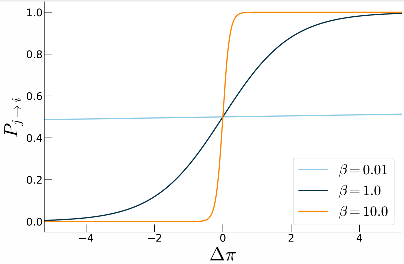
- Moran, P. A. P. *The Statistical Processes of Evolutionary Theory* (1962).

Pairwise games on networks

Update Rules

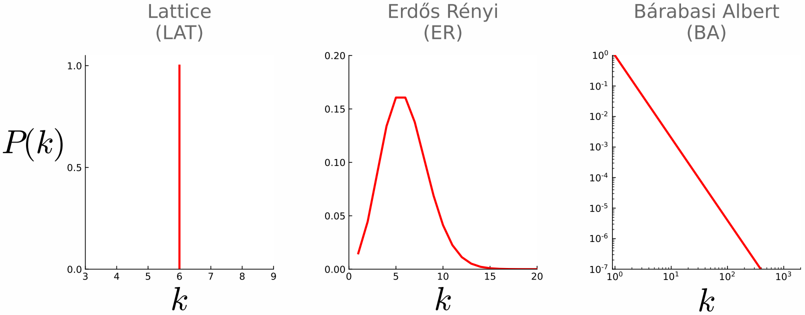
Fermi Rule (FER): Player i chooses at random one of her neighbors, j, compare their payoffs, and copy her strategy with probability:

$$P_{j\to i} = \frac{1}{1+e^{-\beta(\pi_j-\pi_l)}} .$$



- Blume, L. E. Games and Economic Behavior, **5**, 387 (1993).

Pairwise games on networks

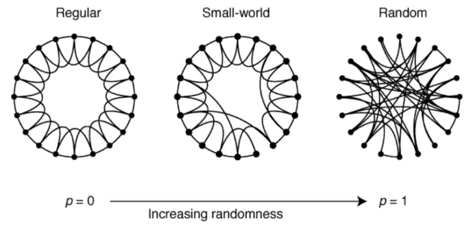


What are the effects of introducing **degree heterogeneity**?

Pairwise games on networks

Setup

- Consider a PD game with UI update.
- Measure the effects of topology considering a Watts-Strogatz network with rewiring probability ε



- Abramson, G., and Kuperman, M. Phys. Rev. E, **63**, 030901 (2001).

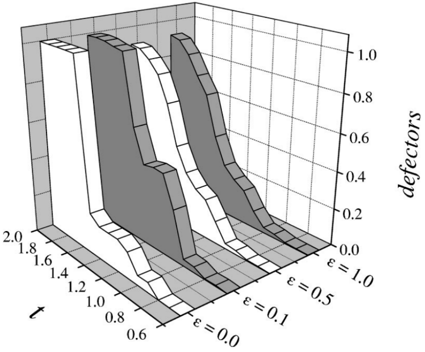
Pairwise games on networks

Main results

As we move from a lattice network ($\varepsilon = 0$) to an ER network ($\varepsilon = 1$), defection emerges for higher values of the temptation t.

Note

values of $t < 1$ do not correspond to the PD game.



- Abramson, G., and Kuperman, M. Phys. Rev. E, **63**, 030901 (2001).

Pairwise games on networks

Setup

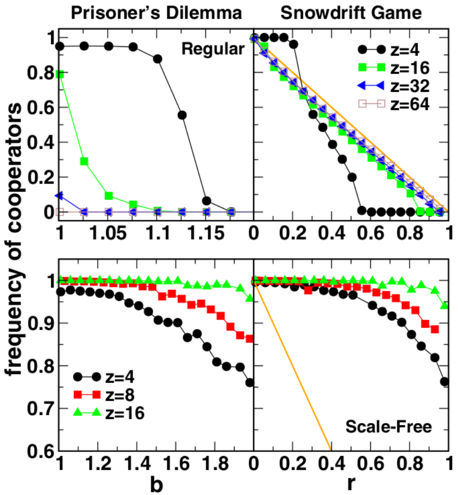
- Consider a PD and Snowdrift (SG) games with replicator update.
- Consider different scale-free BA networks with different average degree $\langle k \rangle = z$.
- They compare the effects of degree heterogeneity running the dynamics also on regular lattices.

- Santos, F., and Pacheco, J. Phys. Rev. Lett., **95**, 098104. (2005)

Pairwise games on networks

Main results

- The presence of hubs stimulates the emergence of cooperation in all the region of the parameter space (*b* for PD and *r* for SG).
- Increasing the value of *z* has a positive effect on cooperation in BA networks.



Note

They also test size effects, as well as the role of degree correlations (by using different models to generate scale-free networks).

- Santos, F., and Pacheco, J. Phys. Rev. Lett., **95**, 098104. (2005)

Pairwise games on networks

Setup

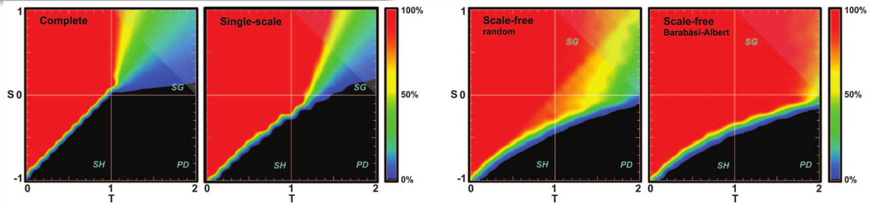
- Explore the behavior of four games (HG, HD, PD, and SH) spanning the T, S space. Update the strategies via a replicator rule.
- Consider four network types: complete (*i.e.*, mean-field), single-scale (Gaussian degree distribution), scale-free random (*i.e.*, configuration model), and scale-free (BA).

- Santos, F., Pacheco, J., and Lenaerts, T. Proc. Nat. Acad. Sci. USA, **103**, 3490 (2006).

Pairwise games on networks

Main results

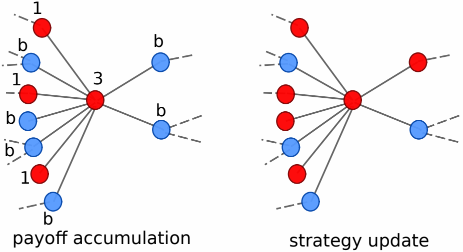
- Degree heterogeneity (*i.e.*, hubs) amplify the region of the (T, S) space where cooperation thrives.
- Degree correlations in scale-free networks boost even more cooperation.



- Santos, F., Pacheco, J., and Lenaerts, T. Proc. Nat. Acad. Sci. USA, **103**, 3490 (2006).

Pairwise games on networks

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \end{array} \text{ with } b > 1$$



Even if cooperators are **exploited** by defectors (and accumulate less payoff on a single pairwise interaction), cooperator hubs can accumulate higher payoffs (*additive payoff scheme*) **taking over defectors** and **triggering a cascade of "conversions,"** thus allowing the onset of full cooperation.

- Gómez-Gardeñes, *et al.* Phys. Rev. Lett., **98**, 108103 (2007).

Pairwise games on networks

Payoff schemes

The total payoff of a player i, $\Pi(i)$, is equal to:

Additive The sum of all the payoffs accumulated in each of the games played, $\pi(i,j)$: $\Pi(i) = \sum_j \pi(i,j)$.

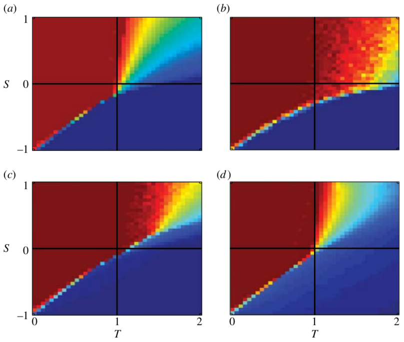
Average The average of the payoffs accumulated in each of the games played: $\Pi(i) = \frac{1}{k_i} \sum_j \pi(i,j)$.

Note

Alternatively, instead of computing the average of the payoffs one can introduce a "participation cost" h.

- Masuda, N. Proc. R. Soc. B., **274**, 1815 (2007).

Pairwise games on networks



Results

- Increasing the participation cost, *h*, (top right, bottom right, bottom left) is detrimental for cooperation.
- We recover the mean-field cooperation diagram (top-left).

• Masuda, N. Proc. R. Soc. B., **274**, 1815 (2007).

Group games

Group games

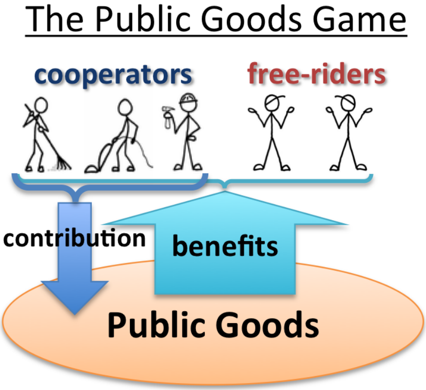
Public Good Game

- Consider a population of N players with two possible strategies, s: cooperation ($s = 1$) and defection ($s = 0$).

Group games

Public Good Game

- Consider a population of N players with two possible strategies, s: cooperation ($s = 1$) and defection ($s = 0$).
- At each round, select a group of $m < N$ players. Cooperators will donate c to the **public good** whereas defectors give anything.



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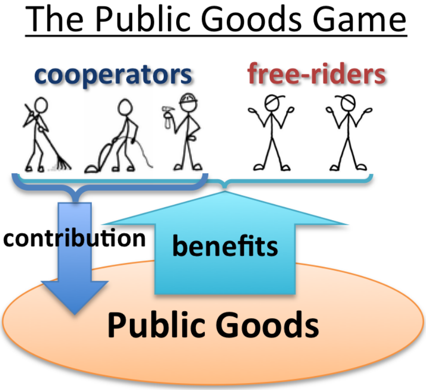

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Payoff

$$\pi_i = \begin{cases} b_i - c & \text{if } s_i = 1 \\ b_i & \text{if } s_i = 0 \end{cases}$$

where

Benefit

$$b_i = \frac{\eta \left(\sum_j^{m-1} s_j \, c + s_i \, c \right)}{m} .$$

Group games

Science

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ARTICLE

The Tragedy of the Commons: The popula

nical solution; it requires a fundamental extension in morality.

GARRETT HARDIN

SCIENCE • 13 Dec 1968 • Vol 162, Issue 3859 • pp. 1243-1248 • DOI: 10.1126/science.162.3859.1243

- Hardin, G. Science, **162**, 1243 (1968).

Group games

- Consider N players interacting via a graph (either a scale-free or a regular lattice). The groups correspond to the each node's neighbourhood. The enhancement factor is $\eta = \dfrac{r}{\langle k \rangle + 1} \in [0, \infty[$.

• Santos, F. C., Santos, M. D., and Pacheco, J. Nature, **454**, 213 (2008).

Group games

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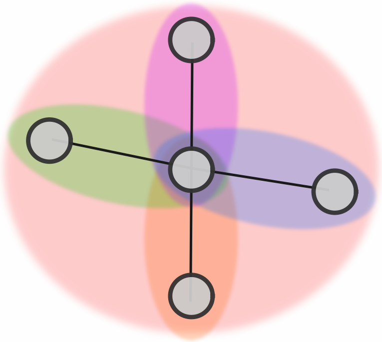

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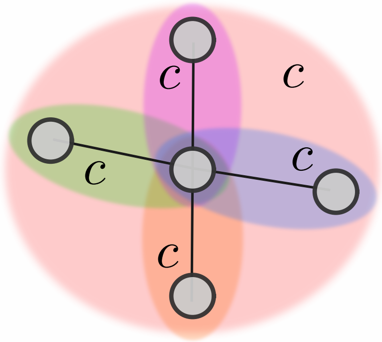

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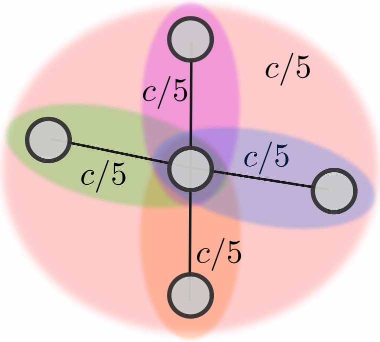

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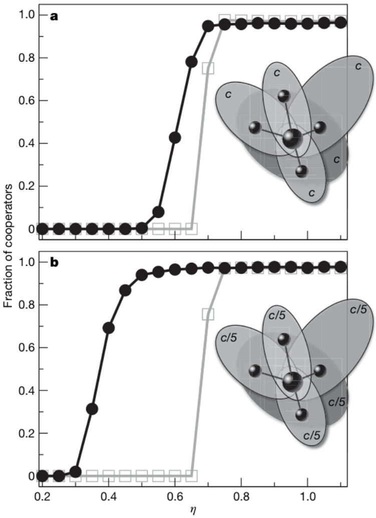

- Santos, F. C., Santos, M. D., and Pacheco, J. Nature, **454**, 213 (2008).

What's next?

What’s next?


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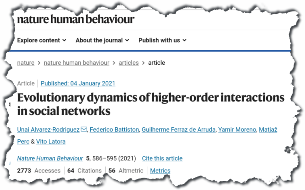
Alessio Cardillo, Giovanni Petri, Vincenzo Nicosia, Roberta Sinatra, Jesús Gómez-Gardeñes, and Vito Latora

Phys. Rev. E **90**, 052825 – Published 25 November 2014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Article | Published: 04 January 2021

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Nature Human Behaviour 5, 586–595 (2021) | Cite this article

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High order

- Gómez-Gardeñes, J. *et al.* Sci. Rep. **2**, 620 (2012).
- Cardillo, A., *et al.* Phys. Rev. E, **90**, 052825 (2014).
- Alvarez-Rodriguez, U. *et al.* Nat. Hum. Behav. **5**, 586 (2021).

What’s next?

- Grujić, J., *et al.* PLoS ONE **5**, e13749 (2010).
- Gracia-Lazaro, C. *et al.* Proc. Natl. Acad. Sci. USA, **109**, 12922 (2012).
- Szekely, A., *et al.* Nat. Comm. **12**, 5452 (2021).
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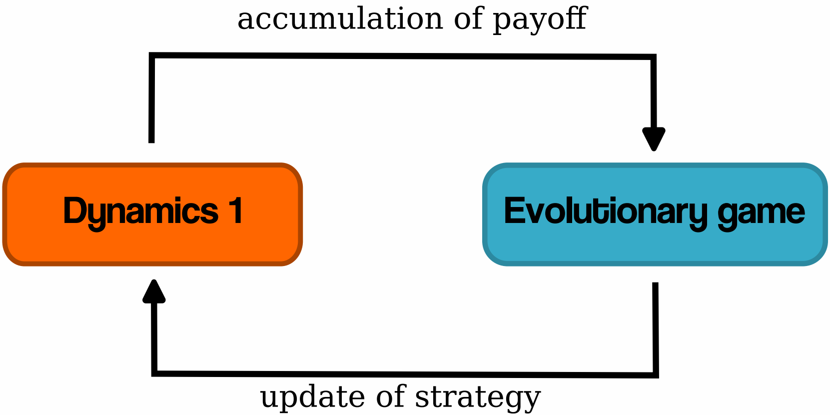
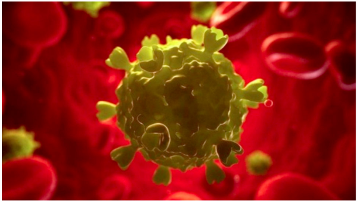
What’s next?


accumulation of payoff
Dynamics 1
Evolutionary game
update of strategy

What’s next?

Motion coordination
(synchro + games)

Vaccination
(epidemic + games)

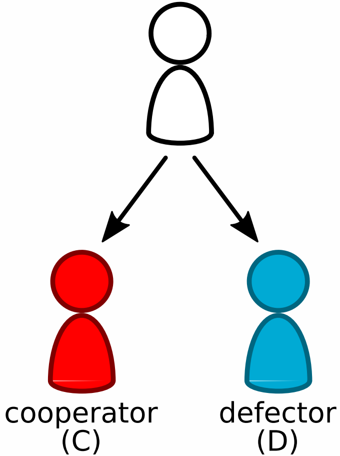


Comorbidity
(epidemic + games)

- Bauch, C. T., and Earn, D. J. D. Proc. Natl. Acad. Sci. USA, **101**, 13391 (2004).
- S., Bohl, *et al.* Mol. BioSyst., **10**, 3044 (2014).
- Antonioni, A., and Cardillo, A. Phys. Rev. Lett., **118**, 238301 (2017).

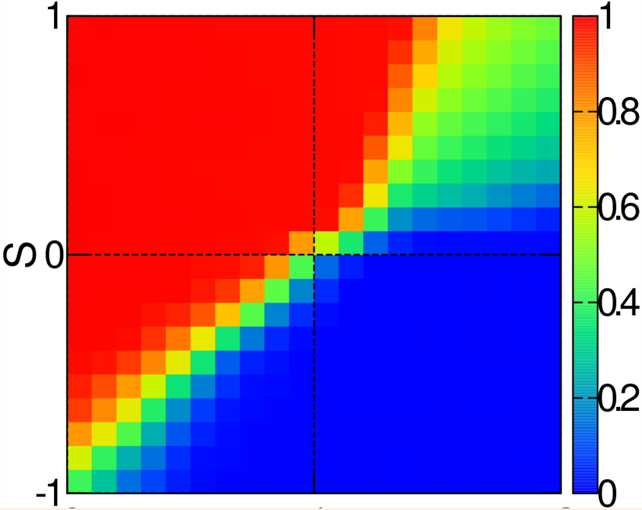
Summing up ...

Take home messages



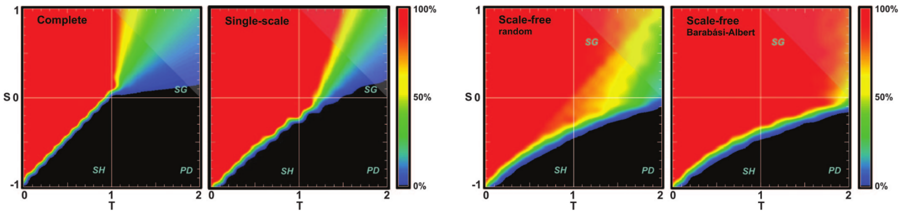
Game theory as a way to model **rational decisions**.

Take home messages



Evolutionary game theory as a way to model **evolution under variable** (frequency dependent) **reproduction's rate**.

Take home messages



Scale-free networks act like **a catalyzer** for cooperation.

Contacts

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