Co-evolution of strategies and update rules in the prisoner's dilemma game on complex networks

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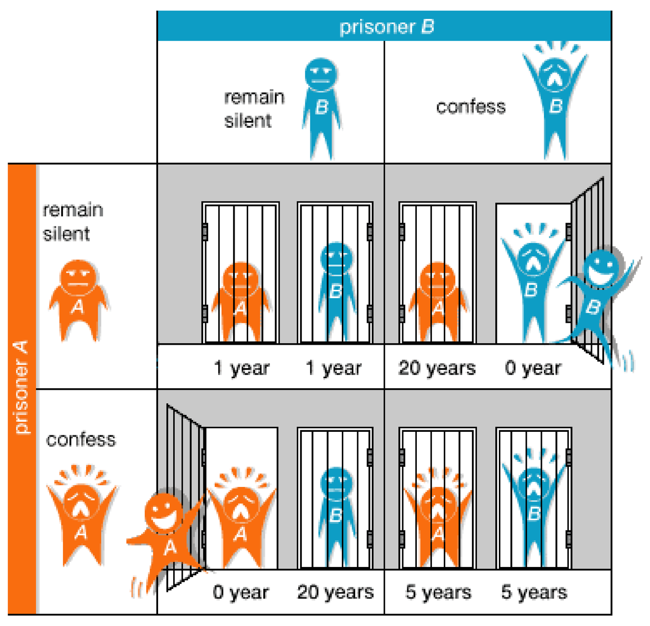
Motivation

The cooperative behavior of a population of agents playing evolutionary Prisoner's Dilemma (PD) depends on many factors such as the underlying topology or the update rule to cite a few [1, 2]. Furthermore, up to few years ago, no one has ever tried to study systems in which both strategy of players and the way they choose it (i.e. the **update rule**) undergo an evolutionary pressure [3].
Using the paradigmatic game of Prisoner's Dilemma, we extend the pioneering work of Moyano and Sanchez on co-evolution of strategy and update rule on regular lattices [4, 5] and examine the emergence of a collective behavior such as cooperation in systems subjected to an evolutionary process, in which players could change both their strategy and update rule. Several scenarios, in terms of both network topologies and update rule, have been considered in order to span the widest set of conditions available.

Introduction on Prisoner's Dilemma

Situation: Two bank robbers are arrested, but the police do not possess enough information for an arrest. Following the separation of the two men, the police offer both a similar deal: if one testifies against his partner (defects), and the other stays quiet (cooperates), the betrayer goes free and the cooperator receives the full twenty-year sentence. If both remain silent, both are sentenced to only five years in jail for a minor charge. If each 'rats out' the other, each receives a one-year sentence. Each prisoner must choose to either betray or remain silent; the decision of each is kept quiet. What should they do?



Describing the game with respect to its Payoff Matrix one has:

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} \mathcal{R} & \mathcal{S} \\ \mathcal{T} & \mathcal{P} \end{pmatrix} \end{array} \quad \text{such that:} \quad \mathcal{T} > \mathcal{R} > \mathcal{P} > \mathcal{S}\,.$$

In our simulations we used a reduced form of the payoff matrix, given by:

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \end{array} \quad \text{with } b \in [1,2]\,.$$

Update Rules

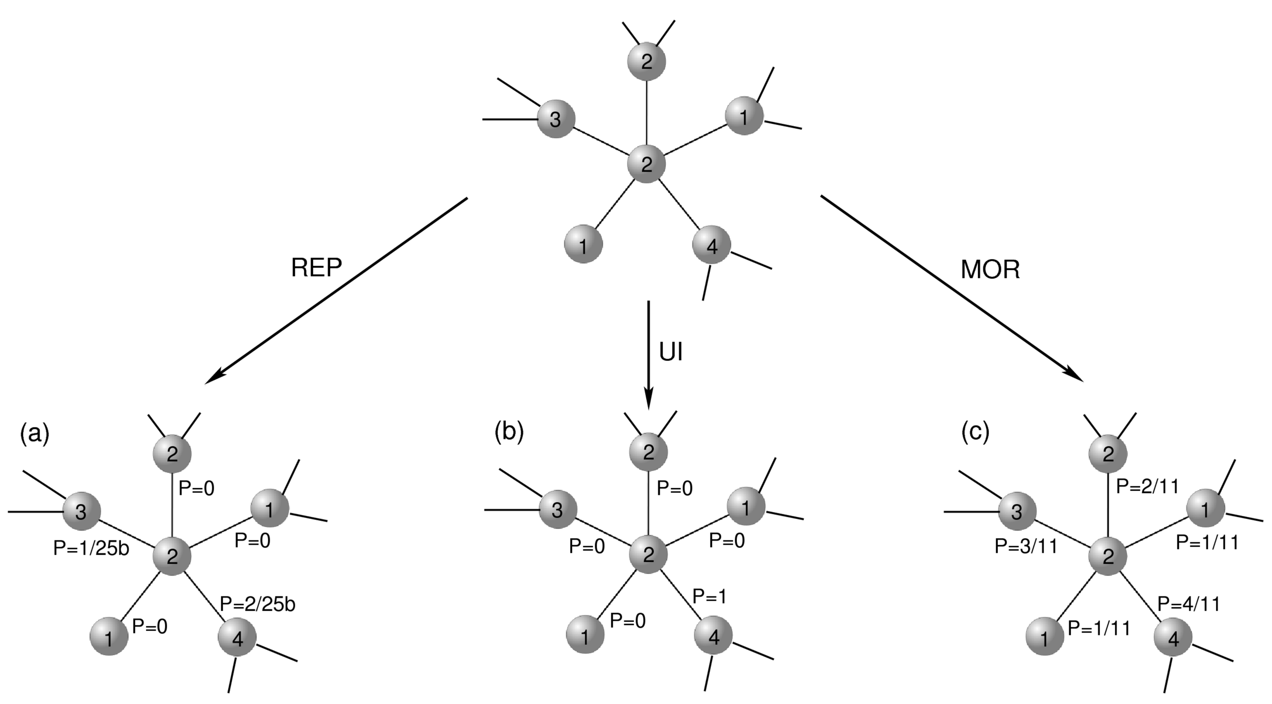
The description of evolutionary systems through the means of game theory alone is not enough. The Evolutionary version of the PD game allow players to play the game repeatedly and change their strategy at the end of each round. This changes are based on the payoff received in the last round of the PD game. To this end, each agent compares her payoff with those of her game mates. We consider three update rules:

Replicator Dynamics (REP): Each agent i chooses one of his neighbors at random, say j, and compares their payoffs. If $f_j > f_i$ agent i will copy strategy and update rule of j with probability:

$$\Pi \propto f_j - f_i\,.$$

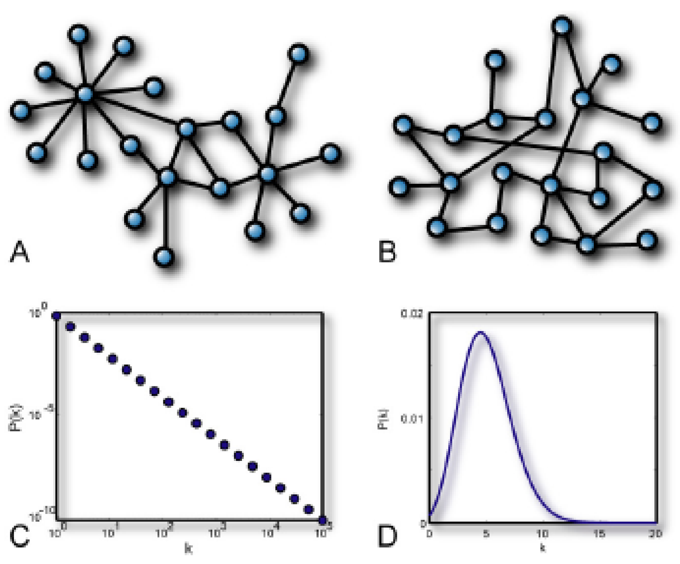
Unconditional Imitator (UI): Each agent i looks at all his neighbors j, choose the one with the highest payoff and if $f_j > f_i$ he will copy both strategy a and update rule of j or maintains its own otherwise.

Moran Rule (MOR): Each agent i chooses one of his neighbors proportionally to his payoff and changes his state to the one of the chosen one.



Complex Networks

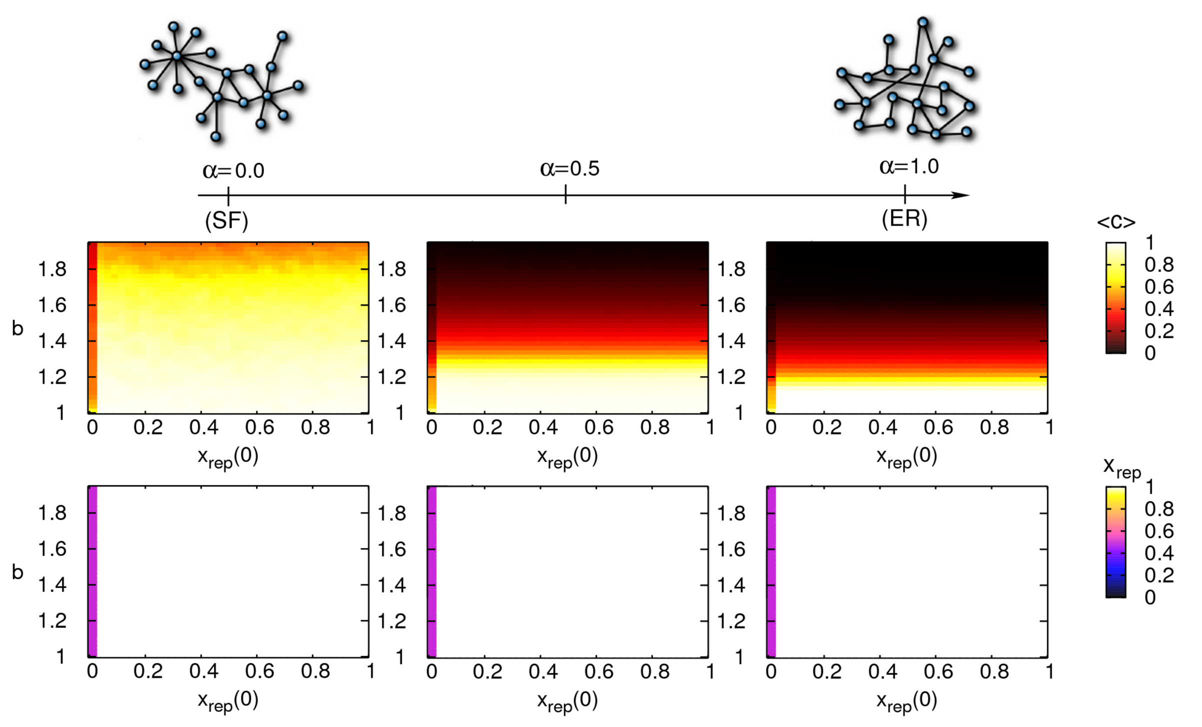
Two of the most common topologies used are: Scale Free (SF) and Erdős-Rényi (ER) graphs. Using the evolutionary model of [6] we can build networks spanning from ER to SF passing through intermediate case.



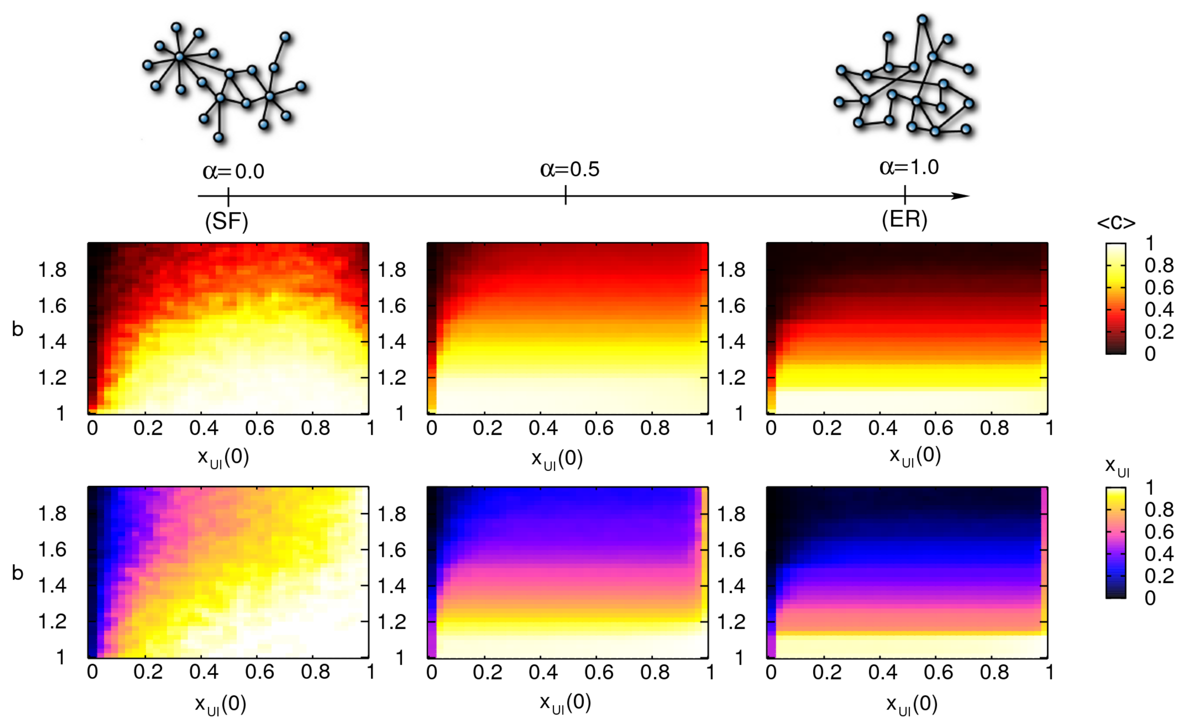
Result

Since there are not any a-priori reasons to fix the update rule or to impose one over the others, we treat update rule in the same manner as strategy: we let it evolve, allowing the system itself choose what he "likes" most.
We want to see if the coexistence of different update rules, in association with different underlying topologies, changes the overall cooperative behavior present in literature.
Results are shown in terms of average fraction of cooperators $\langle C\rangle$, and average final fraction of players with a certain update rule $\langle x_{\mathrm{rule}}\rangle$.

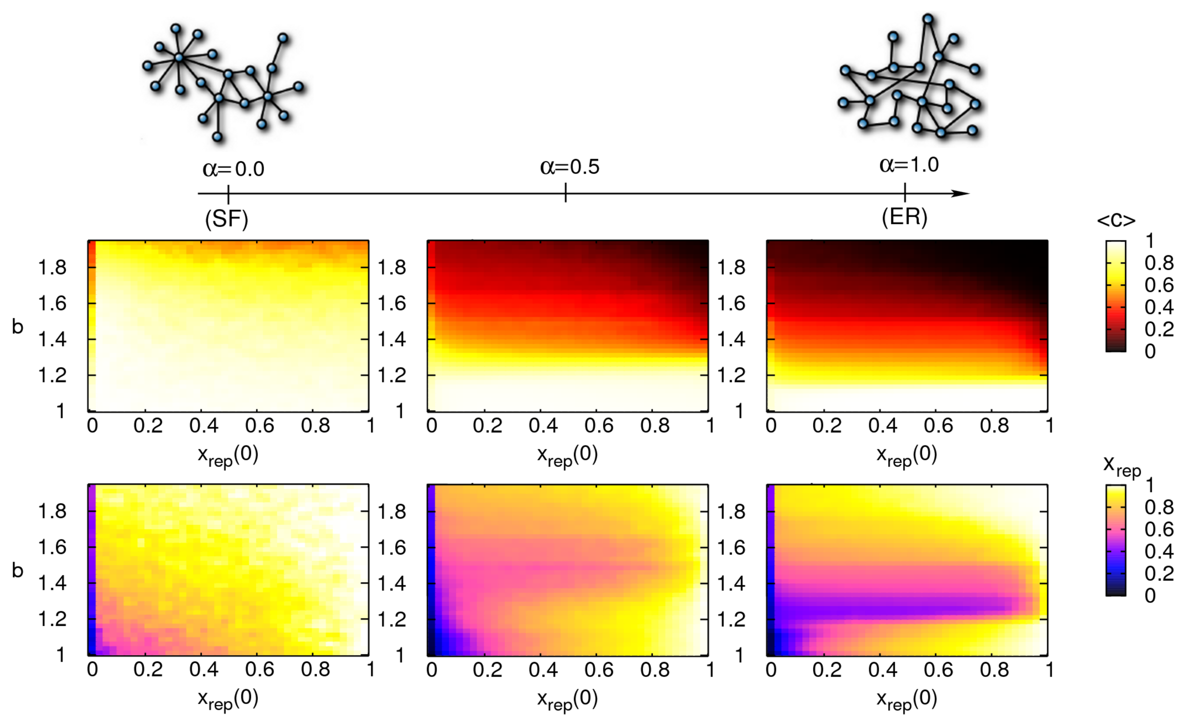
Result I: REP vs MOR



Result II: UI vs MOR



Result III: REP vs UI



Conclusion

- In MOR vs REP, Moran players are wiped out regardless of the underlying topology and the temptation to defect b;
- In UI vs MOR and SF graphs, the average cooperation level $\langle C\rangle$ shows a maximum for populations made of players with both kind of update rules;
- In REP vs UI and SF graphs, again, the cooperation level has a maximum for "mixed populations";
- In REP vs UI for both intermediate and ER topologies, the presence of a region in which UI players survive more reflects into a survival of cooperation for values of temptation higher than in the corresponding case of REP vs MOR;
- The surviving species is always responsible for the overall cooperation level of the system;
- Moran players are more keen to extinguish when playing with players using other update rules;
- Co-evolution shows that is possible to obtain relatively large cooperation values when two update rule coexist in contrast with the single rule case;

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