Coevolution of Synchronization and Cooperation in Costly Networked Interactions

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ClabB meeting, Universitat de Barcelona, Wednesday 18 October 2017

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Motivation

A world of synchronization...

A world of synchronization. . .

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Foreword


What happens to the synchronization when the
interactions are regulated by the cost/benefit ratio?


Why we do observe only fireflies
that flash in synchrony?

Summary

- ~~Motivation~~
- Crash course on synchronization and evolutionary game theory on networks
- The Evolutionary Kuramoto's Dilemma
- Results
- Conclusion

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Introduction to Kuramoto model & evolutionary game theory

Kuramoto model on networks

REVIEWS OF MODERN PHYSICS

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The Kuramoto model: A simple paradigm for synchronization phenomena

Juan A. Acebrón, L. L. Bonilla, Conrad J. Pérez Vicente, Félix Ritort, and Renato Spigler
Rev. Mod. Phys. **77**, 137 – Published 7 April 2005

ELSEVIER

Physics Reports

Volume 469, Issue 3, December 2008, Pages 93-153

Synchronization in complex networks

Alex Arenas [a, b], Albert Díaz-Guilera [c, b], Jurgen Kurths [d, e], Yamir Moreno [b, f], Changsong Zhou [g]

ELSEVIER

Physics Reports

Volume 610, 26 January 2016, Pages 1-98

The Kuramoto model in complex networks

Francisco A. Rodrigues [a], Thomas K. DM. Peron [b, c], Peng Ji [c, d], Jürgen Kurths [c, d, e, f]

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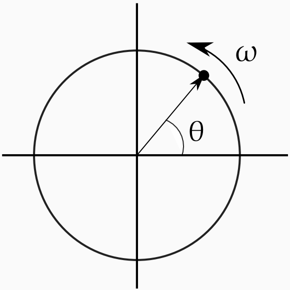
Kuramoto model on networks



$\theta \in [0, 2\pi]$ Phase

$\omega \in [0, 2\pi]$ Natural frequency

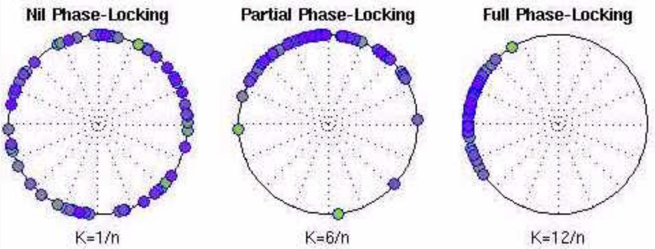
$\lambda \geq 0$ Coupling

$$\dot{\theta}_l = \omega_l + \lambda \sum_{j=1}^{N} a_{lj} \sin(\theta_j - \theta_l)$$

- Kuramoto, Y. (1984). Progress of Theoretical Physics Supplement, 79, 223–240.

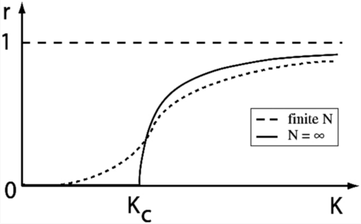
Global order parameter

$$r_G \, e^{i\Psi} = \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j} \qquad r_G \in [0, 1]$$



• Kuramoto, Y. (1984). Progress of Theoretical Physics Supplement, 79, 223–240.

Kuramoto model on networks



Critical coupling

$$\lambda_c = \lambda_c^{MF} \frac{\langle k \rangle}{\langle k^2 \rangle}$$

- Kuramoto, Y. (1984). Progress of Theoretical Physics Supplement, 79, 223–240.
- Arenas, A. *et al.* (2008). Physics Reports, 469, 93–153.

Evolutionary game theory on networks

Physics Reports
Volume 446, Issues 4–6, July 2007, Pages 97-216

Evolutionary games on graphs

György Szabó [a], Gábor Fáth [b]

Physics of Life Reviews
Volume 6, Issue 4, December 2009, Pages 208-249

Review

Evolutionary game theory: Temporal and spatial effects beyond replicator dynamics

Carlos P. Roca [a], José A. Cuesta [a], Angel Sánchez [a, b, c]

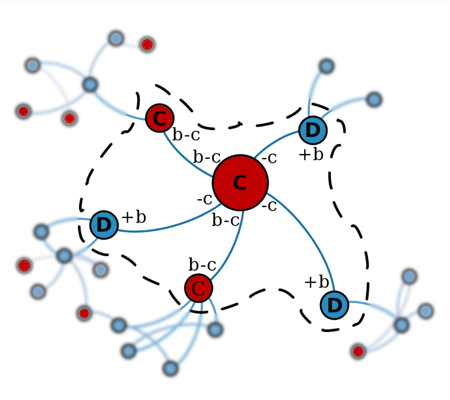
Biosystems
Volume 99, Issue 2, February 2010, Pages 109-125

Coevolutionary games—A mini review

Matjaž Perc [a], Attila Szolnoki [b]

Evolutionary game theory on networks

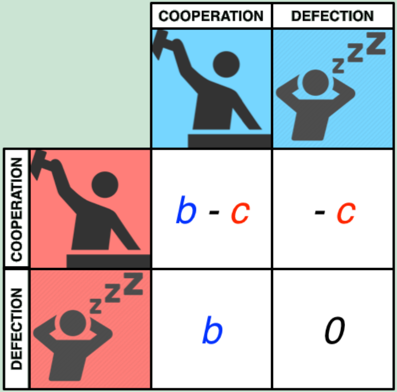
Agents' states correspond to their strategies: cooperation and defection. Agents interact in a pairwise manner, and accumulate a payoff p according to the payoff matrix of the game.

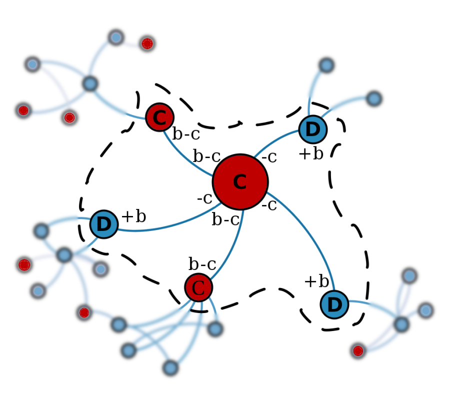


- Roca, C. P., ***et al.*** (2009). Phys. of Life Rev., 6, 208.
- Szabó, G., & Fáth, G. (2007). Evolutionary games on graphs. Phys. Rep., 446, 97–216.

Prisoner's Dilemma game

benefit: $b > 0$; cost: $c > 0$ ($b > c$)



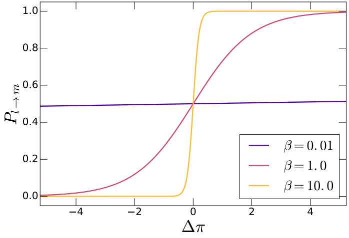


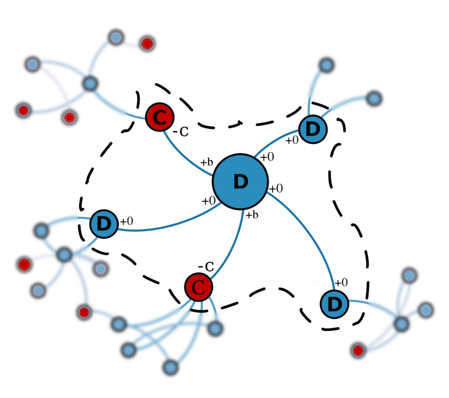
- Roca, C. P., *et al.* (2009). Phys. of Life Rev., 6, 208.
- Szabó, G., & Fáth, G. (2007). Evolutionary games on graphs. Phys. Rep., 446, 97–216.

Evolutionary game theory on networks

Agents update their strategies according to some rule.

$$P_{l\to m} = \frac{1}{1+e^{-\beta(p_m - p_l)}}\ .$$

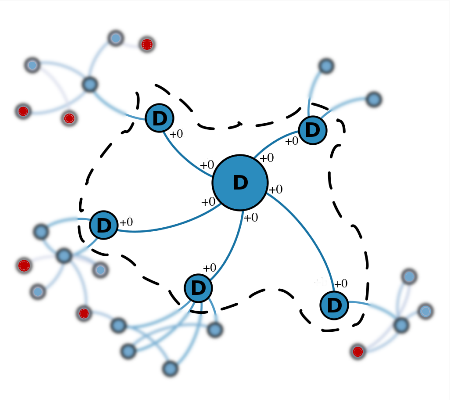




- Roca, C. P., ***et al.*** (2009). Phys. of Life Rev., 6, 208.
- Szabó, G., & Fáth, G. (2007). Evolutionary games on graphs. Phys. Rep., 446, 97–216.

Evolutionary game theory on networks

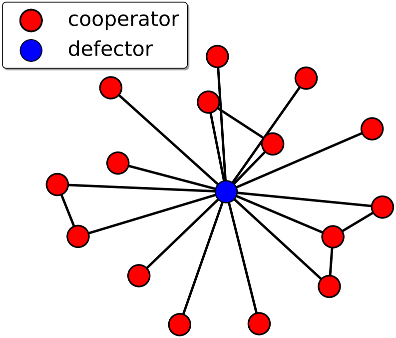
Repeat until stationary state



- Roca, C. P., ***et al.*** (2009). Phys. of Life Rev., 6, 208.
- Szabó, G., & Fáth, G. (2007). Evolutionary games on graphs. Phys. Rep., 446, 97–216.

The Evolutionary Kuramoto's Dilemma

The Evolutionary Kuramoto's Dilemma



Phase

$$\theta_I \in [0, 2\pi]$$

Strategy

$$s_I = \begin{cases} 1 & \text{if } I \text{ is cooperator} \\ 0 & \text{if } I \text{ is defector} \end{cases}$$

The Evolutionary Kuramoto's Dilemma

Kuramoto

$$\dot{\theta}_l = \omega_l + \overbrace{s_l\,\lambda}^{\text{interaction}} \sum_{j=1}^{N} a_{lj} \sin(\theta_j - \theta_l)\,.$$

Payoff

$$p_l = \underbrace{r_{L_l}}_{\text{benefit}} - \alpha \underbrace{\frac{c_l}{2\pi}}_{\text{cost}}$$

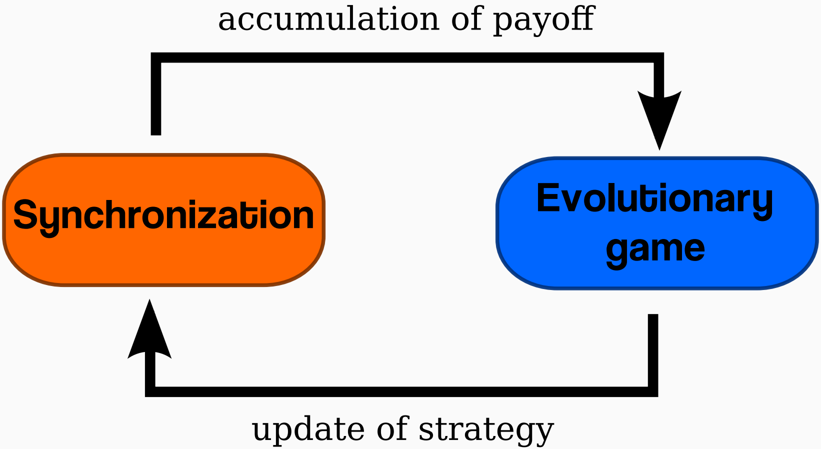
Benefit

$$r_{L_l} = \frac{1}{k_l} \sum_{j=1}^{N} a_{lj} \frac{\left| e^{i\theta_l} + e^{i\theta_j} \right|}{2}$$

$$r_L \in [0, 1],$$

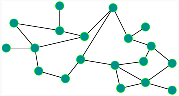
Cost

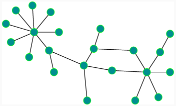
$$c_l = \Delta\dot{\theta}_l = \left| \dot{\theta}_l(t) - \dot{\theta}_l(t-1) \right|$$


accumulation of payoff
Synchronization
Evolutionary game
update of strategy

The Evolutionary Kuramoto's Dilemma

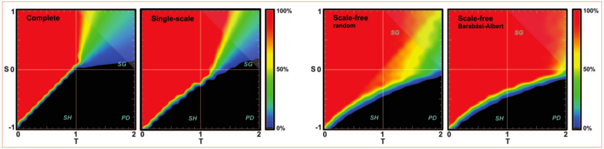
Erdős Rényi

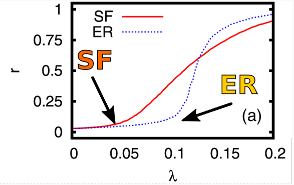




Scale Free

ER **SF**





- Santos, F., *et al.* (2006). Proceedings of the National Academy of Sciences, 103, 3490–3494.
- Gómez-Gardeñes, J., *et al.* (2007). Physical Review Letters, 98, 34101.

Question:

How the underlying topology of the interactions affects the emergence of cooperation/synchronization?

Question:

How the underlying topology of the interactions affects the emergence of cooperation/synchronization?

Answer

We consider three different topologies:

- **ER** Erdős-Rényi random graphs
- **RGG** Random Geometric Graph
- **BA** Barabási-Albert scale-free

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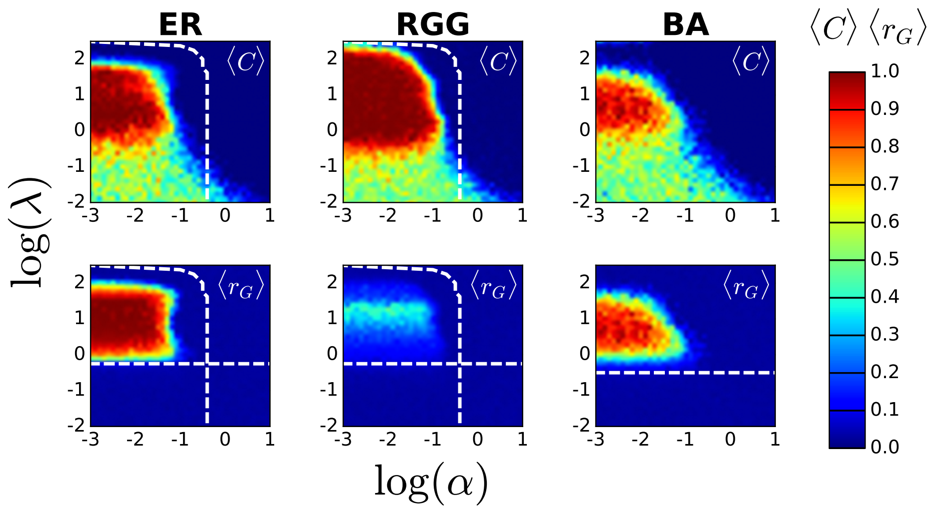
Results



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Macroscopic behaviour

Lower bound

$$\lambda_c = \lambda_c^{MF} \frac{\langle k \rangle}{\langle k^2 \rangle}$$

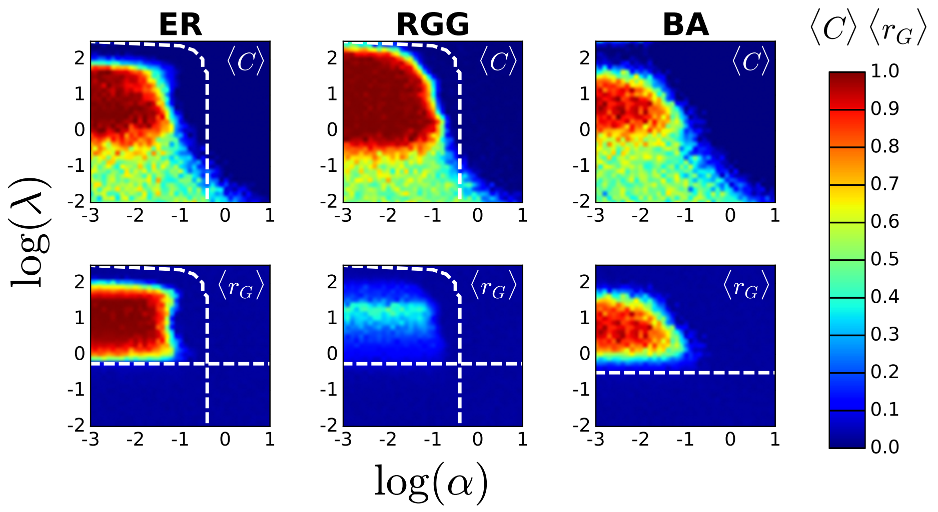
Upper bound

$$\frac{\Delta b}{\Delta c} = \frac{b_{Coop} - b_{Def}}{c} > \langle k \rangle$$

$$\frac{\sqrt{2\left[1 + \sin(\varepsilon\lambda)\right]} - \sqrt{2}}{\varepsilon\lambda\langle k \rangle}\pi > \alpha \, .$$

- Arenas, A., *et al.* (2008). Physics Reports, 469, 93–153.
- Ohtsuki, H. *et al.* (2006). Nature, 441, 502–505.

Macroscopic behaviour



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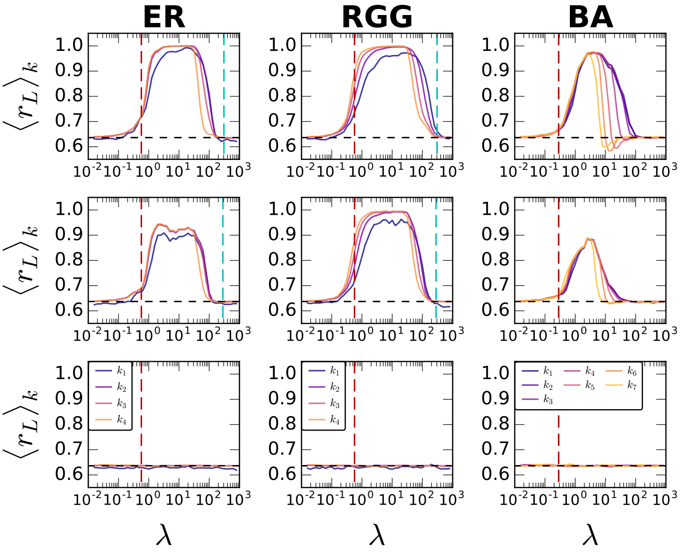
Microscopic behaviour



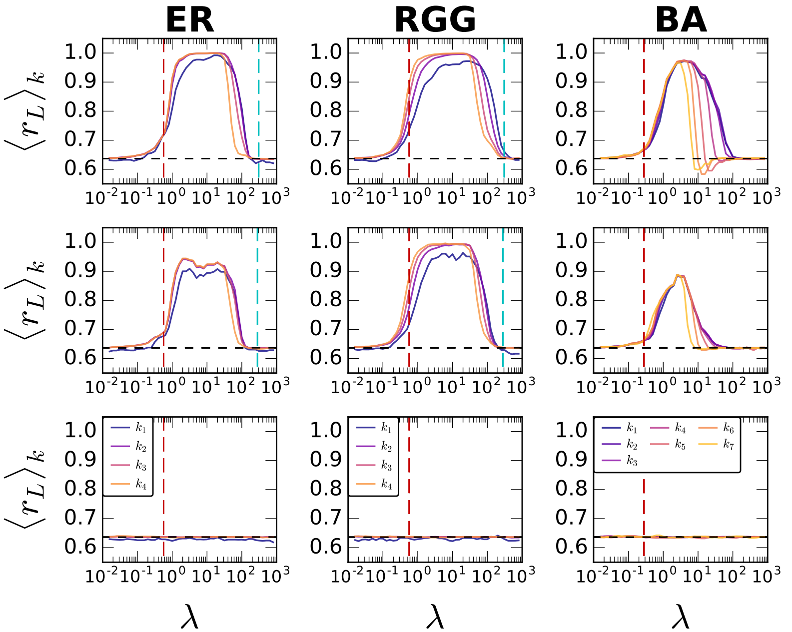
Three regimes of relative cost α:

10^{-3} Cheap

$10^{-1.4}$ Medium

10^{0} Expensive

Microscopic behaviour



Microscopic behaviour

average pairwise order parameter

$$\overline{r_{lm}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\left\|1 + e^{i\theta}\right\|}{2}\, d\theta =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\|1 + \cos\theta + i\sin\theta\|}{2}\, d\theta =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\sqrt{[1 + \cos\theta]^2 + \sin^2\theta}}{2}\, d\theta = \frac{4}{2\pi} = \frac{2}{\pi} \sim 0.6366\,.$$

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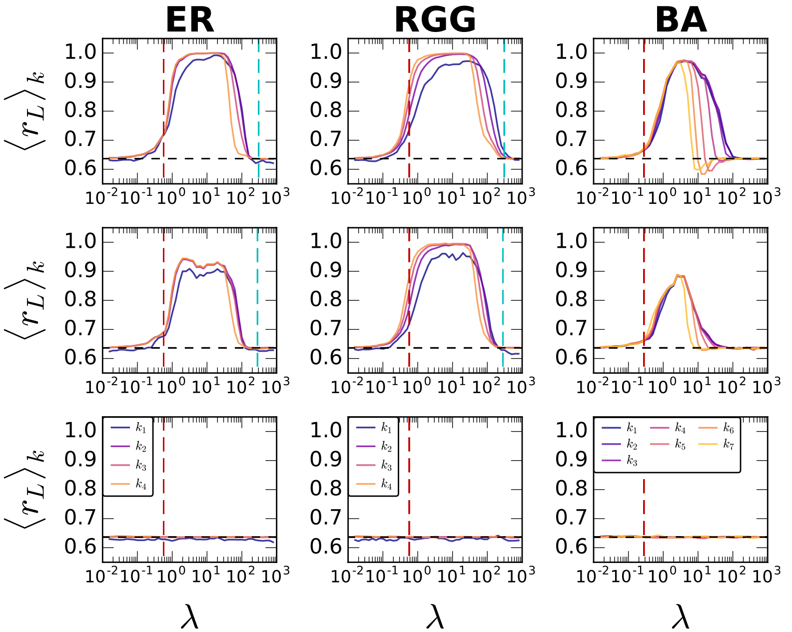
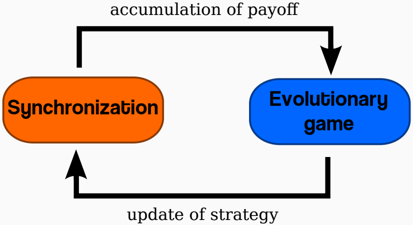
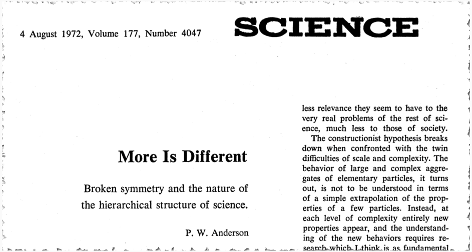
Microscopic behaviour



Conclusions

Coevolutionary model (Evolutionary Kuramoto's Dilemma) based on synchronization and evolutionary game theory.





4 August 1972, Volume 177, Number 4047

SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

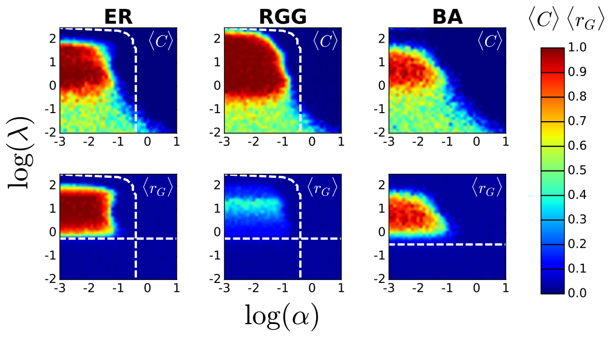
less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental

- Anderson, P. W. (1972). More Is Different. Science, 177, 393–396.

Take home messages

Role of the underlying topology in the emergence of cooperation/synchronization.



Take home messages

The synchronization of fireflies can be interpreted as the result of Darwinian selection

- Sumpter, D. J. T. (2006). The principles of collective animal behaviour. Phil. Trans. Roy. Soc. B: Biological Sciences, 361, 5–22.

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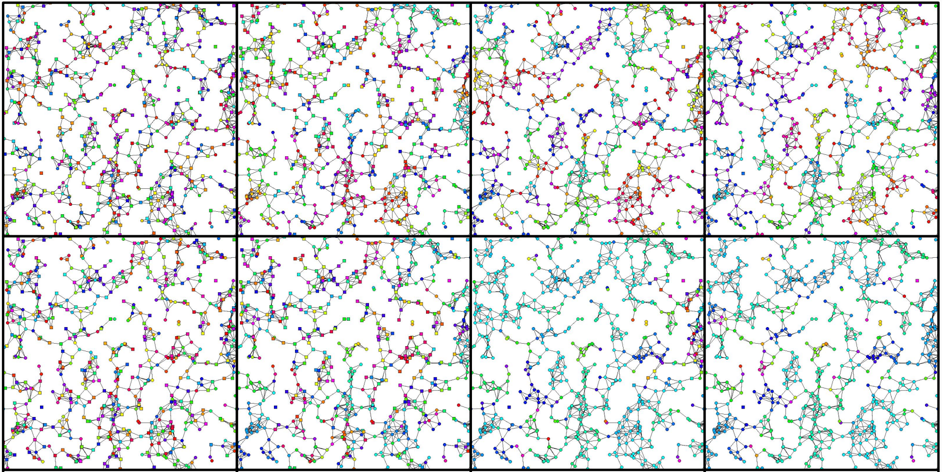
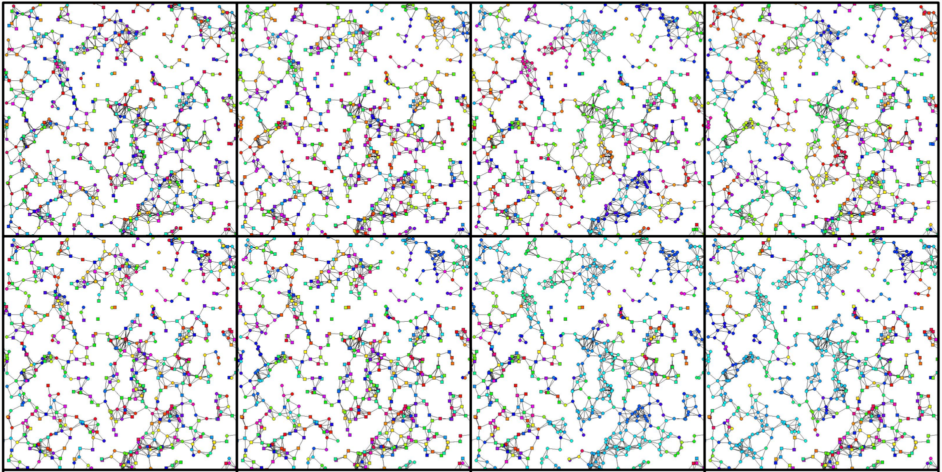
Extra contents

Microscopic behaviour in RGG



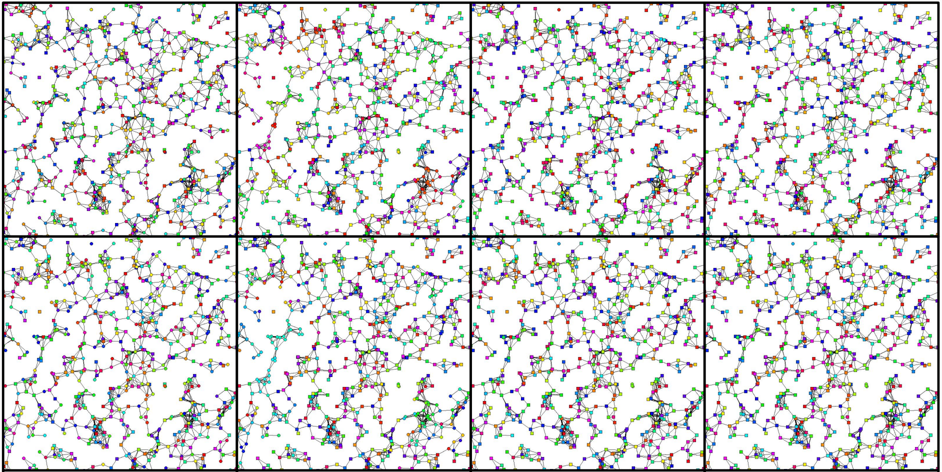
$t = 0$	$t = 200$	$t = 2000$	$t = 5000$
$r_G = 0.011$	$r_G = 0.054$	$r_G = 0.114$	$r_G = 0.070$
$r_L = 0.640$	$r_L = 0.787$	$r_L = 0.913$	$r_L = 0.918$
coop = 0.500	coop = 0.565	coop = 0.937	coop = 0.998

Microscopic behaviour in RGG



$t = 0$	$t = 200$	$t = 2000$	$t = 5000$
$r_G = 0.039$	$r_G = 0.097$	$r_G = 0.042$	$r_G = 0.094$
$r_L = 0.641$	$r_L = 0.773$	$r_L = 0.840$	$r_L = 0.822$
coop = 0.500	coop = 0.479	coop = 0.690	coop = 0.604

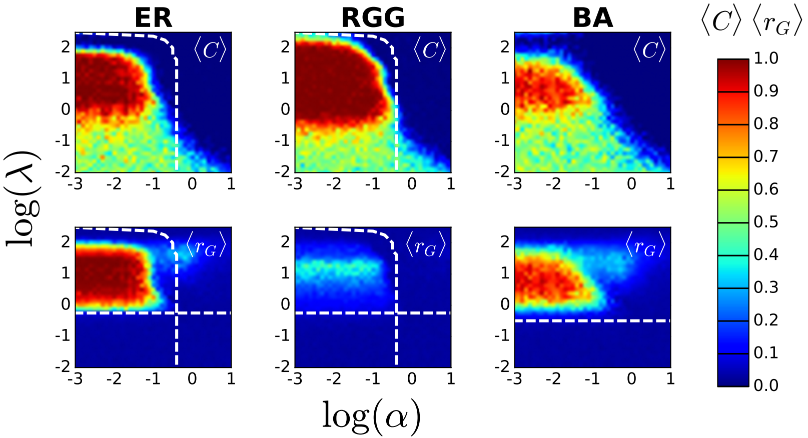
Microscopic behaviour in RGG



t	= 0	= 200	= 2000	= 5000
r_G	= 0.033	= 0.048	= 0.053	= 0.042
r_L	= 0.631	= 0.738	= 0.647	= 0.635
coop	= 0.500	= 0.322	= 0.017	= 0.008

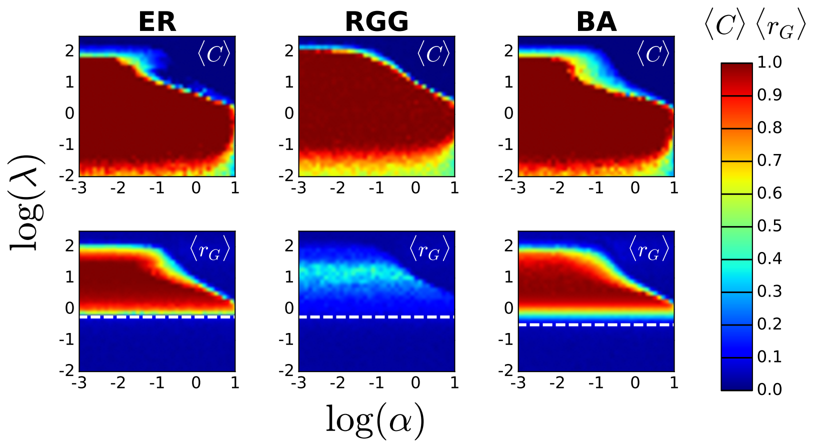
Other update rules

Asynchronous Fermi



Other update rules

Synchronous Imitation of the best



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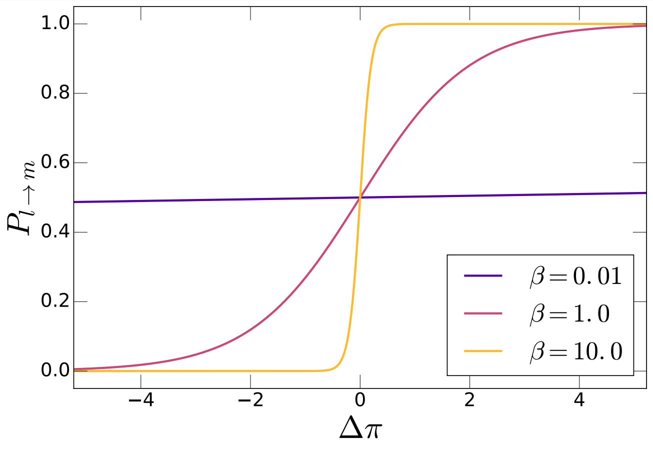
Fermi's Rule

$$P_{l\to m} = \frac{1}{1 + e^{-\beta(p_m - p_l)}}\,.$$



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