Beyond simple complex-networks: coevolution, multiplexity, and time-varying interactions

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&
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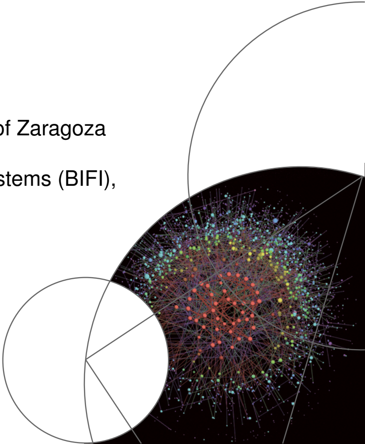
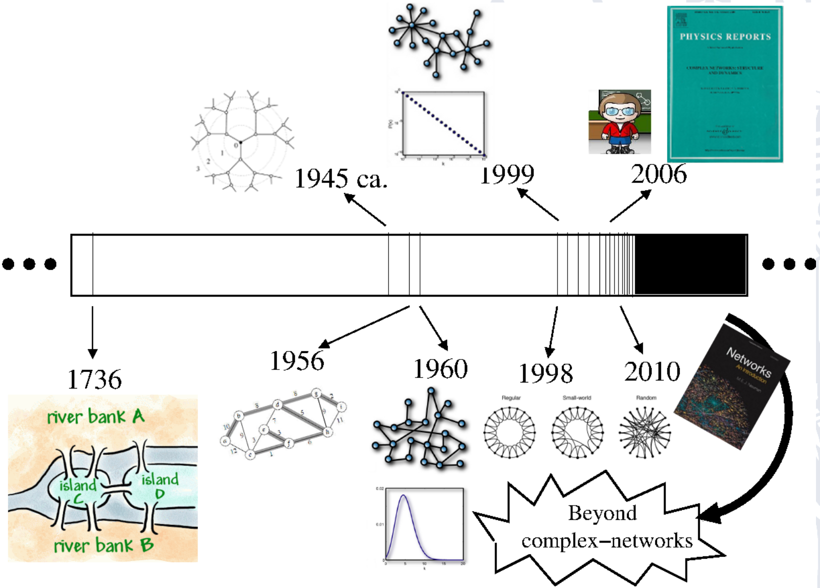
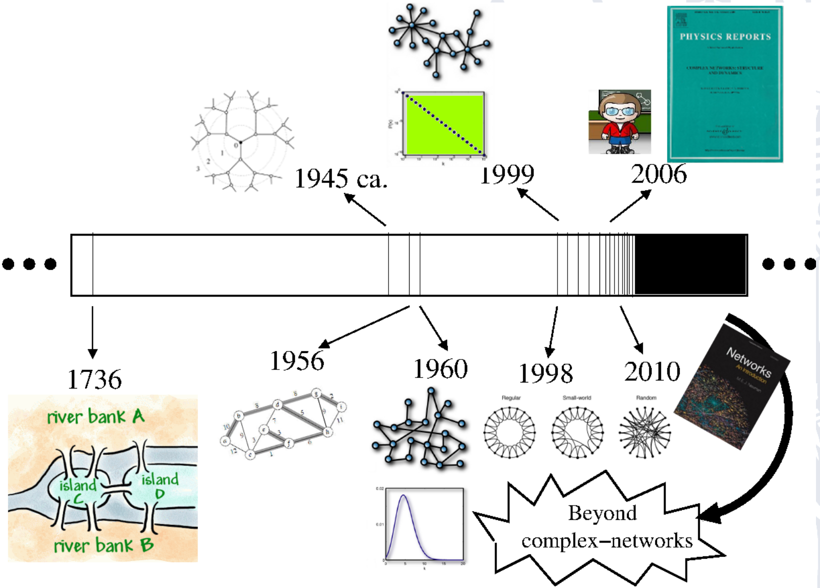
http://bifi.es/~cardillo/



Once upon a time . . .



Once upon a time . . .



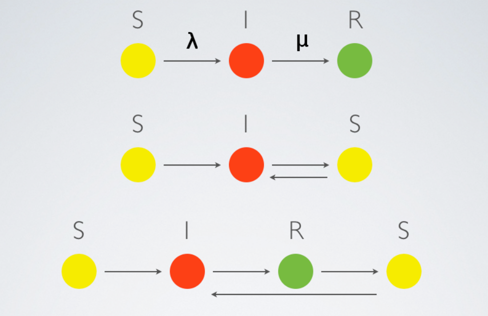
Section 1

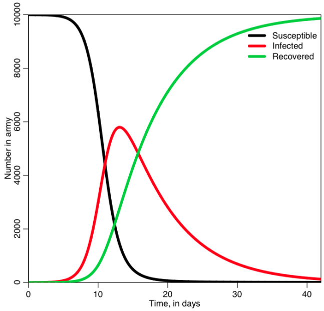
Success story

Spreading of infections

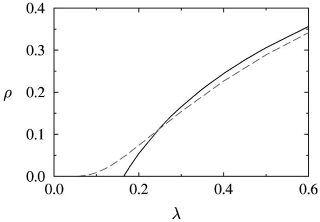
Spreading of infections

Compartmental models





Spreading of infections



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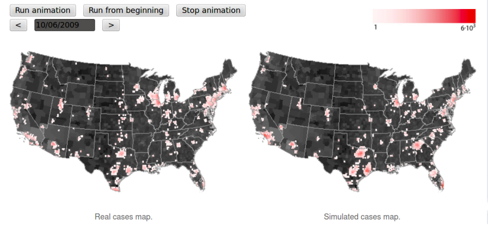
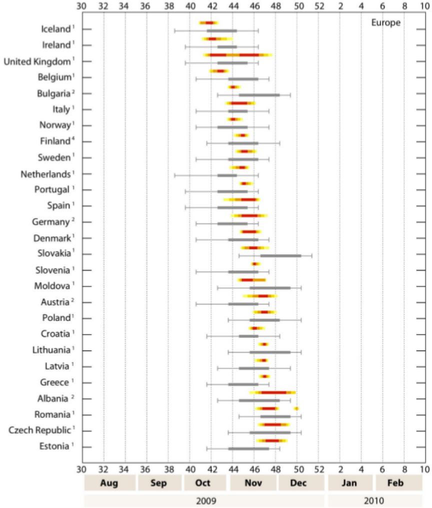


Spreading of infections

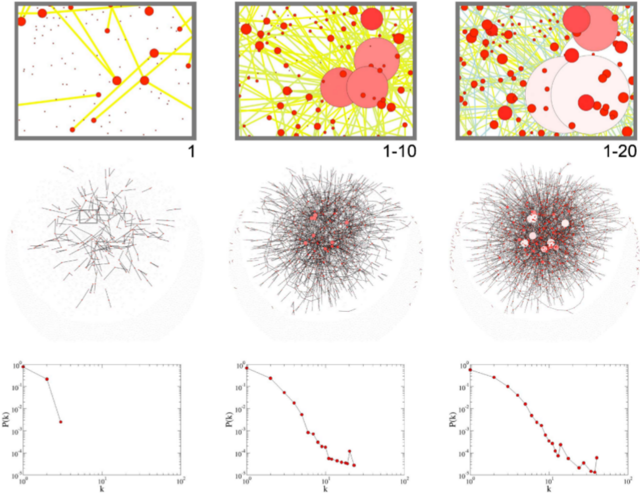


- Tizzoni *et al.* *Real-time numerical forecast of global epidemic spreading: case study of 2009 AH1N1pdm.* BMC Medicine, **10**, 165 (2012).

Section 2

Methods

Time Varying Graphs



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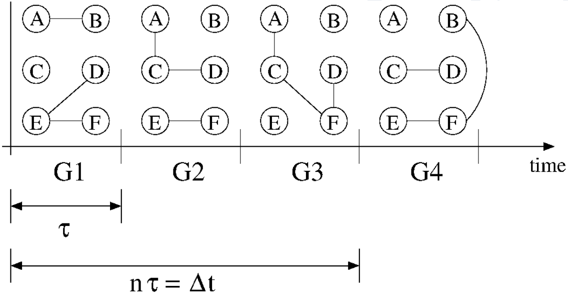
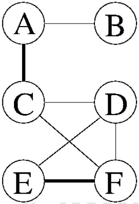
Time Varying Graphs





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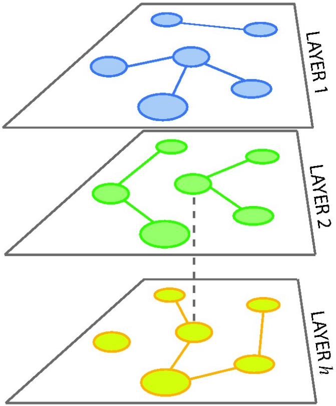
Multiplex networks



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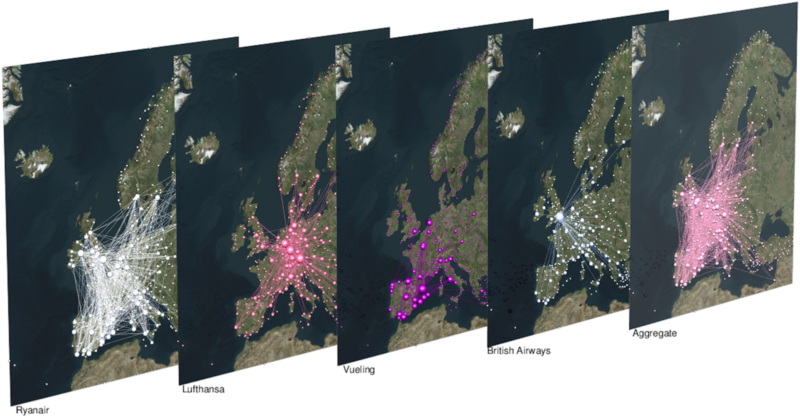
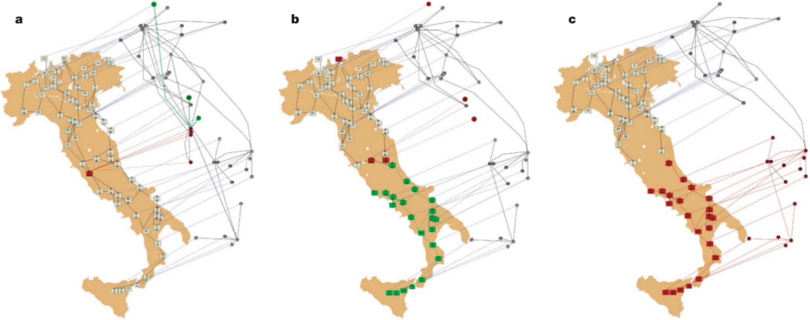
Multiplex networks



- https://github.com/manlius/muxViz

Interdependent networks



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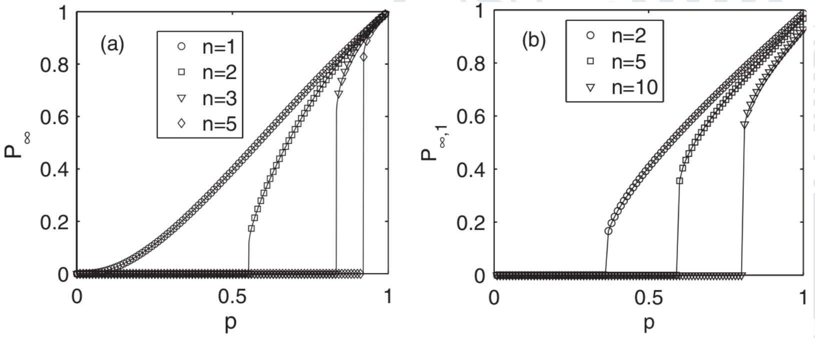
Interdependent networks



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Section 3

Results

Cooperation & time correlations

Cooperation & time correlations

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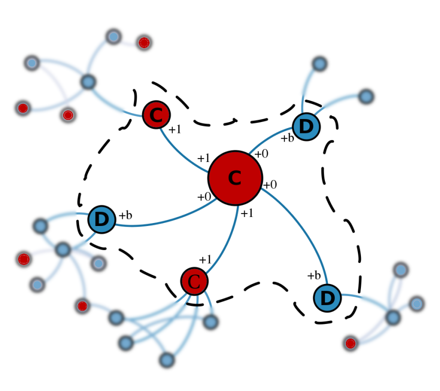
Evolutionary dynamics of time-resolved social interactions

Alessio Cardillo, Giovanni Petri, Vincenzo Nicosia, Roberta Sinatra, Jesús Gómez-Gardeñes, Vito Latora

Cooperation & time correlations

A brief introduction on evolutionary game theory

- Agents states are equal to strategies of a game;

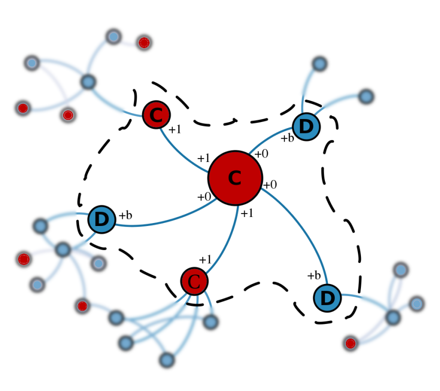


Cooperation & time correlations

A brief introduction on evolutionary game theory

- Consider the following payoff matrix:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \end{array} \text{ with } b > 1\,;$$



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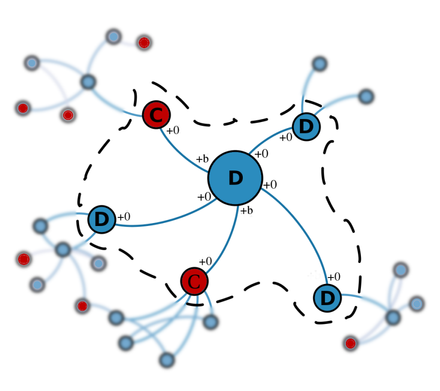

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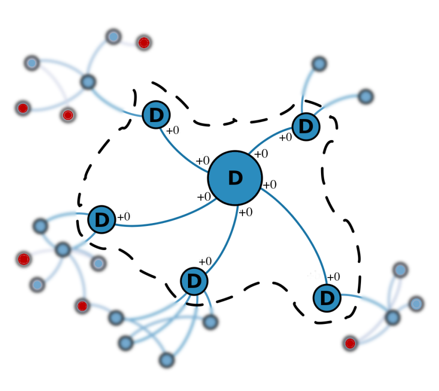
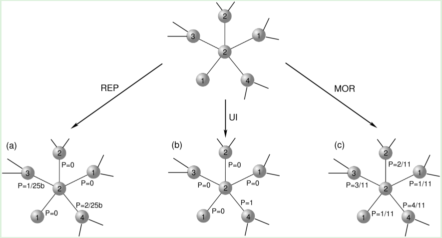


Cooperation & time correlations

A brief introduction on evolutionary game theory

- Agents update their strategy;

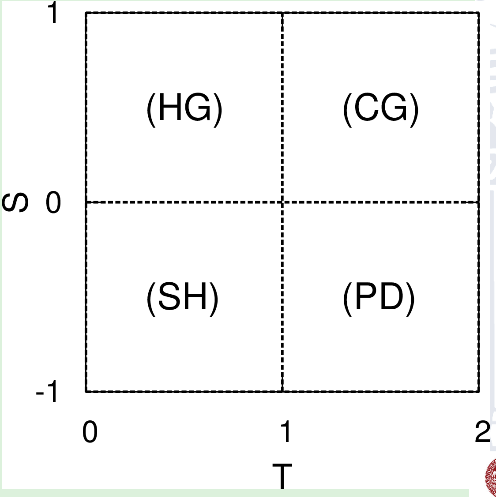


Cooperation & time correlations

A brief introduction on evolutionary game theory

- Four different pairwise games

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & S \\ T & 0 \end{pmatrix} \end{array} ;$$



Cooperation & time correlations

A brief introduction on evolutionary game theory

- Four different pairwise games

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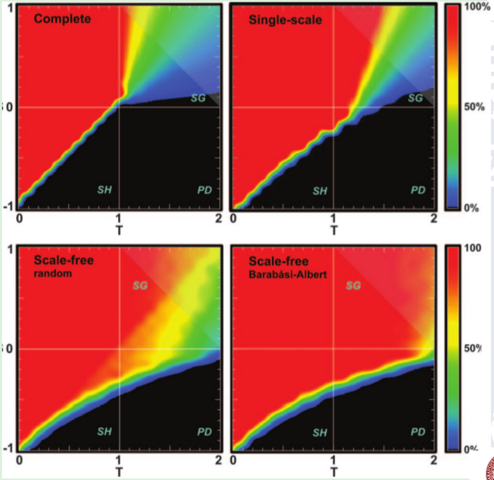
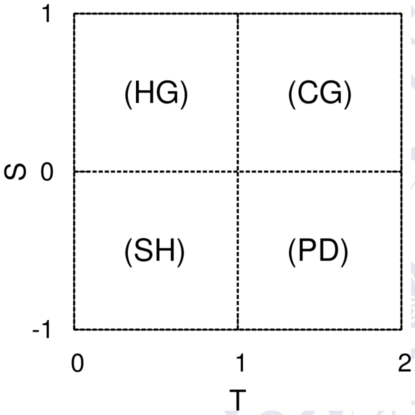


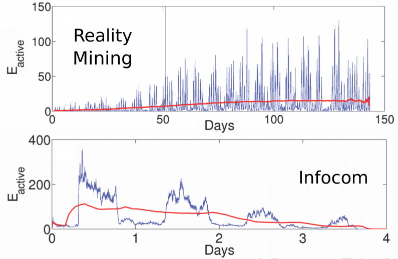
Cooperation & time correlations



The model

- Four different games;

Cooperation & time correlations



The model

- Four different games;
- Two different datasets;

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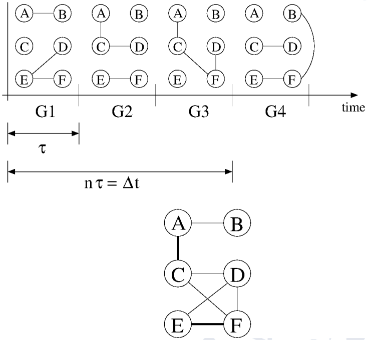
Cooperation & time correlations



The model

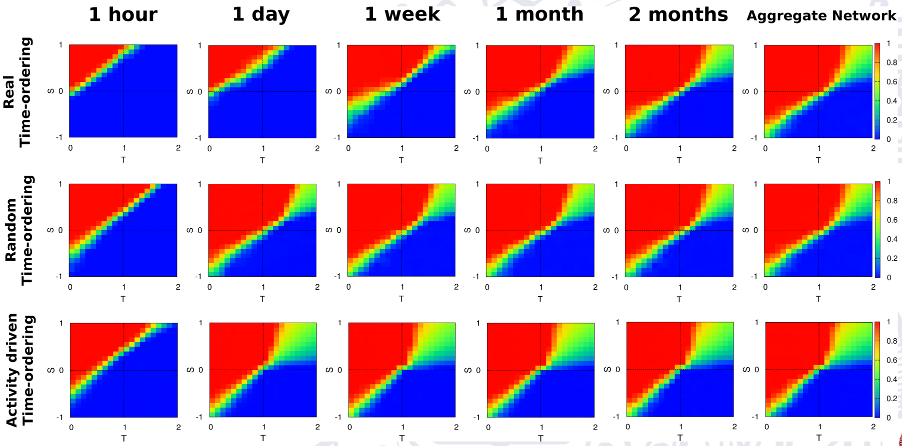
- Four different games;
- Two different datasets;
- Agents interact through the structure of the time-varying network aggregated over a time window Δt;

Cooperation & time correlations

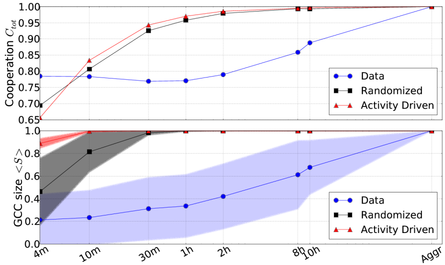
The model

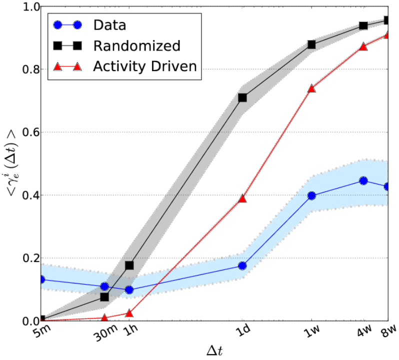
- Four different games;
- Two different datasets;
- Agents interact through the structure of the time-varying network aggregated over a time window Δt;
- Agents play the game and update their strategies.

Cooperation & time correlations



Cooperation & time correlations





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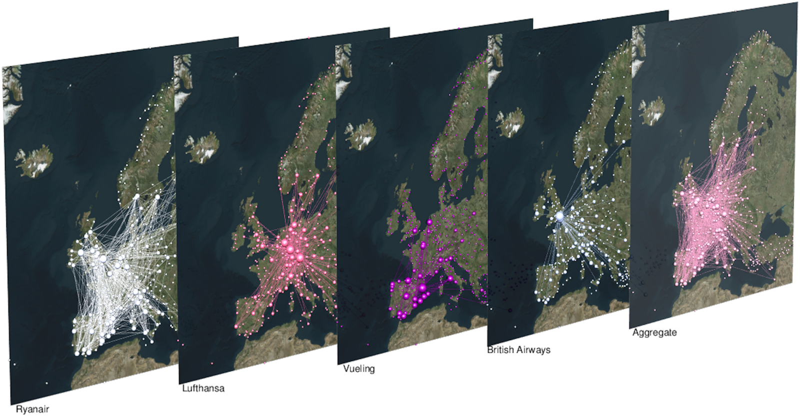
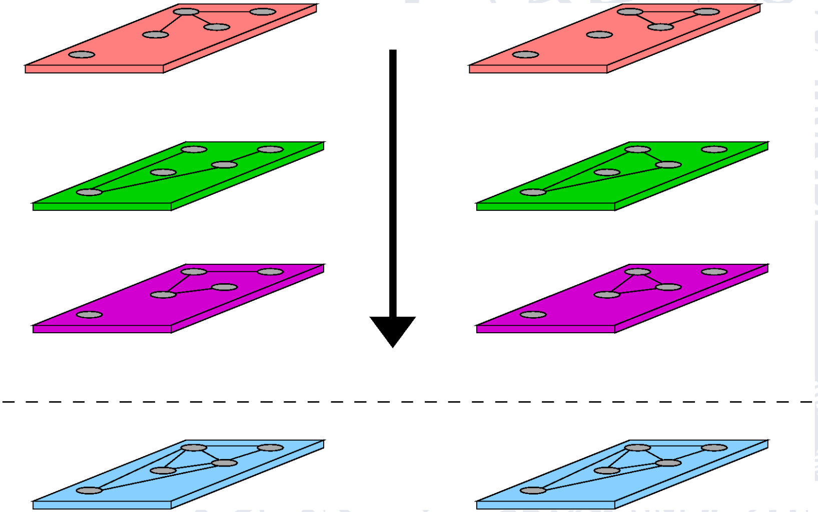
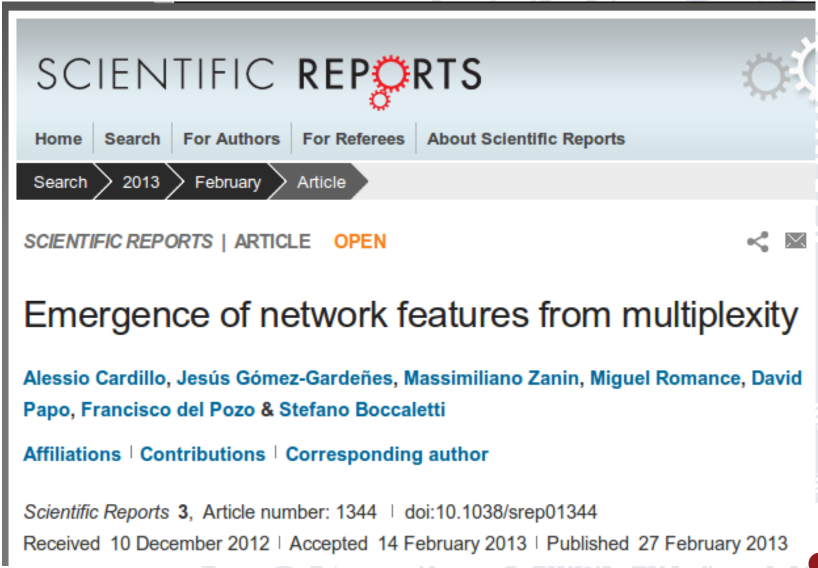


Multiplexity & topological properties



Multiplexity & topological properties



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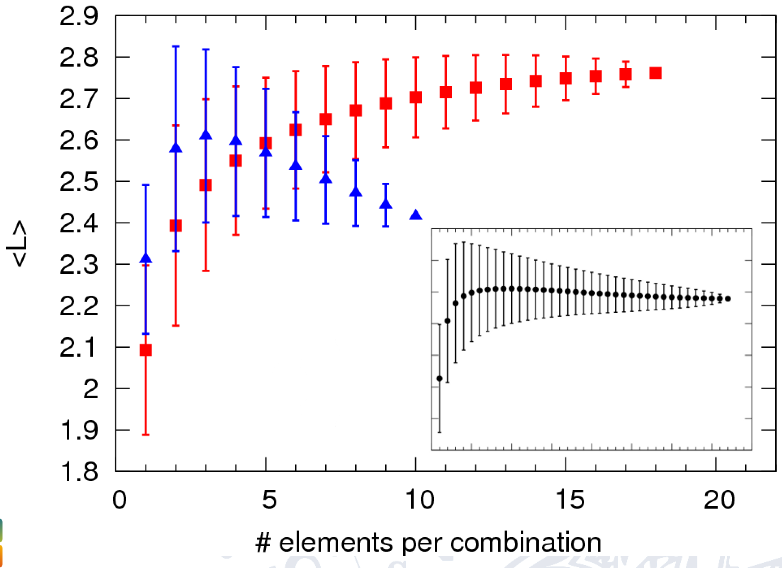
Affiliations | Contributions | Corresponding author

Scientific Reports **3**, Article number: 1344 | doi:10.1038/srep01344

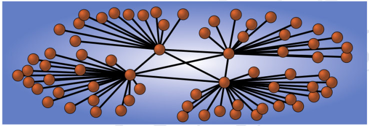
Received 10 December 2012 | Accepted 14 February 2013 | Published 27 February 2013

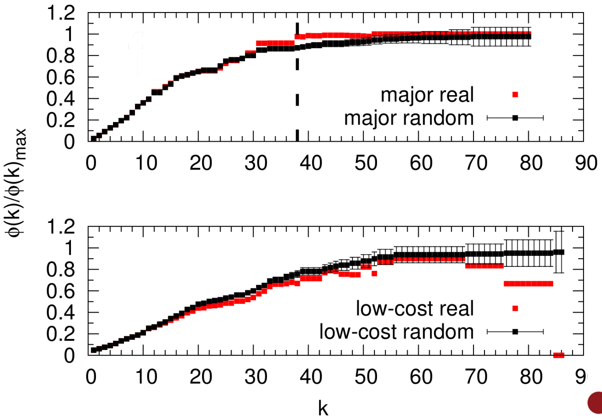
Multiplexity & topological properties

Multiplexity & topological properties



Multiplexity & topological properties





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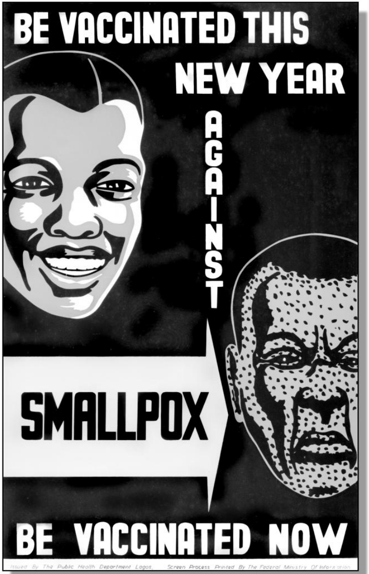
Interdependent processes



Interdependent processes

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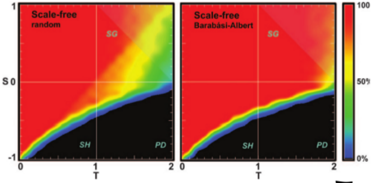
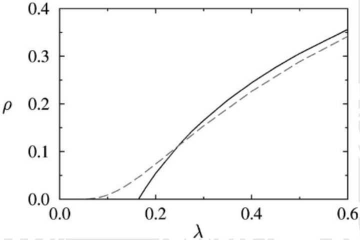


Cooperation

VS

Spreading



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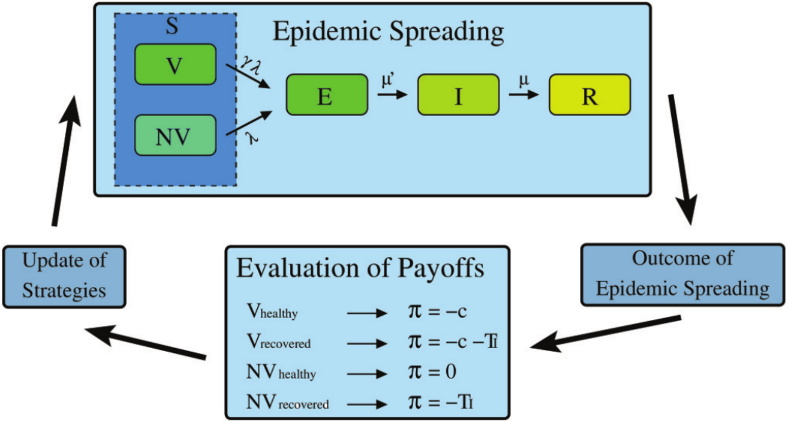
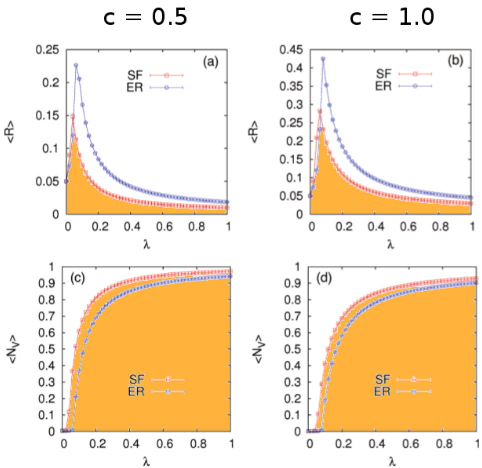


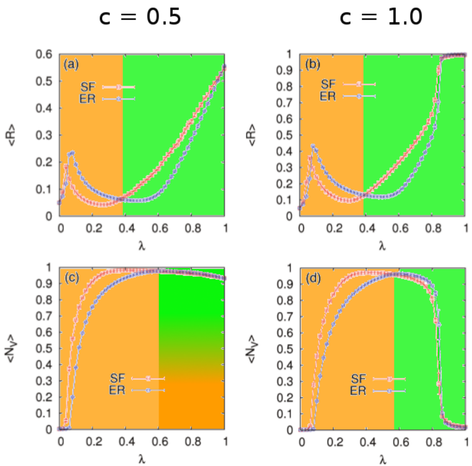
Interdependent processes

ideal case $\gamma = 0$



Interdependent processes

real case $\gamma \neq 0$



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Alessio Cardillo[1,2a], Massimiliano Zanin[3,4,5b], Jesús Gómez-Gardeñes[1,2c], Miguel Romance[3,6d], Alejandro J. García del Amo[3,6e] and Stefano Boccaletti[3f]

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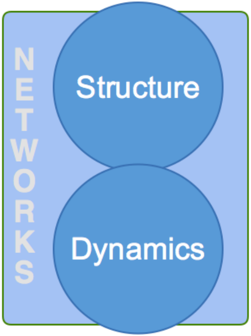
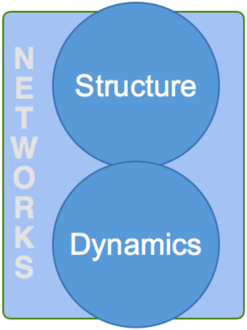
Section 4

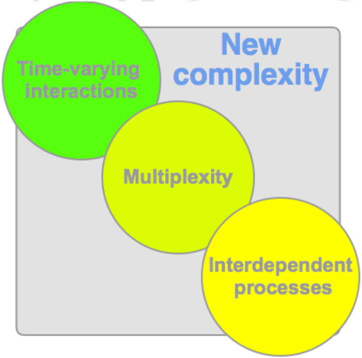
Conclusions

Summing up . . .

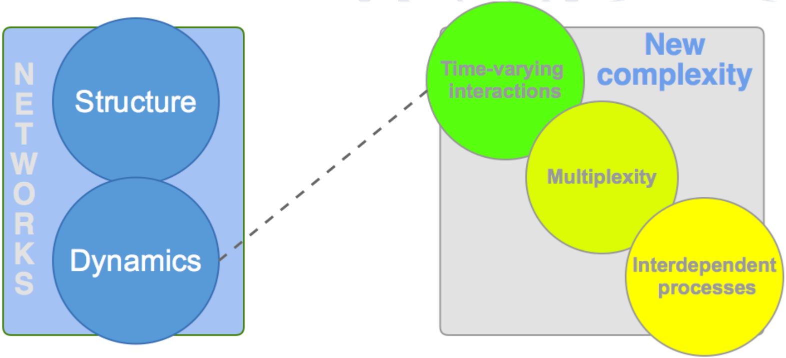


Summing up . . .


NETWORKS
Structure
Dynamics


Time-varying interactions
New complexity
Multiplexity
Interdependent processes

Summing up . . .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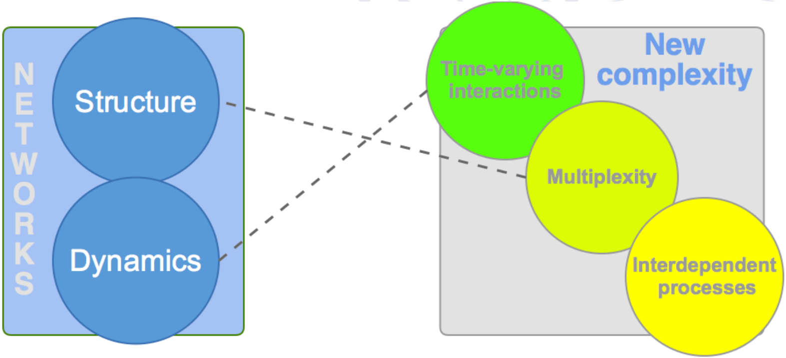
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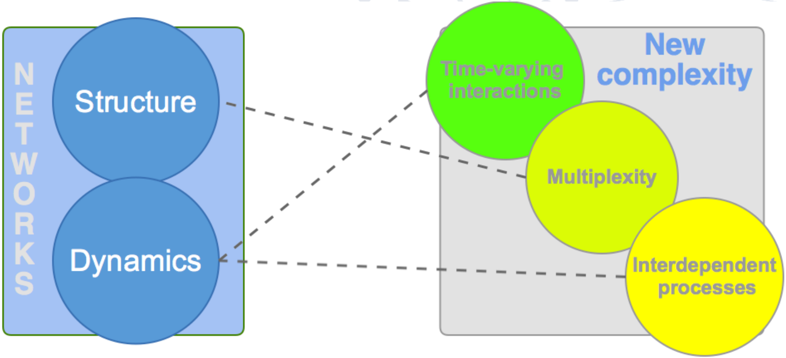
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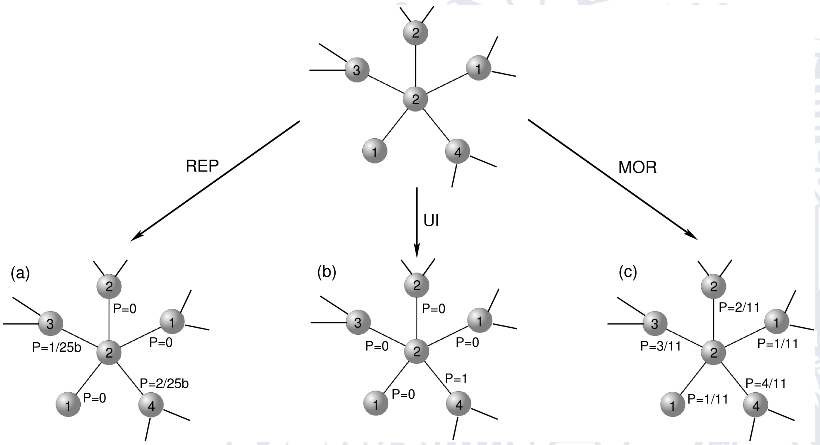
Part II

Appendix

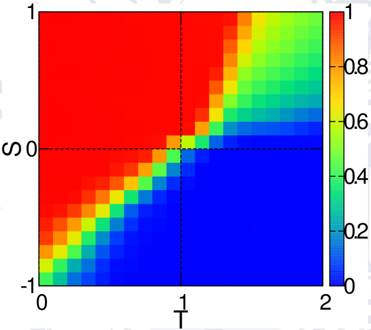
Evolutionary rules



Cooperation diagram

We measure the cooperation level as:

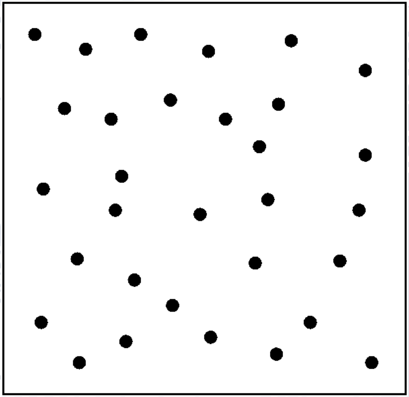
$$\langle C(T,S)_{\Delta t}\rangle = \frac{1}{Q}\sum_{i=1}^{Q}\frac{N_c^i}{N},$$



Cooperation & Time-varying interactions

The model

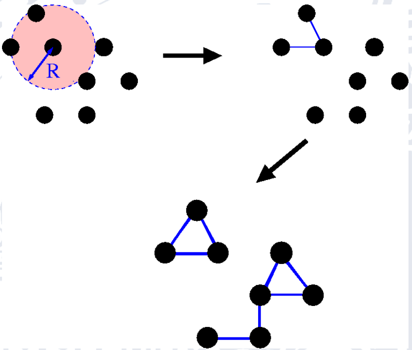
- Agents are scattered randomly on a surface;



Cooperation & Time-varying interactions

The model

- Agents are scattered randomly on a surface;
- Agents interact through proximity with those agents within a radius *R*;

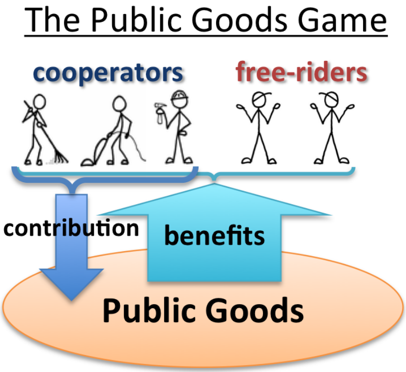


Cooperation & Time-varying interactions

The model

- Agents are scattered randomly on a surface;
- Agents interact through proximity with those agents within a radius R;
- Agents play a public goods game

$$r\, n_{\text{coop}}\, c \rightarrow \begin{cases} \dfrac{r\, n_{\text{coop}}\, c}{N} - c \\ \dfrac{r\, n_{\text{coop}}\, c}{N} \end{cases}$$



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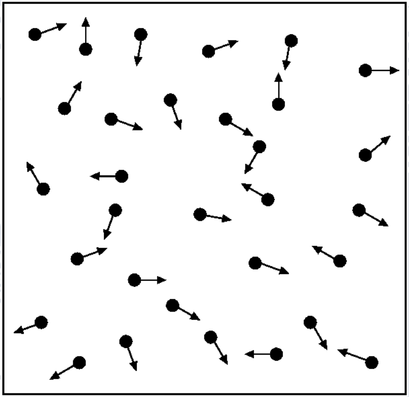
- Agents move at random.



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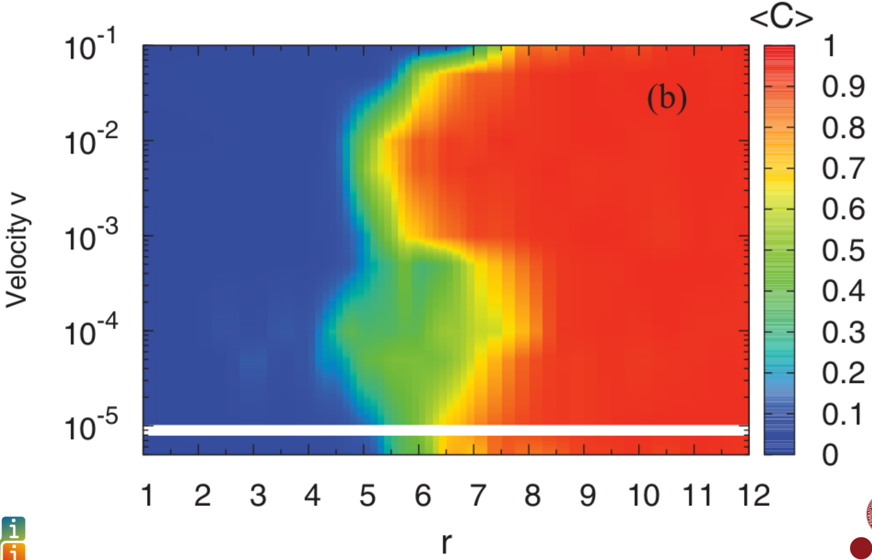
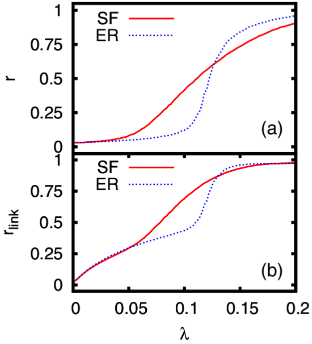


Synchronization



- Arenas A, Díaz-Guilera A, Kurths J, Moreno Y, & Zhou C. *Synchronization in complex networks*. Phys. Rep., **469**, 93 (2008).
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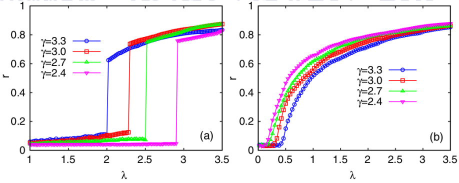


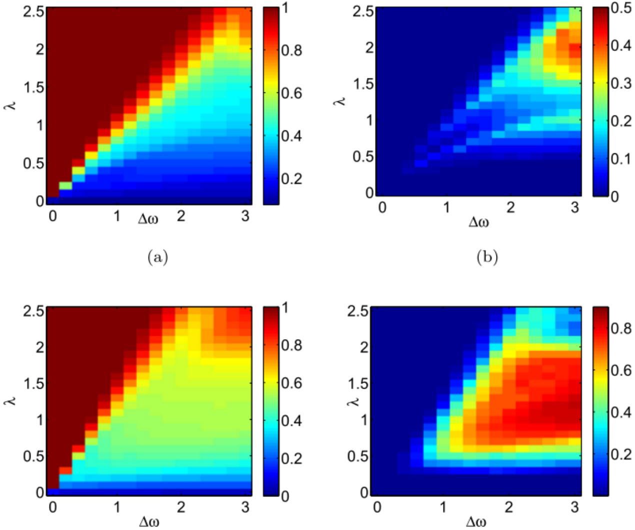
Kuramoto Model

Fundamental relations

$$\frac{d\theta_i}{dt} = \omega_i + \frac{\lambda}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i)\,, \qquad i = 1, \ldots, N \qquad (1)$$

$$r\,e^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}\,. \qquad i = 1, \ldots, N \qquad (2)$$

Remote synchronization



Activity driven model

The model

Consider N nodes and assign to each node i an **activity rate** $a_i = \eta x_i$, defined as the probability per unit time to create new contacts or interactions with other individuals. Then, a simple generative process is built, according to the following rules:

- At each discrete time step t the network G_t starts with N disconnected vertices;

- Perra N, *et al.* . "Activity driven modeling of time varying networks", Scientific Reports **2**, 469 (2012).

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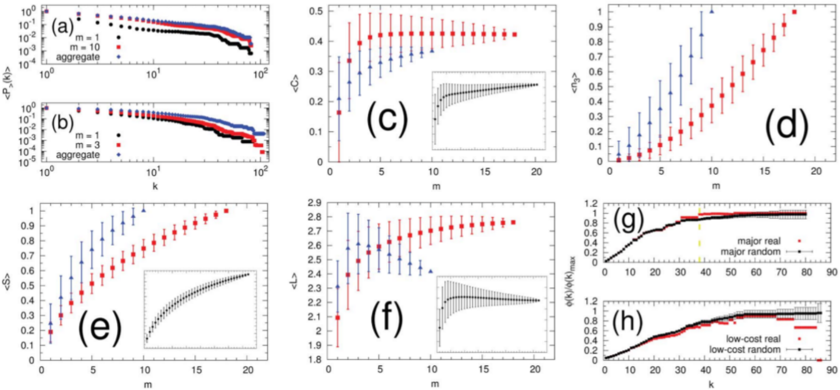
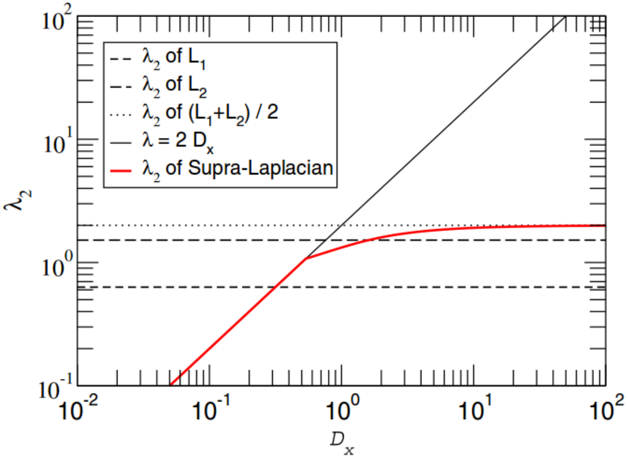
- With probability $a_i \Delta t$ each vertex i becomes active and generates m links that are connected to m other randomly selected vertices. Non-active nodes can still receive connections from other active vertices;
- At the next time step $t + \Delta t$, all the edges in the network G_t are deleted. From this definition it follows that all interactions have a constant duration $t_i = \Delta t$.

- Perra N, *et al.* . "Activity driven modeling of time varying networks", Scientific Reports **2**, 469 (2012).

Airline results



Diffusivity



- Gómez S, Díaz-Guilera A, Gómez-Gardeñes J, Pérez-Vicente C J, Moreno Y, & Arenas A. *Diffusion Dynamics on Multiplex Networks*. Phys. Rev. Lett. **110**, 028701 (2013).