Emergence of synergistic and competitive pathogens in a co-evolutionary spreading model

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MAIN QUESTION

The spreading of multiple pathogens is the norm rather than the exception. In particular, ***comorbidity*** indicates the simultaneous presence of multiple diseases in the same host; whereas ***cross-immunity*** denotes the acquisition of immunity towards a certain disease as a result of infection by another one. Pathogens supporting comorbidity are indicated as ***cooperators***, whereas the others as ***competitors***.

The models developed hitherto assume that the strategy of a pathogen to cooperate - or not - is ***costless***. In the evolutionary game framework, however, cooperation has a rather different meaning and implies always **the payment of some costs**.

How cooperator and competitor pathogens *emerge*?

We envision an evolutionary scenario where interacting pathogens have two strains with cooperative and defective strategies, and evolve while maximizing their own benefit. We do so by **extending the two-strain** susceptible-infected-recovered **(SIR) model** of Chen *et al.* [2] by **intertwining it with an evolutionary infection game**.

1. Evolution of SIR model

Single SIR	**Double SIR**	**Double SIR + strategies**
3 states	9 states	25 states

Pathogens: A, B
+
Strategies: C, D
=
Tot. Species: A_C, A_D, B_C, B_D

2. Co-evolutionary model scheme

Phase 1 (within season T)

Coupled SIR (25 states) dynamics

Single pathogen infection

$$\mathbf{X}_\Lambda + \mathbf{S} \xrightarrow{\alpha} 2\,\mathbf{X}_\Lambda$$

Multiple pathogens infection

$$\mathbf{X}_\Lambda + \mathbf{Y}_\Gamma \xRightarrow{\beta_{\Lambda\Gamma}} \mathbf{X}_\Lambda + \mathbf{X}_\Lambda\mathbf{Y}_\Gamma$$

Accumulation of Payoff

Single pathogen infection

$$\pi_{\mathbf{X}_\Lambda} = 1$$

Multiple path. infection (HD game)

Infecting $\pi_{\mathbf{X}_\Lambda}$

$$\begin{array}{c} \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} \frac{1}{2} & \gamma \\ 1-\gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Infected $\pi_{\mathbf{Y}_\Gamma}$

$$\begin{array}{c} \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} \frac{1}{2} & 1-\gamma \\ \gamma & -\frac{1}{2} \end{pmatrix} \end{array}$$

Evaluation of fitness

New epidemic season

$$\rho_i^{T+1}\Big|_{t_0} = \rho_i^{T}\Big|_{t_0}\left[1 + \Pi_i^T\Big|_{t_\infty} - \overline{\Pi}^T\Big|_{t_\infty}\right]$$

Phase 2: Evolution of concentrations/strategies (between season T and $T+1$)

$\alpha \longrightarrow$ Single infection rate

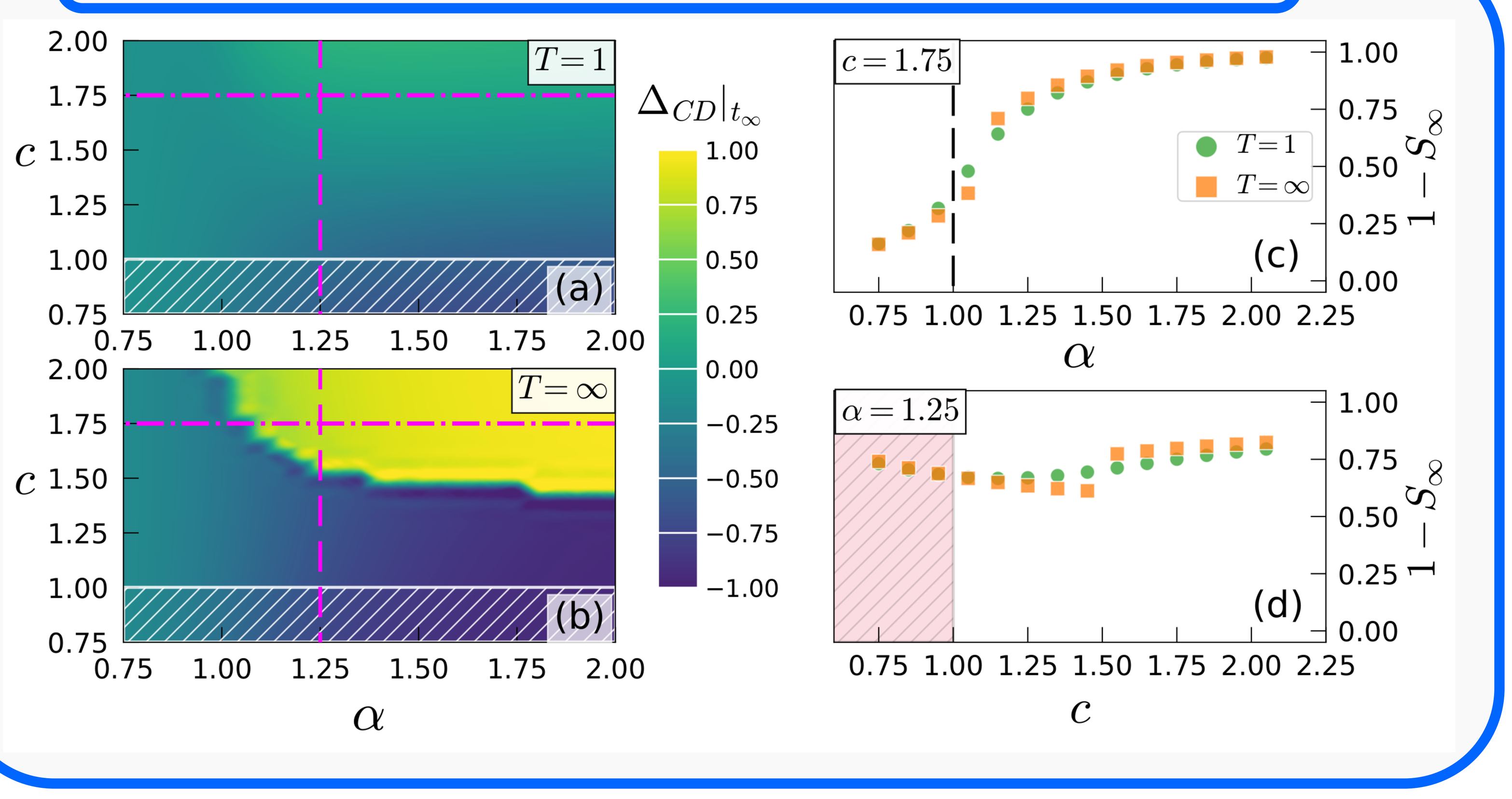
$\beta_{\Lambda\Gamma}$: $\beta_C = c\,\alpha$, $\beta_D = \dfrac{\alpha}{c}$ — Multiple infection rates

$\gamma \in \left[0, \frac{1}{2}\right] \longrightarrow$ Payoff matrix parameter

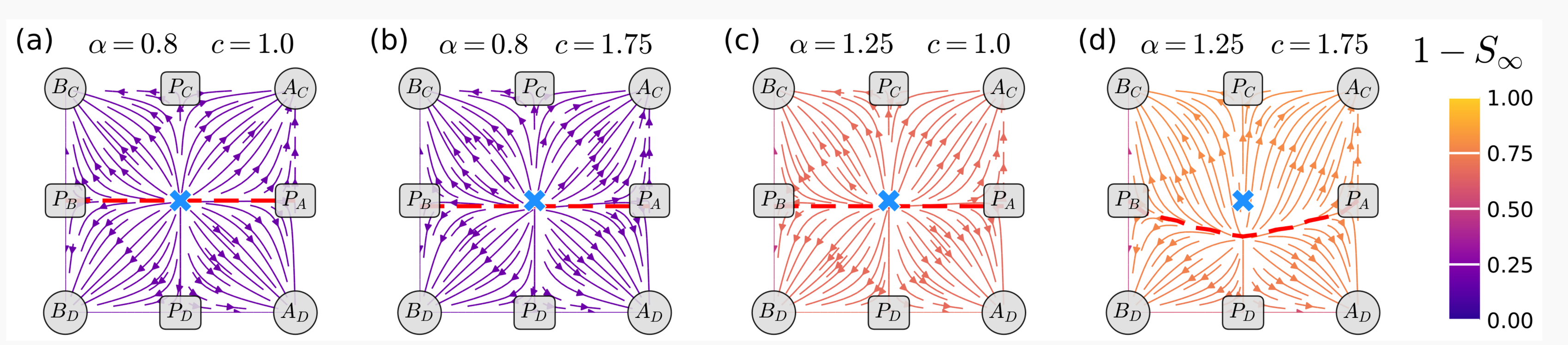
$\rho_i \longrightarrow$ Pathogen's i density in the spreading seed

$\Pi_i \longrightarrow$ Pathogen's i fitness

3. The pathogens' perspective



4. Evolution of the epidemic seed



NOTE: The coordinates of each point correspond to a given mixture of the initial spreading seed expressed in terms of the concentrations of the four pathogen species. The arrows denote the evolution across spreading seasons.

TAKE HOME MESSAGES

1) We have studied the **emergence of synergistic and competitive traits in a co-evolutionary model intertwining epidemic spreading and evolutionary game theory**. The resulting co-evolutionary model displays features that are not present in either spreading or evolutionary game dynamics taken singularly (i.e., "*more is different*").

2) The game dynamics reverberates on the outcome of the spreading dynamics. It **alters the disease prevalence** in the host denoted by the emergence of a region where we observe full cooperation [see the comparison between panels 3(a) and 3(b)]. Moreover, **the game dynamics triggers the emergence of a gap in the function describing the epidemic prevalence in the hosts** [see panels 3(c) and 3(d)].

3) The interplay between the spreading and game dynamics alters the stability of the state space's fixed points of the game. Pure seeds are stable solutions of the dynamics, contrary to the expected stable mixed scenario typical of the hawk-and-dove game [see panel 4]. Interestingly, **we have observed the emergence of a region of the phase space where cooperation thrives even under unfavorable conditions** [see panel 4(d)].

REFERENCES

[1] F. Ghanbarnejad, K. Seegers, A. Cardillo, and P. Hövel *Phys. Rev. E* **105**, 034308 (2022). DOI: 10.1103/PhysRevE.105.034308

[2] L. Chen, F. Ghanbarnejad, W. Cai, and P. Grassberger *Europhysics Letters* **104**, 50001 (2013). DOI: 10.1209/0295-5075/104/50001