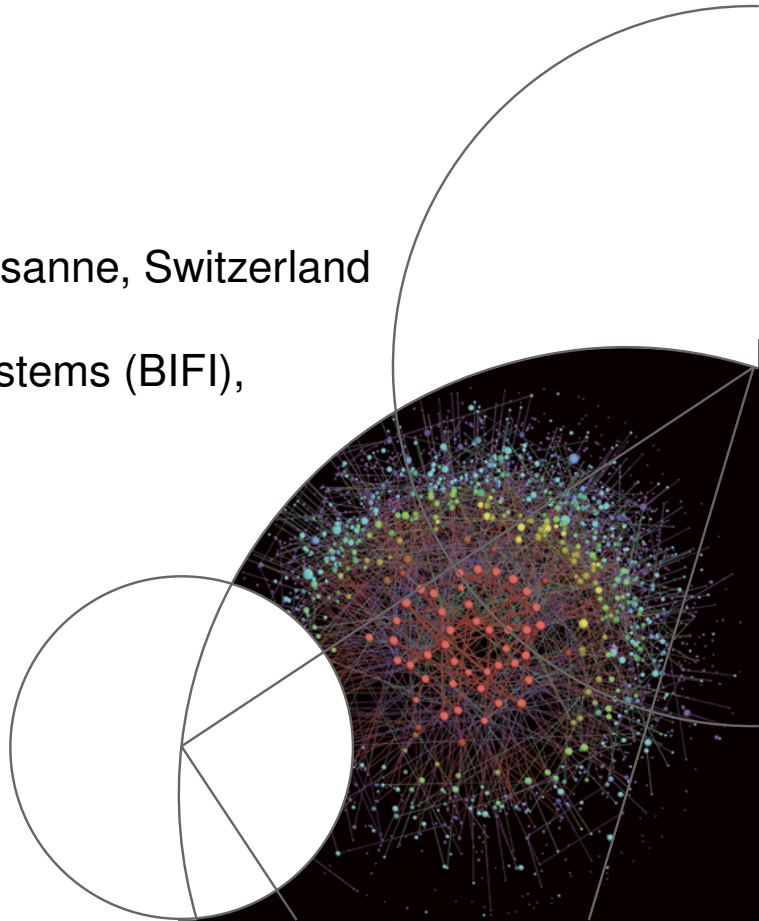
Beyond simple complex-networks: coevolution, multiplexity, and time-varying interactions

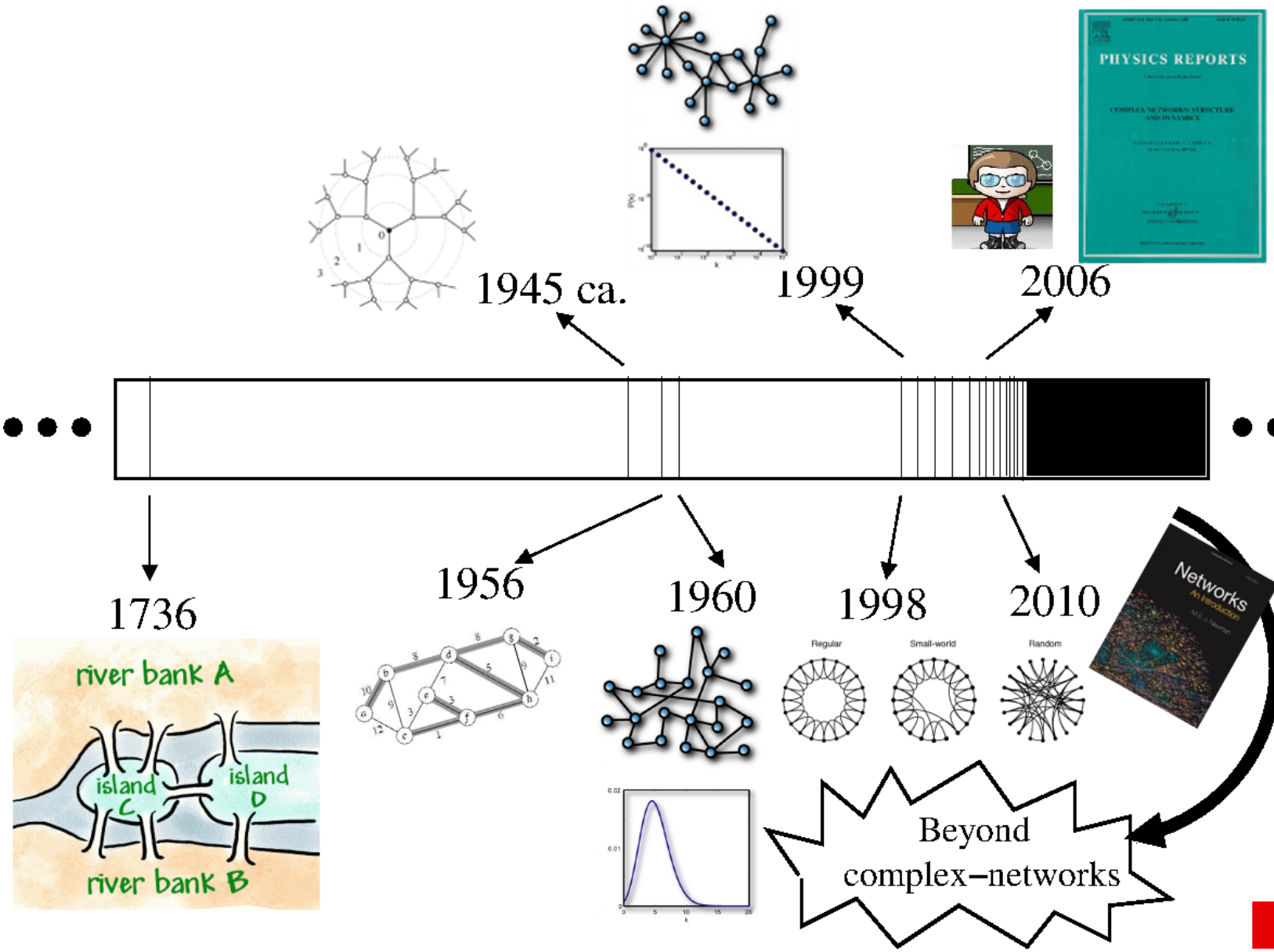
Alessio Cardillo
École polytechnique fédérale de Lausanne (EPFL), Lausanne, Switzerland
&
Institute for Biocomputation and Physics of Complex Systems (BIFI), Zaragoza, Spain

http://bifi.es/~cardillo/



Tuesday 12-th May 2015, Sguardi sulla complessità, Firenze, Italy

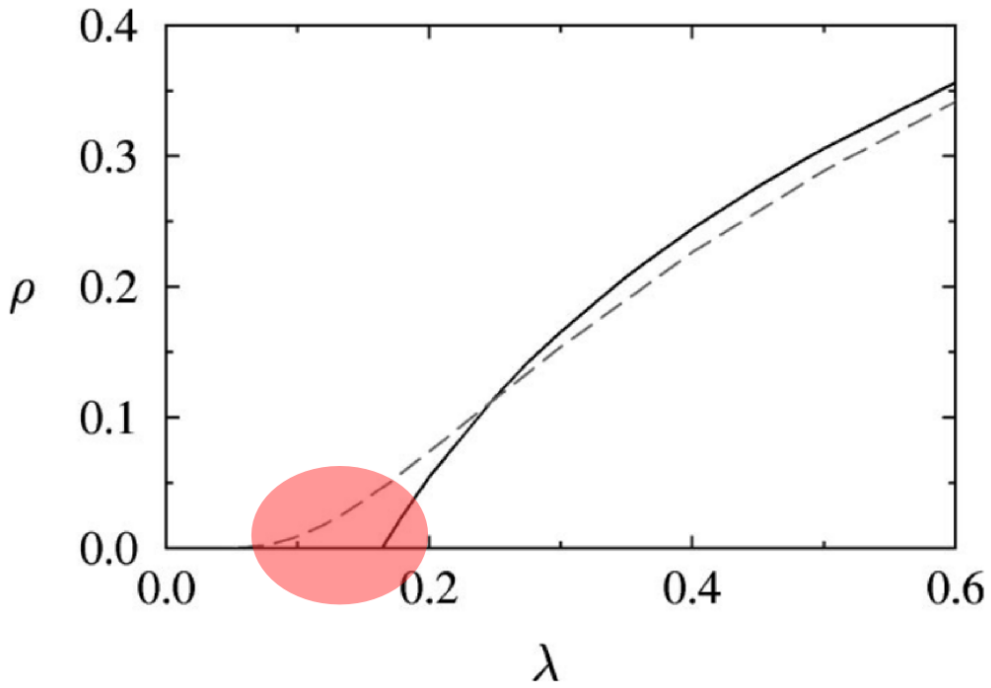
Once upon a time . . .



Success stories

Some examples

- Spreading of infections



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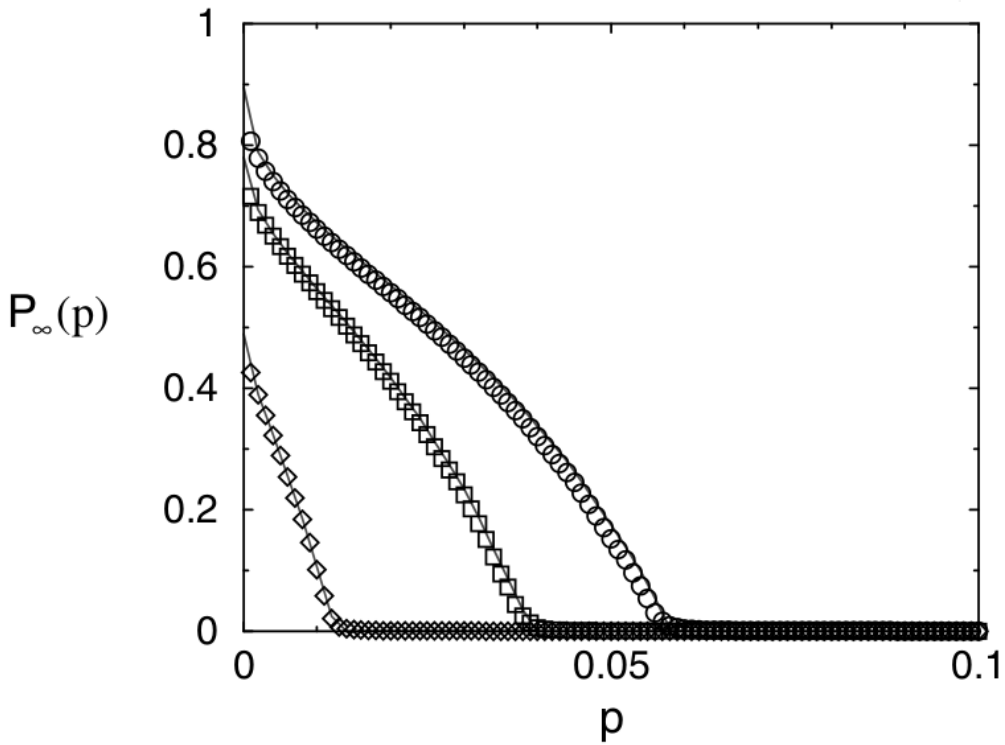
- Pastor-Satorras R, & Vespignani A. *Epidemic dynamics and endemic states in complex networks*. Phys. Rev. E, **63**, 066117 (2001).

Success stories

Some examples

- Spreading of infections
- Percolation/critical infrastructures



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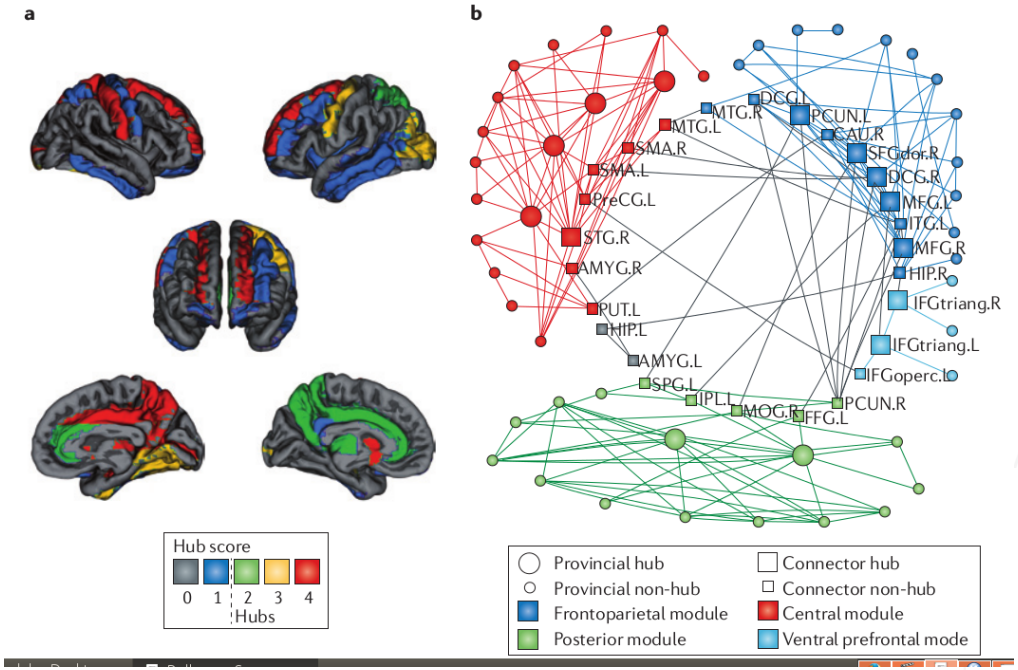
Success stories

Some examples

- Spreading of infections
- Percolation/critical infrastructures
- Biological networks



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Complex networks are not enough . . .

- Networks cannot explain certain phenomena

Complex networks are not enough . . .

Model

Reality

- Networks cannot explain certain phenomena
- Models are not too representative of real systems.

Complex networks are not enough . . .

- Networks cannot explain certain phenomena
- Models are not too representative of real systems.
- Big Data

Making one leap forward

1. Time-varying interactions $\rightarrow$ Time Varying Graph.

Making one leap forward

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2. Multiple kind of interactions $\rightarrow$ Multiplex Networks.

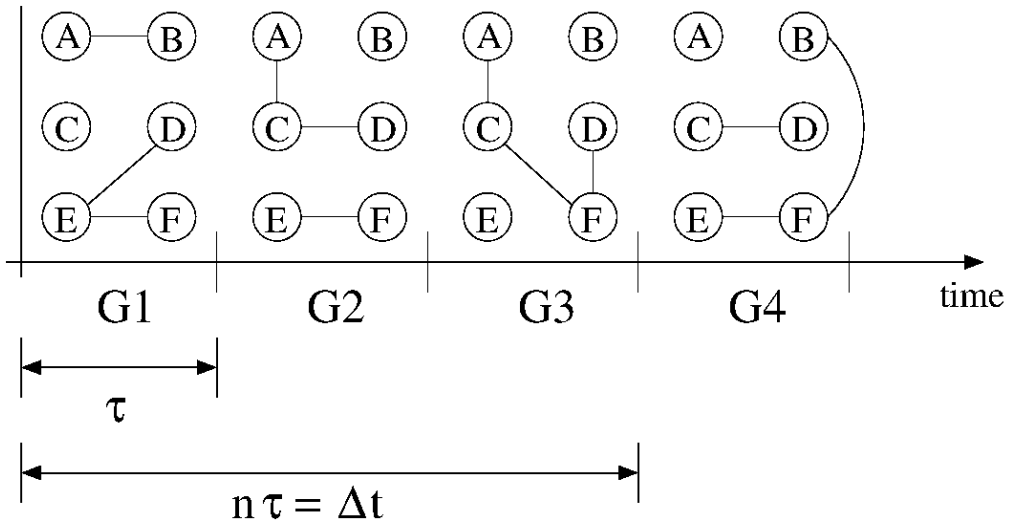
Making one leap forward

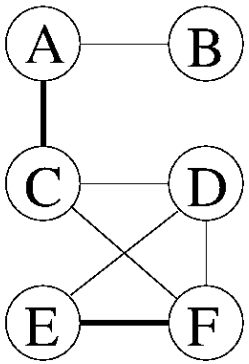
1. Time-varying interactions $\rightarrow$ Time Varying Graph.
2. Multiple kind of interactions $\rightarrow$ Multiplex Networks.
3. Coevolution $\rightarrow$ Interdependent Dynamics.

Section 1

Time-varying interactions

Time Varying Graphs





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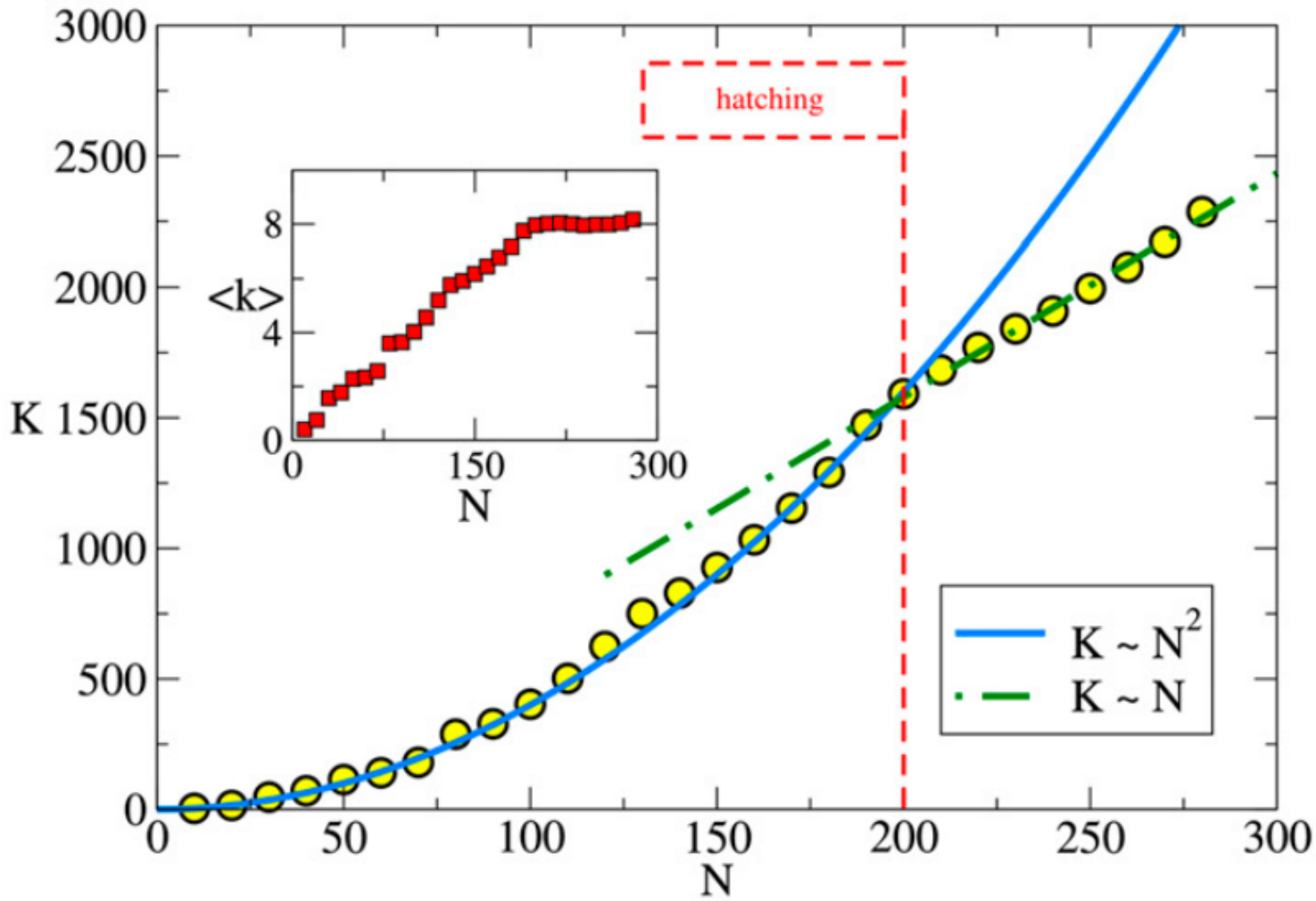
Time Varying Graphs



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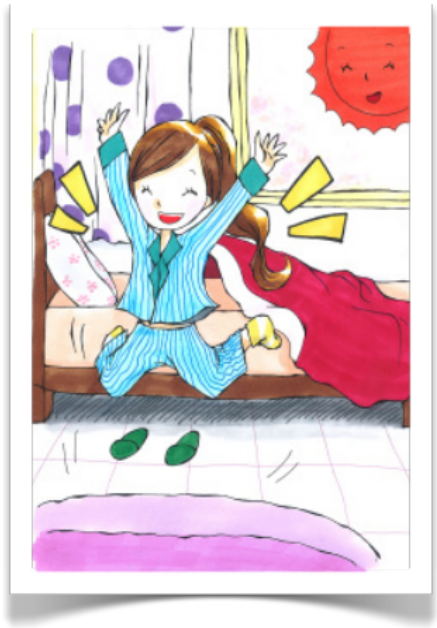
Spreading of infections on time-varying graph

Spreading of infections on time-varying graph

Compartmental models

S - Susceptible (Healty)

I - Infected (and infectious)

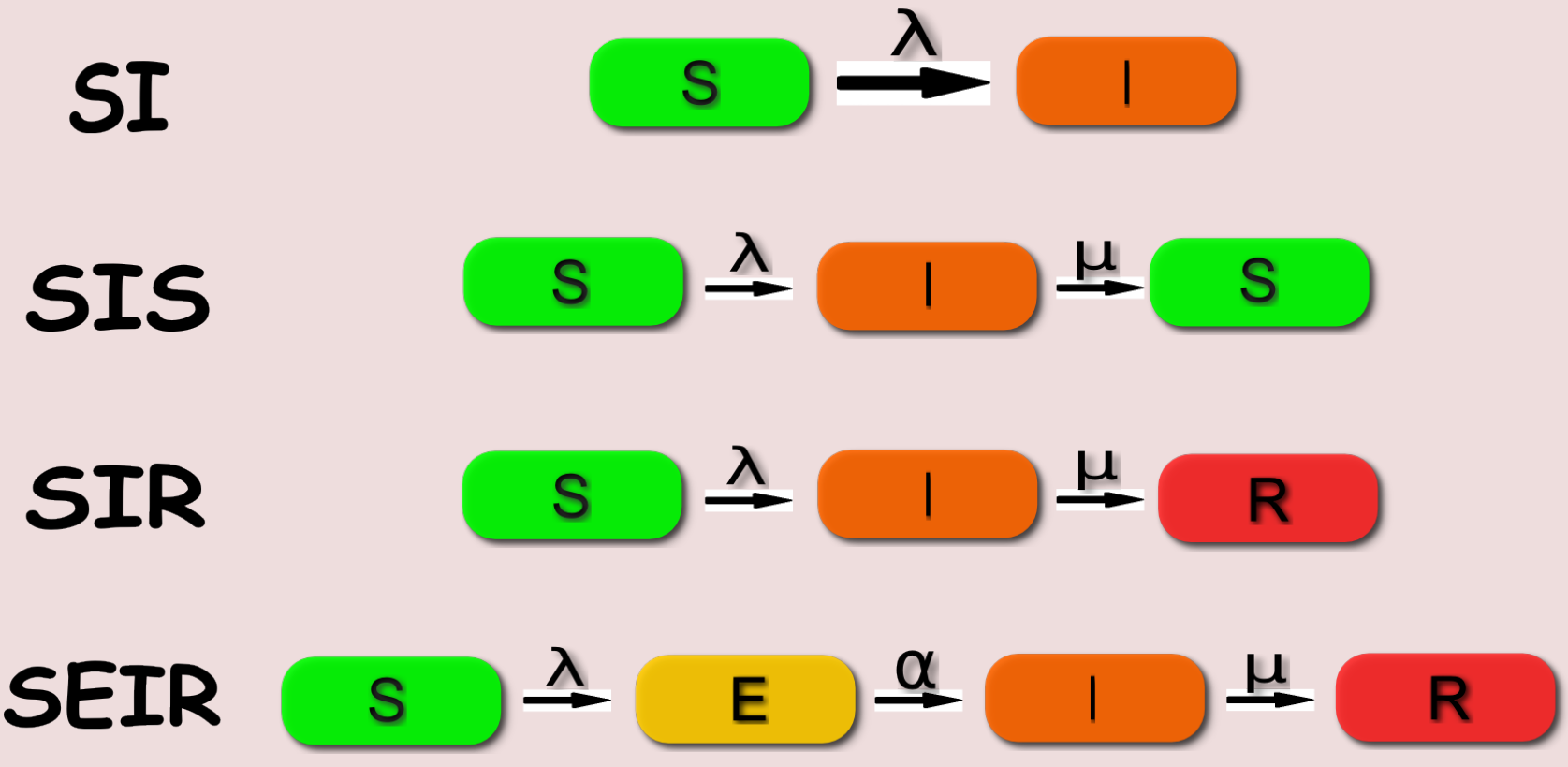


R - Recovered (immune/dead)

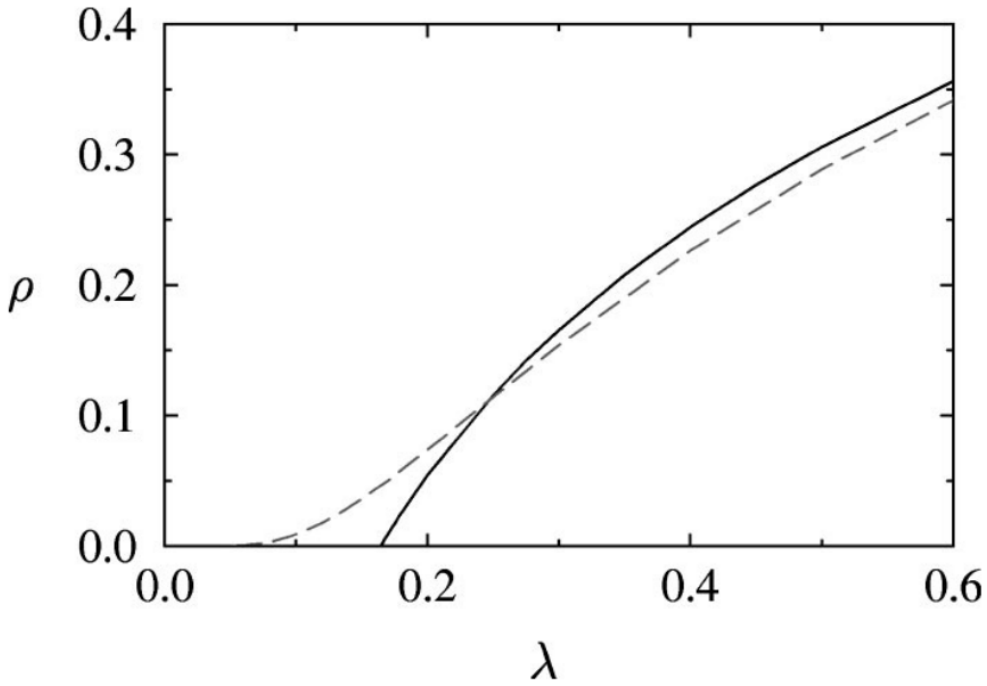
(From Petter Holme's blog)

Spreading of infections on time-varying graph

Compartmental models



Spreading of infections on time-varying graph



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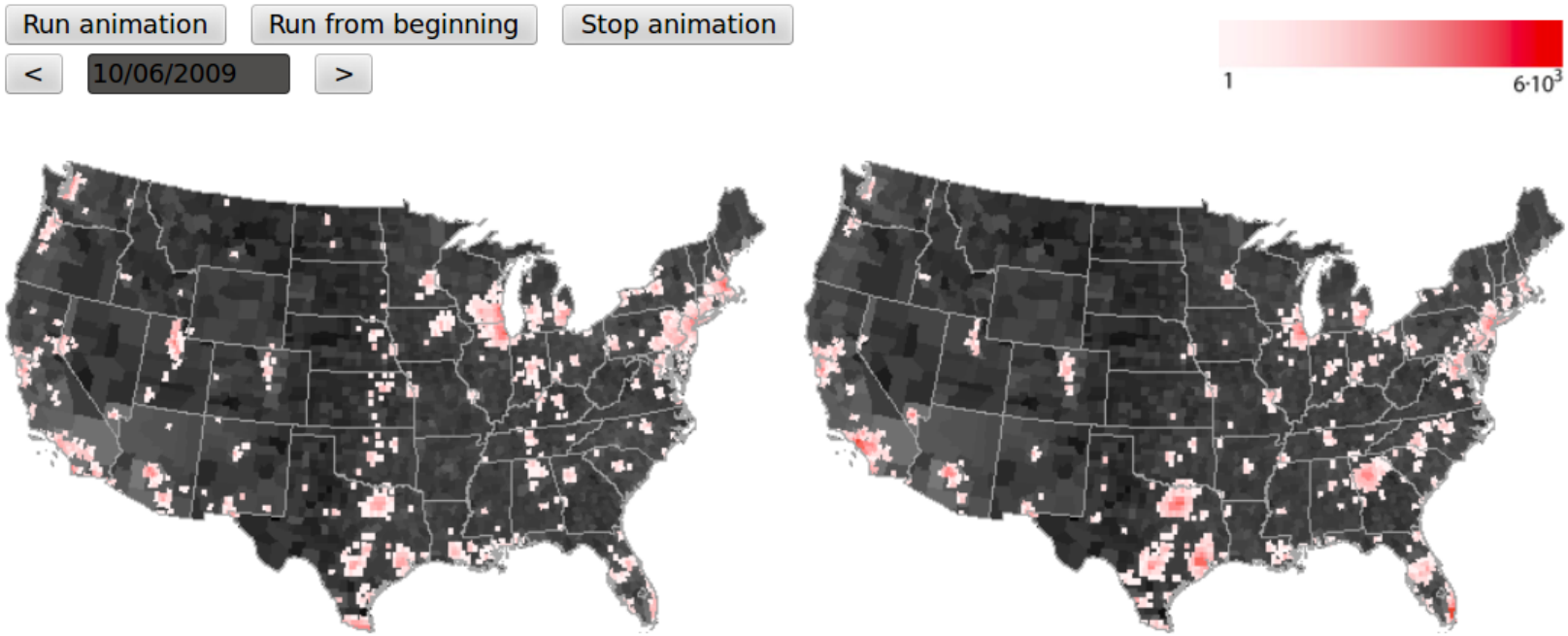
Spreading of infections on time-varying graph

Time-varying features

- Topology
- Location of agents
- Propagation dynamic

Spreading of infections on time-varying graph

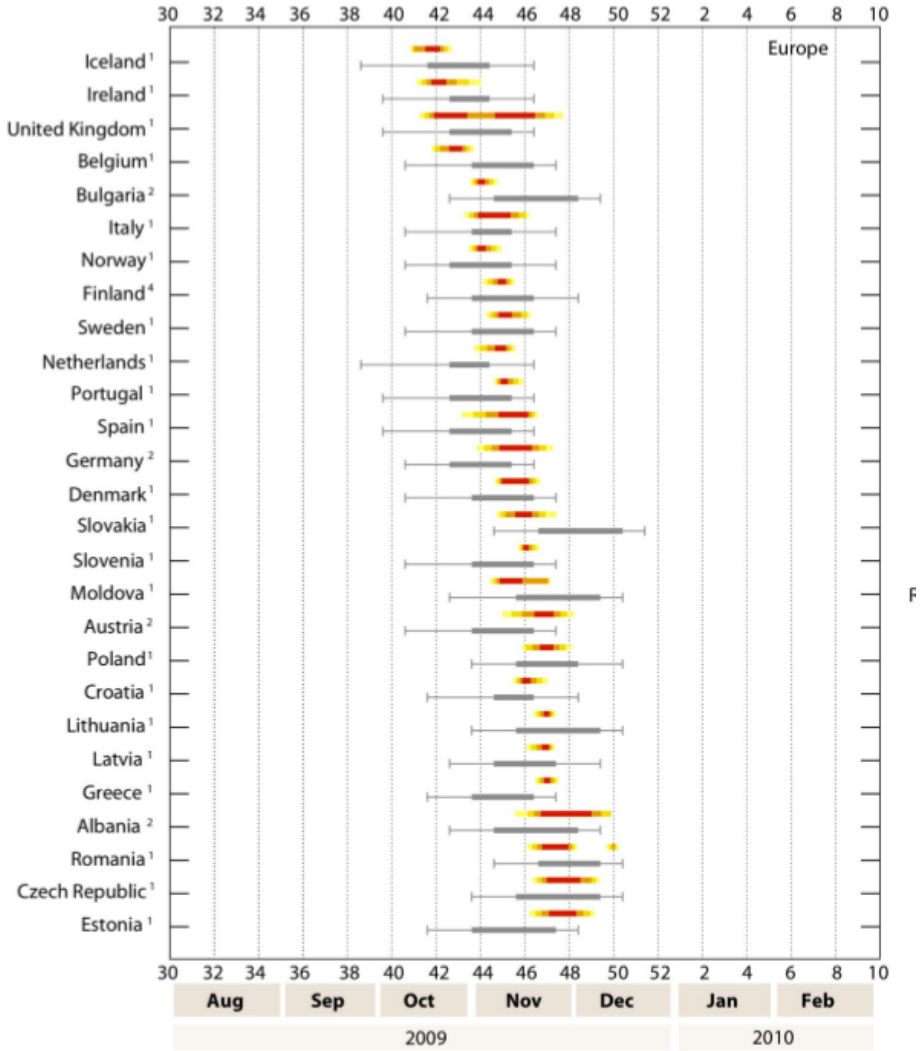


Real cases map.

Simulated cases map.

- Tizzoni *et al. Real-time numerical forecast of global epidemic spreading: case study of 2009 AH1N1pdm*. BMC Medicine, **10**, 165 (2012).
- Starnini, M., *et al. Immunization strategies for epidemic processes in time-varying contact networks*. Jour. Theo. Bio., **337C**, 89–100. (2013).

Spreading of infections on time-varying graph

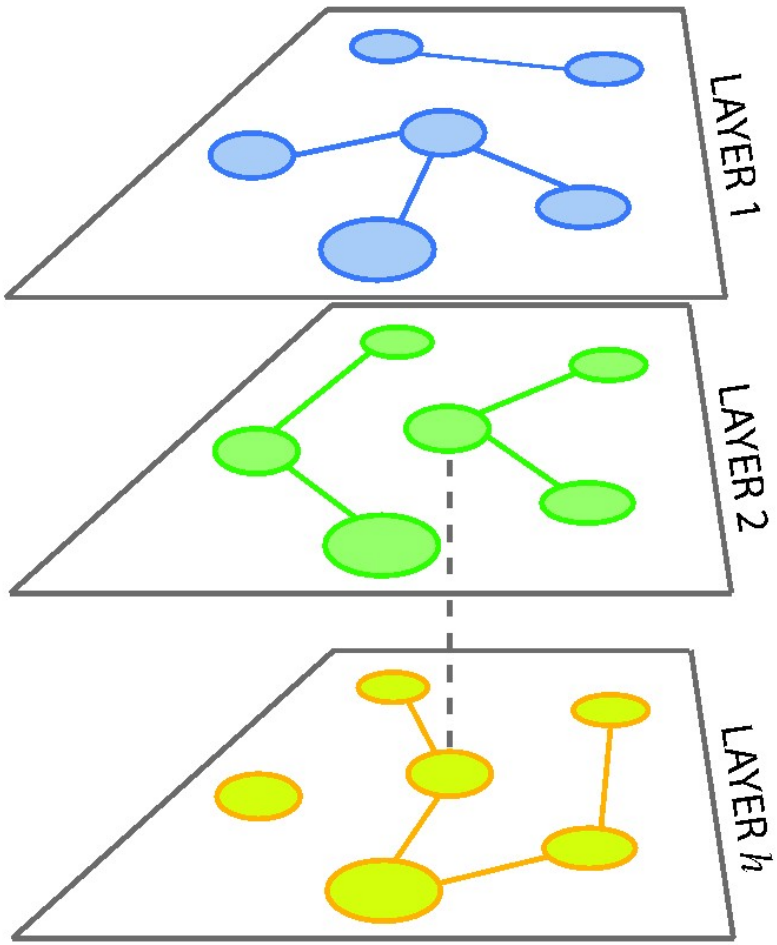


- Tizzoni *et al. Real-time numerical forecast of global epidemic spreading: case study of 2009 AH1N1pdm.* BMC Medicine, **10**, 165 (2012).

Section 2

Multiplexity – Networks of Networks

Multiplex networks



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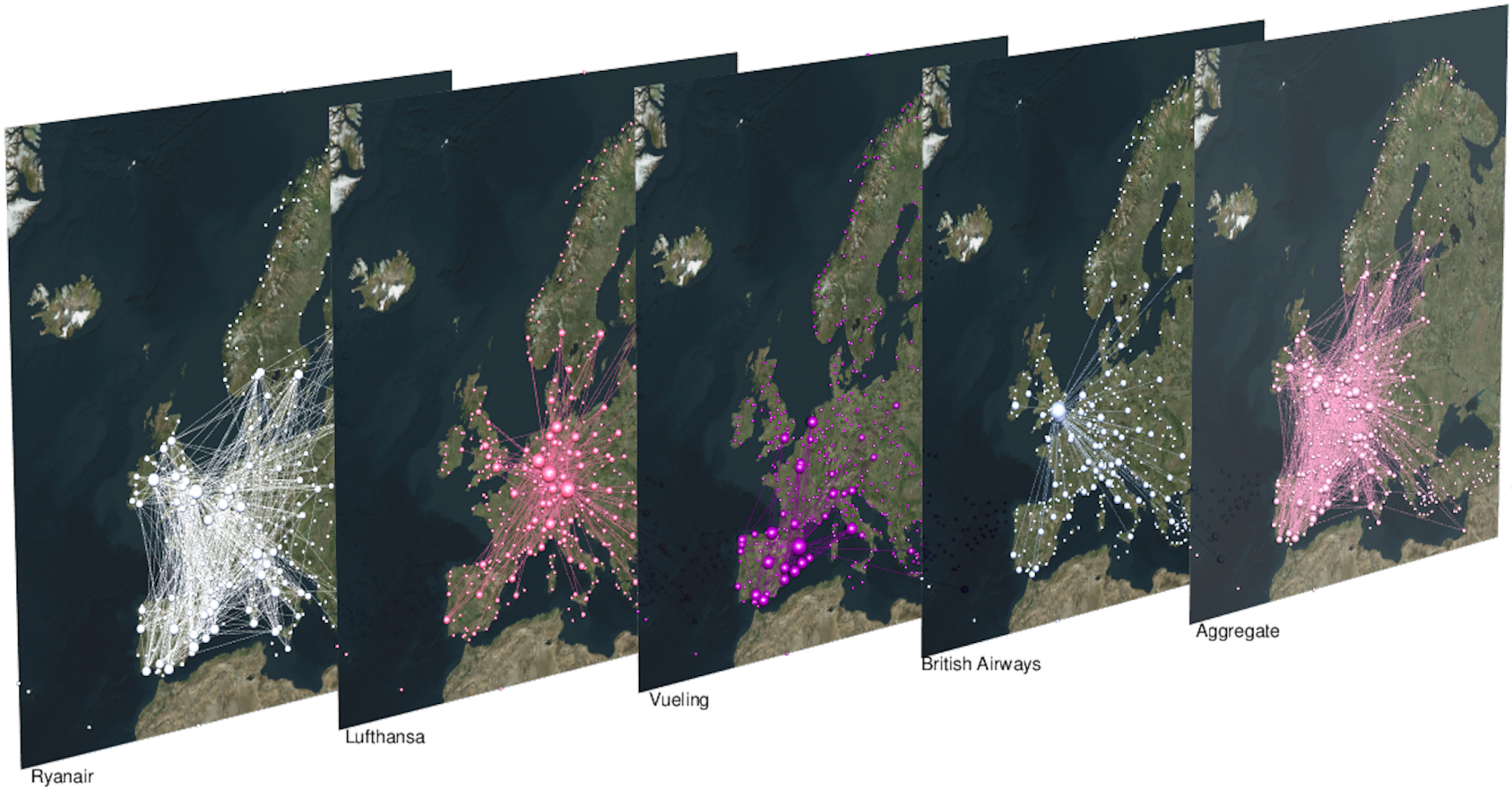
Multiplex networks



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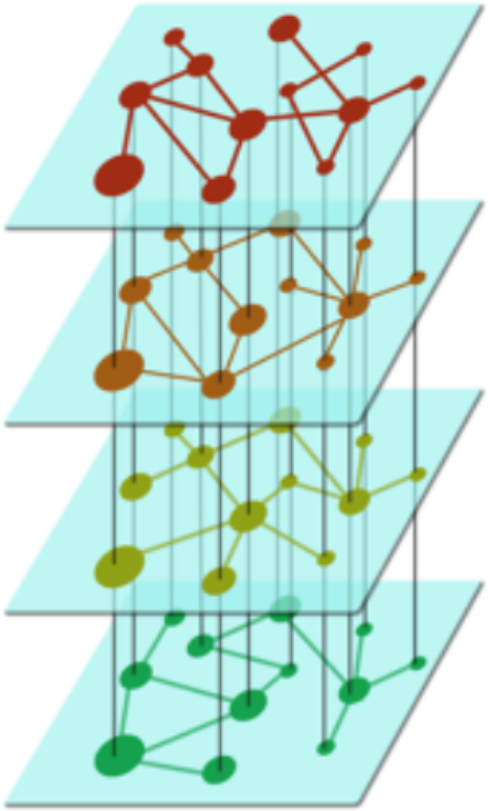
- `https://github.com/manlius/muxViz`

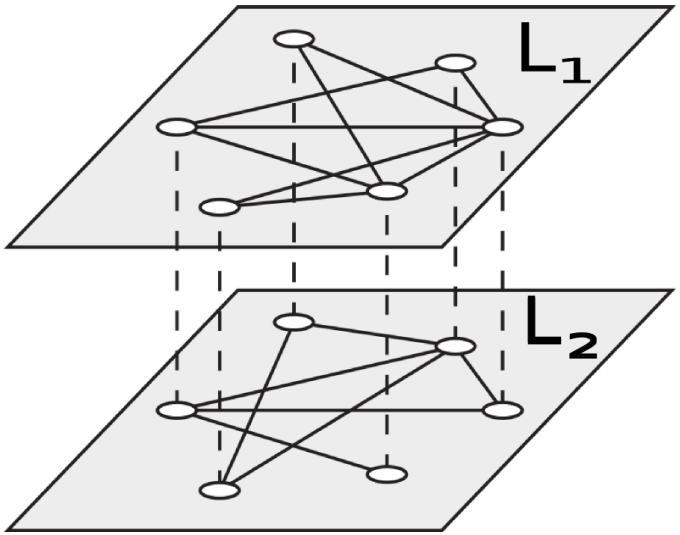
Multiplex networks



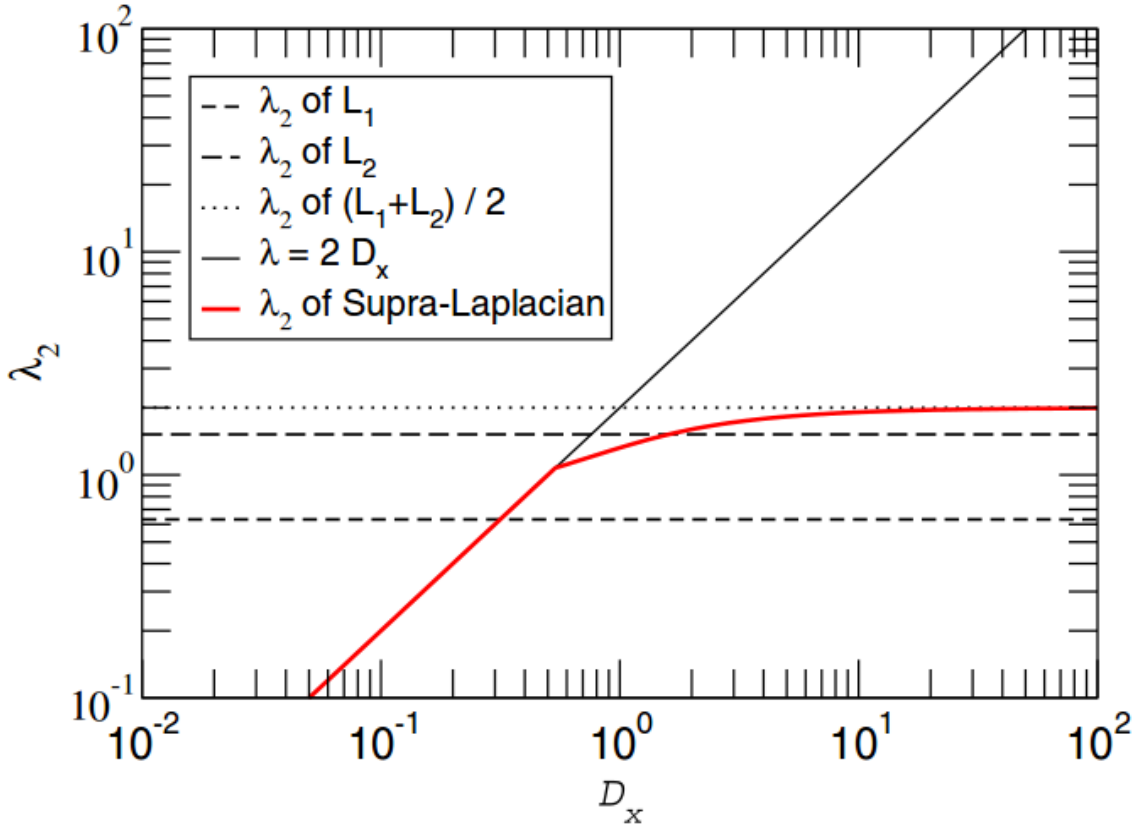
Supra-adjacency matrix

$$\mathcal{A} = \begin{pmatrix} A_1 & \mathbb{I} & 0 & 0 \\ \mathbb{I} & A_2 & \mathbb{I} & 0 \\ 0 & \mathbb{I} & A_1 & \mathbb{I} \\ 0 & 0 & \mathbb{I} & A_4 \end{pmatrix}$$

Diffusivity on multiplex



$$\tau \propto \frac{1}{\lambda_2}$$



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Networks of Networks (NoN)

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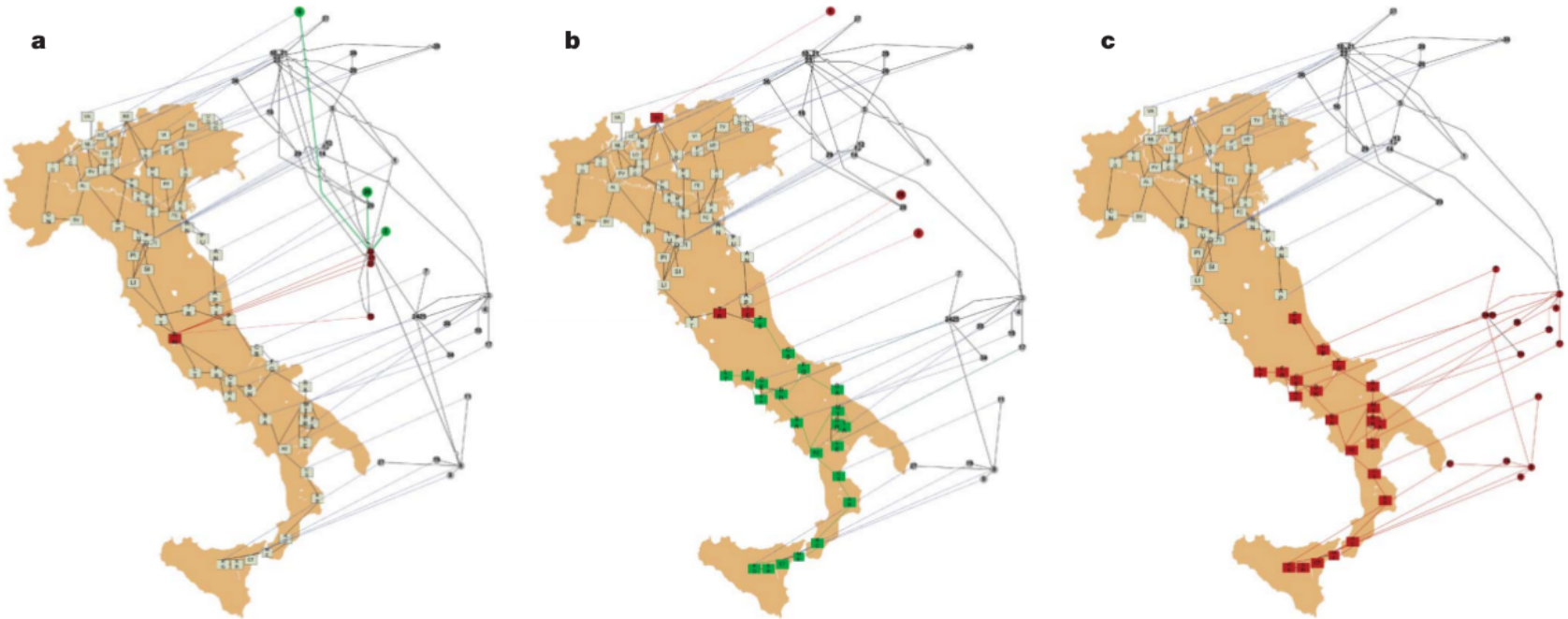
Differences with multiplex

- The same node may not be present in each sub-network.
- Each sub-network could belong to different kind of systems.
- Absence of self-connections between layers.

Networks of Networks (NoN)



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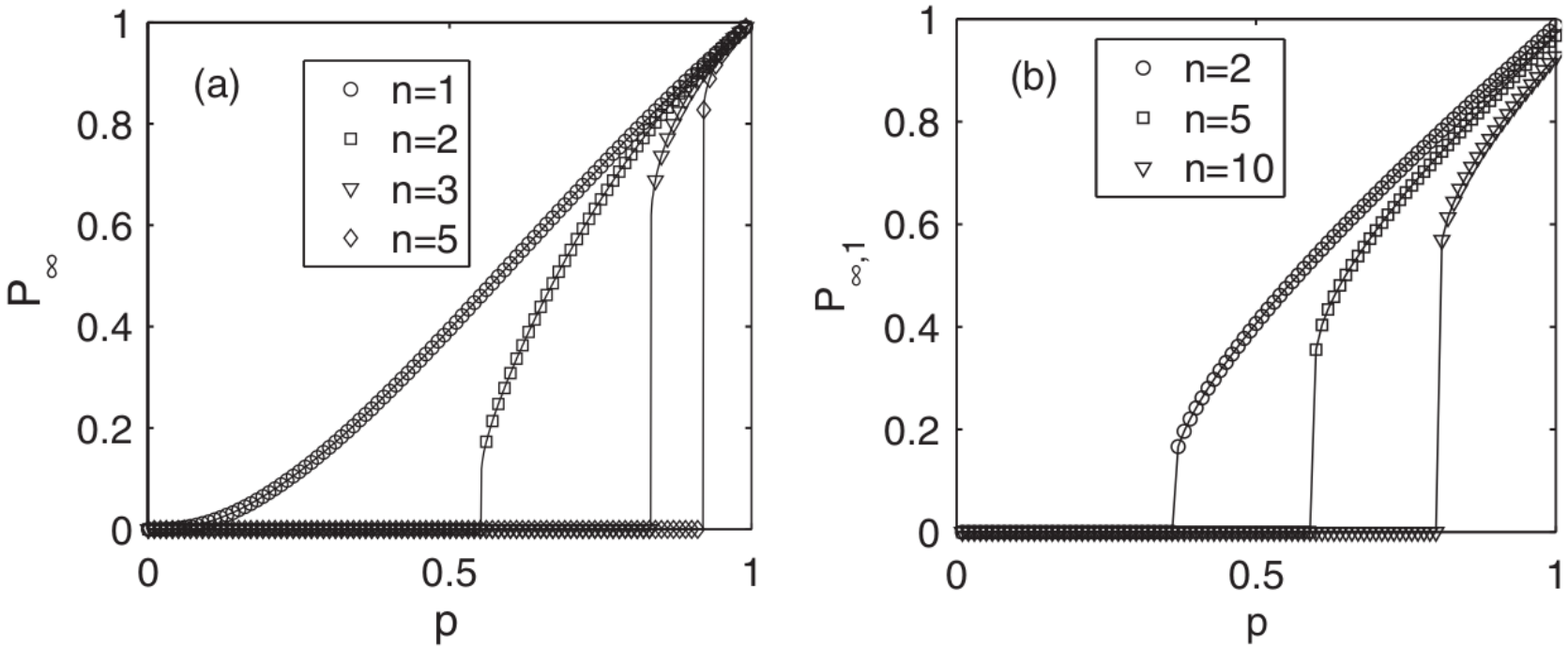
Networks of Networks (NoN)



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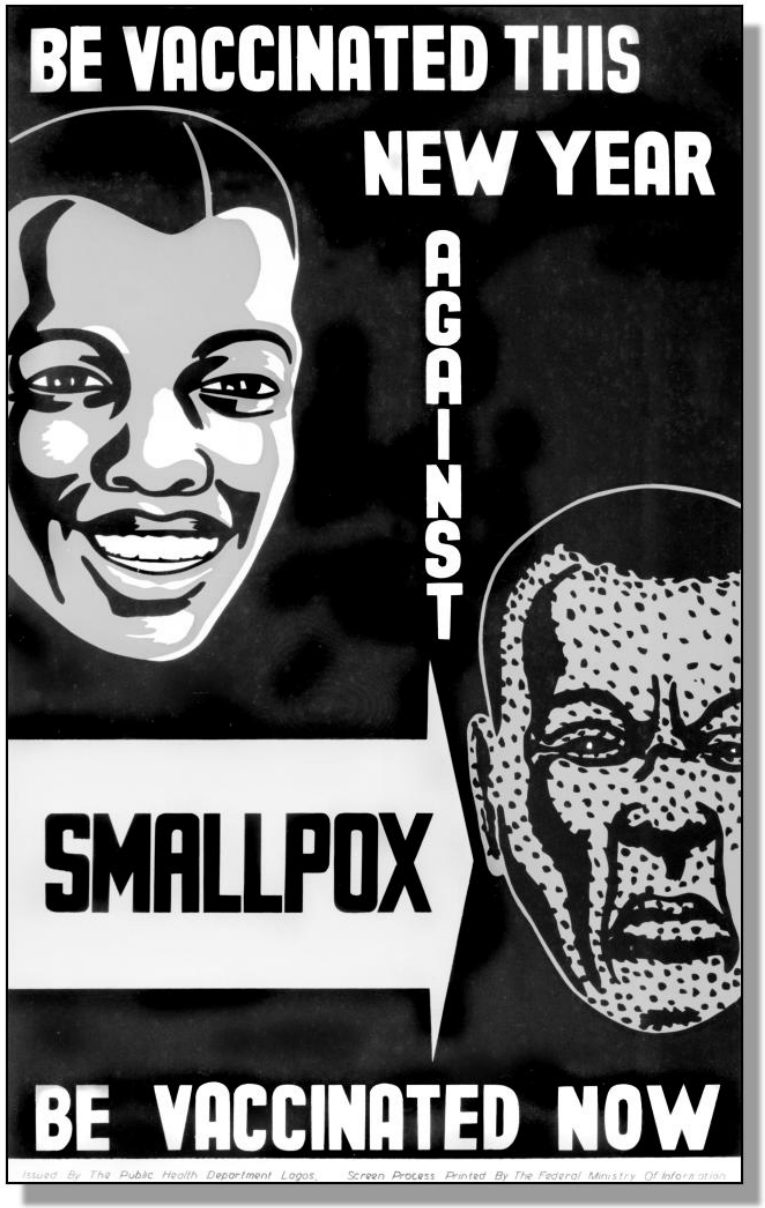
Section 3

Coevolution

Coevolutionary/interdependent dynamics

Evolutionary vaccination



Evolutionary vaccination

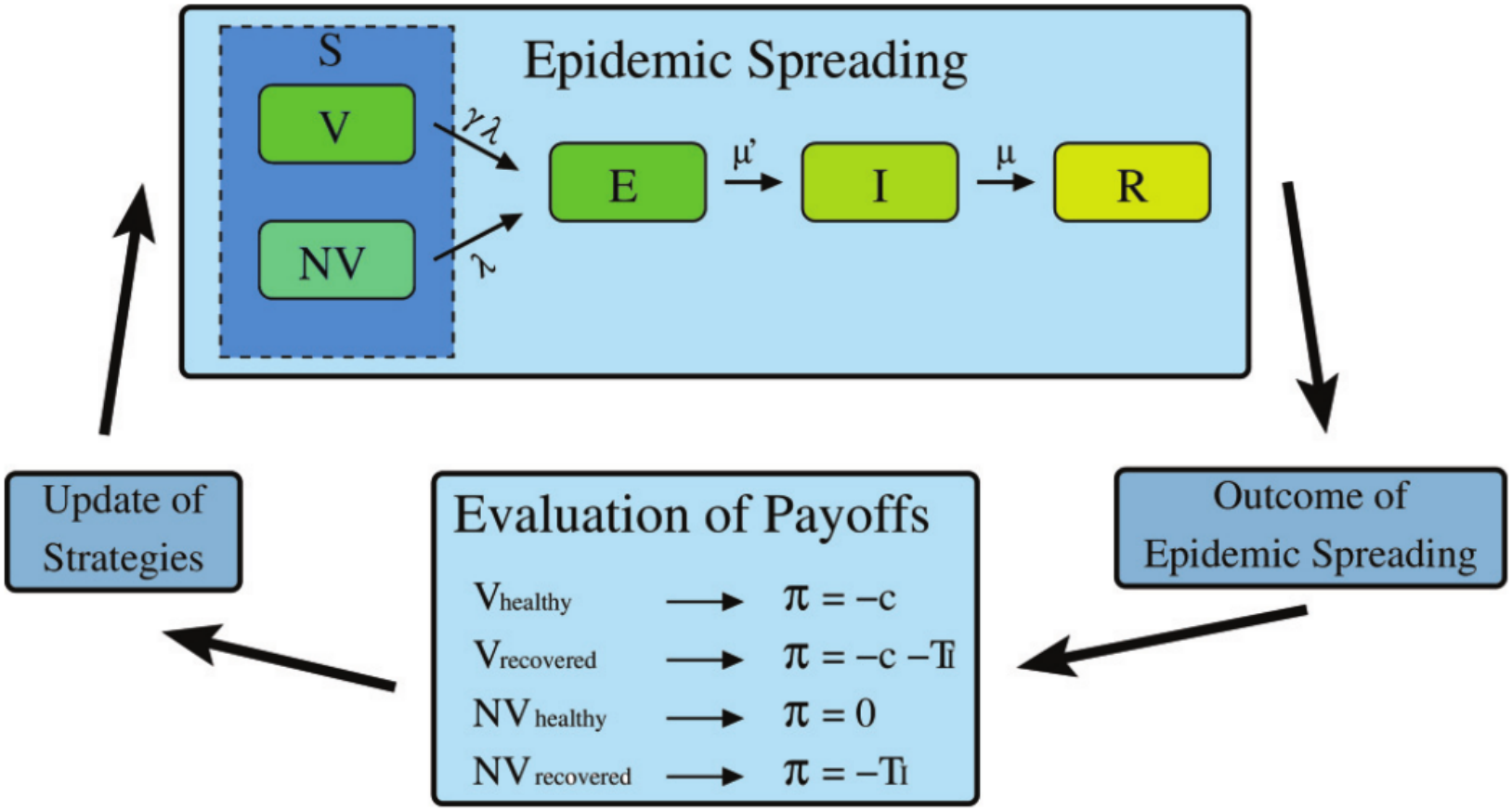
- Immunization strategy $\rightarrow$ spreading
- decision process $\rightarrow$ evolutionary game theory

Both processes are enhanced in scale-free networks!

Evolutionary vaccination

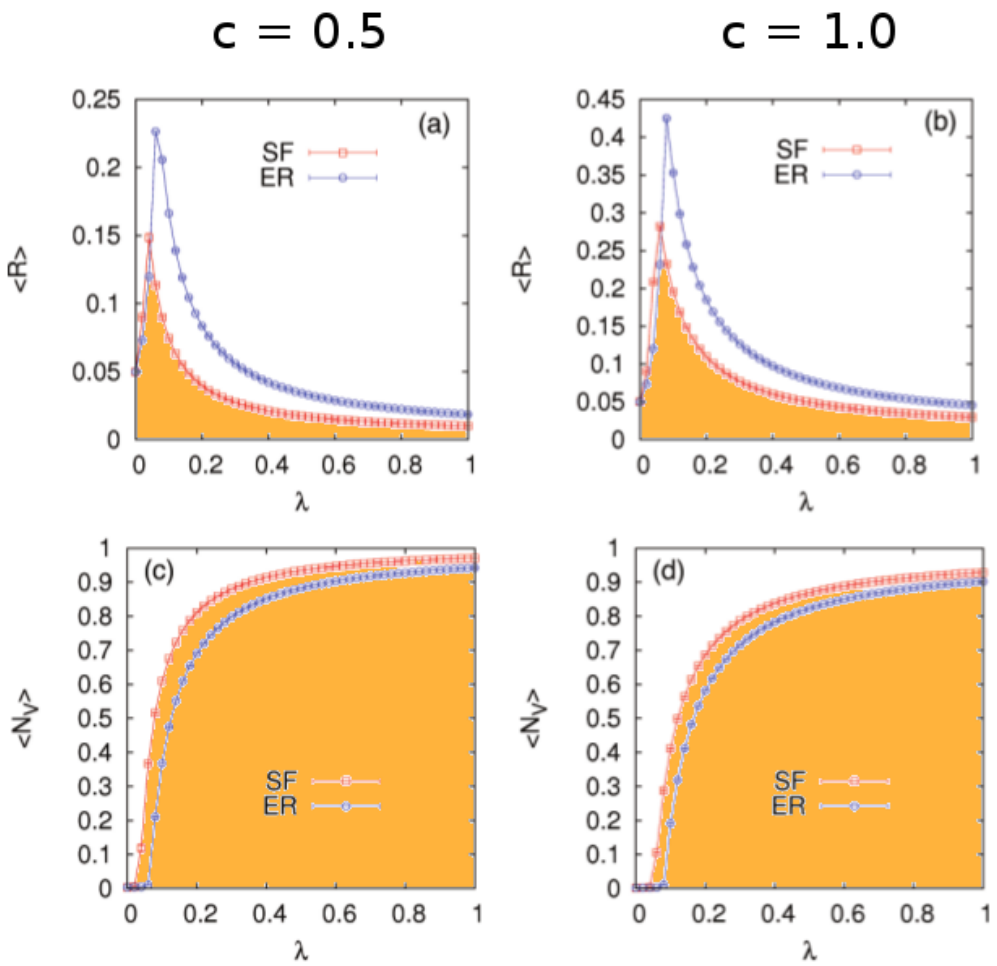
Cardillo A, *et al. Evolutionary vaccination dilemma in complex networks*. Phys. Rev. E, **88**, 032803 (2013).

Evolutionary vaccination



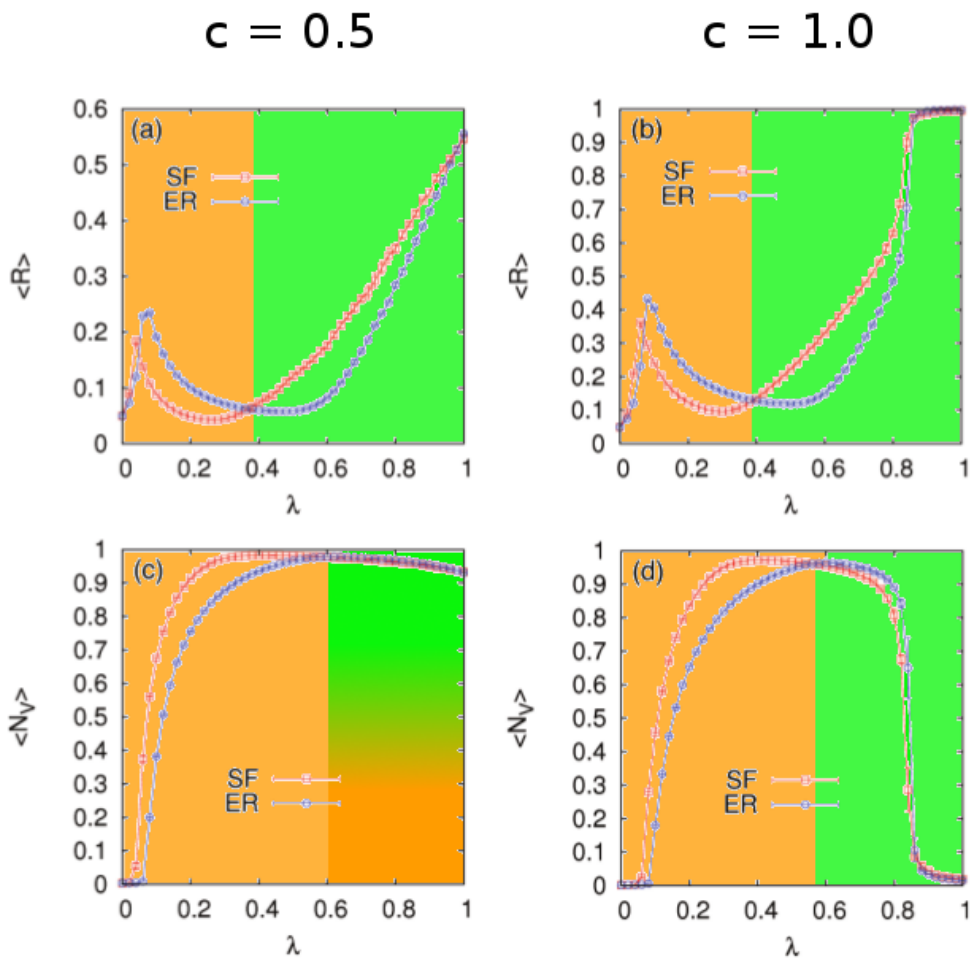
Evolutionary vaccination

ideal case $\gamma = 0$



Evolutionary vaccination

real case $\gamma \neq 0$



Section 4

Conclusions

Summing up . . .

Take home messages

- Inclusion of time dimension in the topology of interactions.
- Possibility to have more than one kind of interaction/links of different nature.
- Evolution of more than one single trait of a system.

Summing up . . .

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- Inclusion of time dimension in the topology of interactions.
- Possibility to have more than one kind of interaction/links of different nature.
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Open problems

- Combination of more than one feature (*i.e.* time-varying + multiplex).
- Coevolutionary emergent behavior.
- Exploit of big data.