A coevolutionary model combining game theory and synchronization: the Evolutionary Kuramoto's Dilemma

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Applied Mathematics Seminar — University College Cork
Thursday, March 19th 2020

UNIVERSITAT ROVIRA i VIRGILI

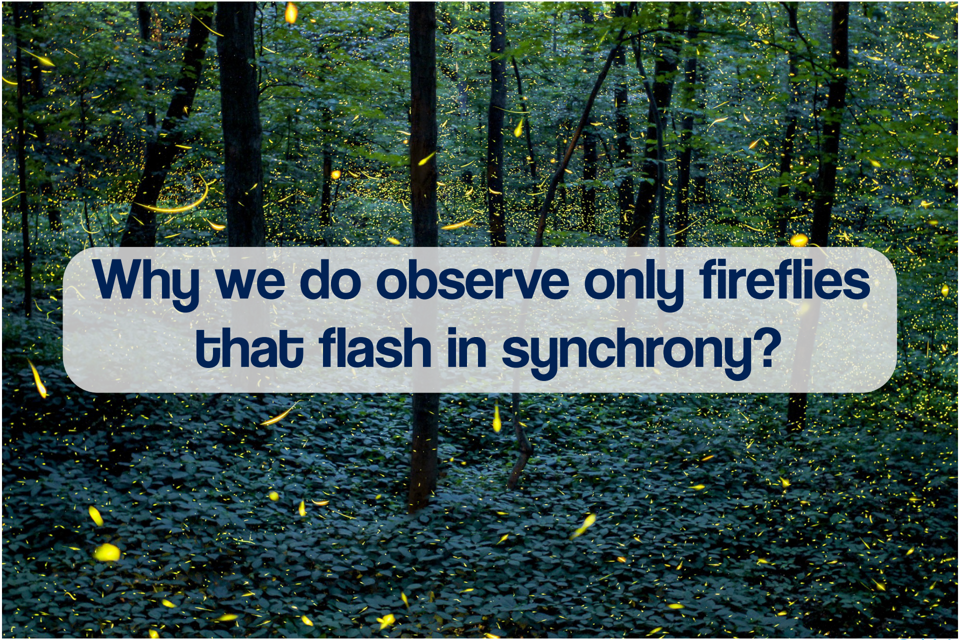
A world of synchronization...

A world of synchronization. . .

Foreword

Foreword

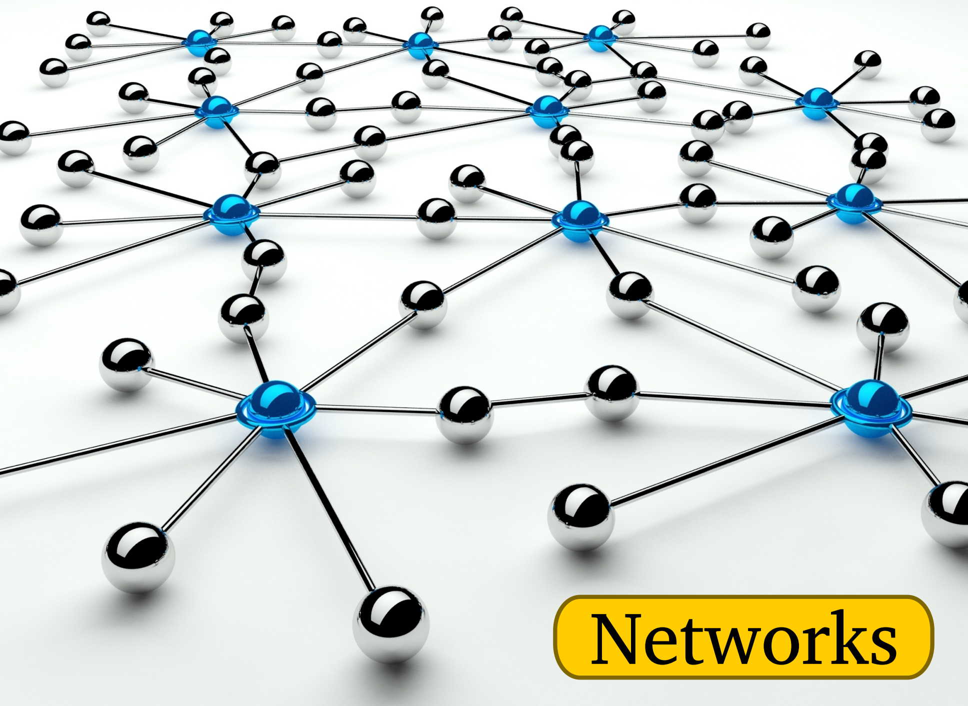
What happens to the synchronization when the interactions are regulated by the cost/benefit ratio?


Why we do observe only fireflies
that flash in synchrony?

Summary

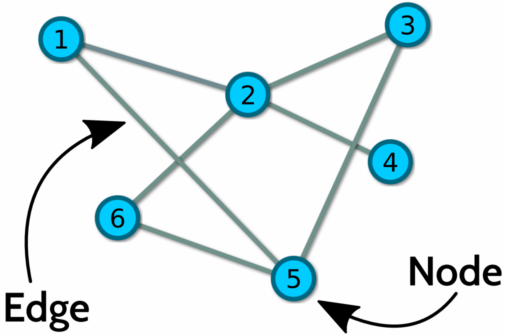
- ~~Motivation~~
- Crash course on synchronization and evolutionary game theory on networks
- The Evolutionary Kuramoto's Dilemma
- Results
- Conclusion

Crash Course on Synchronization and Evolutionary Game Theory on Networks



Networks

Networks



$N \times N$ Adjacency matrix

$$\mathcal{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Degree of a node

$$k_i = \sum_{j=1}^{N} a_{ij}$$

- V. Latora *et al.* *Complex Networks: Principles, Methods and Applications* Cambridge University Press (2017).

Synchronization

Kuramoto model on networks

REVIEWS OF MODERN PHYSICS

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The Kuramoto model: A simple paradigm for synchronization phenomena

Juan A. Acebrón, L. L. Bonilla, Conrad J. Pérez Vicente, Félix Ritort, and Renato Spigler
Rev. Mod. Phys. **77**, 137 – Published 7 April 2005

ELSEVIER

Physics Reports

Volume 469, Issue 3, December 2008, Pages 93-153

Synchronization in complex networks

Alex Arenas [a, b], Albert Díaz-Guilera [c, b], Jurgen Kurths [d, e], Yamir Moreno [b, f], Changsong Zhou [g]

ELSEVIER

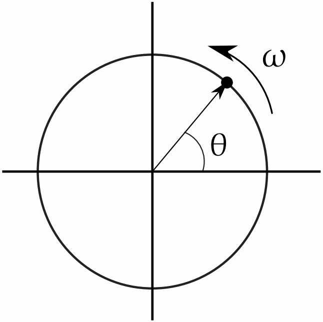
Physics Reports

Volume 610, 26 January 2016, Pages 1-98

The Kuramoto model in complex networks

Francisco A. Rodrigues [a], Thomas K. DM. Peron [b, c], Peng Ji [c, d], Jürgen Kurths [c, d, e, f]

Kuramoto model on networks



$\theta \in [0, 2\pi]$ Phase

$\omega \in [0, 2\pi]$ Natural frequency

$\lambda \geq 0$ Coupling

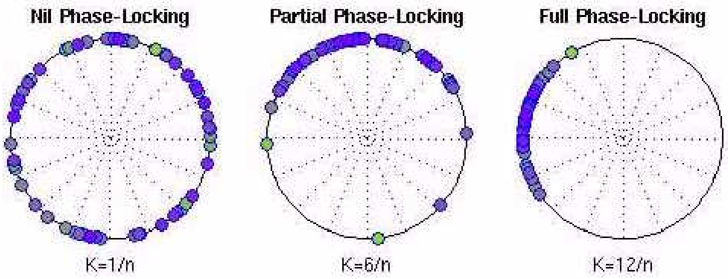
$$\dot{\theta}_l = \omega_l + \lambda \sum_{j=1}^{N} a_{lj} \sin(\theta_j - \theta_l)$$

- Kuramoto, Y. (1984). Progress of Theoretical Physics Supplement, 79, 223–240.

Kuramoto model on networks

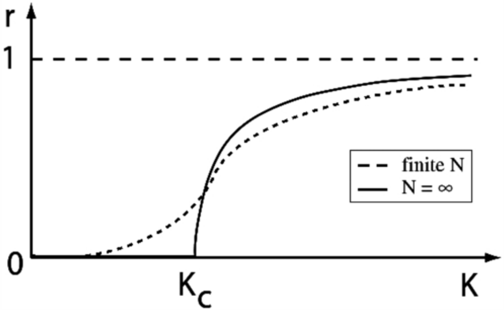
Global order parameter

$$r_G\, e^{i\Psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j} \qquad r_G \in [0, 1]$$



- Kuramoto, Y. (1984). Progress of Theoretical Physics Supplement, **79**, 223–240.

Kuramoto model on networks



Critical coupling

$$\lambda_c = \lambda_c^{MF} \frac{\langle k \rangle}{\langle k^2 \rangle}$$

- Kuramoto, Y. (1984). Progress of Theoretical Physics Supplement, 79, 223–240.
- Arenas, A. *et al.* (2008). Physics Reports, 469, 93–153.

Evolutionary Games

Evolutionary game theory on networks

ELSEVIER

Physics Reports

Volume 446, Issues 4–6, July 2007, Pages 97-216

Evolutionary games on graphs

György Szabó [a], Gábor Fáth [b]

ELSEVIER

Physics of Life Reviews

Volume 6, Issue 4, December 2009, Pages 208-249

Review

Evolutionary game theory: Temporal and spatial effects beyond replicator dynamics

Carlos P. Roca [a], José A. Cuesta [a], Angel Sánchez [a, b, c]

ELSEVIER

Biosystems

Volume 99, Issue 2, February 2010, Pages 109-125

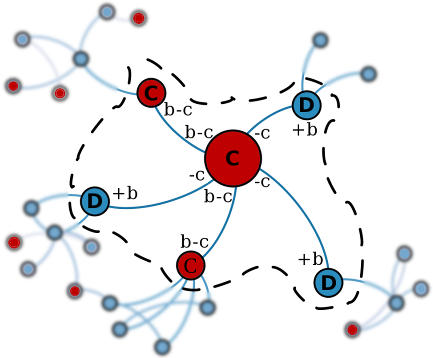
Coevolutionary games—A mini review

Matjaž Perc [a], Attila Szolnoki [b]

Evolutionary game theory on networks

Agents' states correspond to their strategies s: cooperation $(s = 1)$ defection $(s = 0)$.

Agents interact in a pairwise manner, and accumulate a payoff p according to the game's payoff matrix.



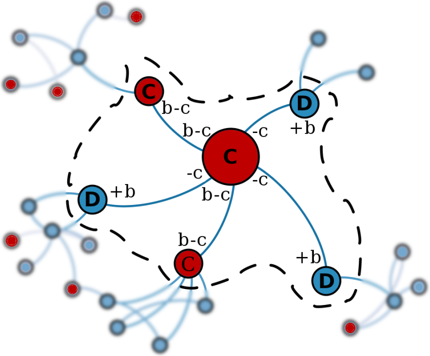
- Roca, C. P., *et al.* (2009). Phys. of Life Rev., 6, 208.
- Szabó, G., & Fáth, G. (2007). Evolutionary games on graphs. Phys. Rep., 446, 97–216.

Evolutionary game theory on networks

Prisoner's Dilemma game

benefit: $b > 0$; cost: $c > 0$ $(b > c)$

	COOPERATION	DEFECTION
COOPERATION	$b - c$	$-c$
DEFECTION	b	0

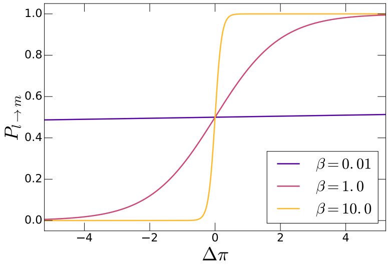


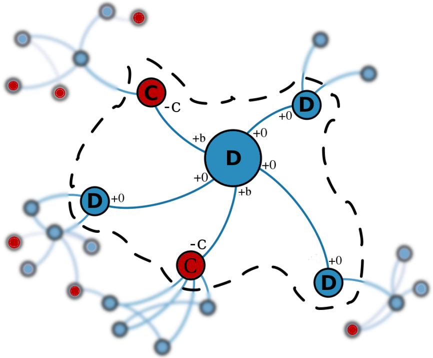
- Roca, C. P., *et al.* (2009). Phys. of Life Rev., 6, 208.
- Szabó, G., & Fáth, G. (2007). Evolutionary games on graphs. Phys. Rep., 446, 97–216.

Evolutionary game theory on networks

Agents update their strategies according to some rule.

$$P_{l\to m} = \frac{1}{1+e^{-\beta(p_m-p_l)}}\,.$$



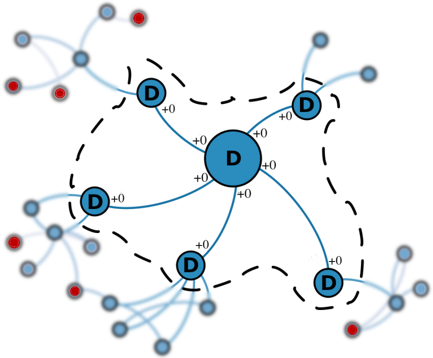


- Roca, C. P., *et al.* (2009). Phys. of Life Rev., 6, 208.
- Szabó, G., & Fáth, G. (2007). Evolutionary games on graphs. Phys. Rep., 446, 97–216.

Evolutionary game theory on networks

Repeat until stationary state, then measure the average cooperation

$$\langle C \rangle = \frac{1}{N} \sum_i s_i \ \in [0, 1]$$



- Roca, C. P., *et al.* (2009). Phys. of Life Rev., 6, 208.
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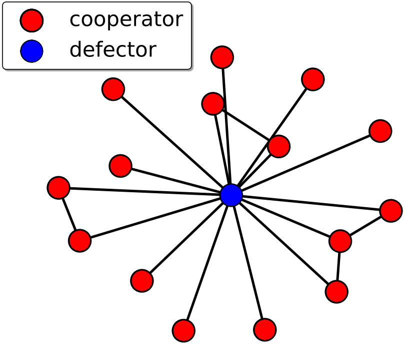
The Evolutionary Kuramoto's Dilemma

The Evolutionary Kuramoto's Dilemma



Phase

$$\theta_l \in [0, 2\pi]$$

Strategy

$$s_l = \begin{cases} 1 & \text{if } l \text{ is cooperator} \\ 0 & \text{if } l \text{ is defector} \end{cases}$$

The Evolutionary Kuramoto's Dilemma

Kuramoto

$$\dot{\theta}_l = \omega_l + \overbrace{s_l\,\lambda}^{\text{interaction}} \sum_{j=1}^{N} a_{lj} \sin(\theta_j - \theta_l)\,.$$

The Evolutionary Kuramoto's Dilemma

Payoff

$$p_l = \underbrace{r_{L_l}}_{\text{benefit}} - \underbrace{\alpha \frac{c_l}{2\pi}}_{\text{cost}}$$

$$\alpha \in]0, \infty[$$

The Evolutionary Kuramoto's Dilemma

Payoff

$$p_l = \underbrace{r_{L_l}}_{\text{benefit}} - \alpha \underbrace{\frac{c_l}{2\pi}}_{\text{cost}}$$

$$\alpha \in \,]0, \infty[$$

Benefit

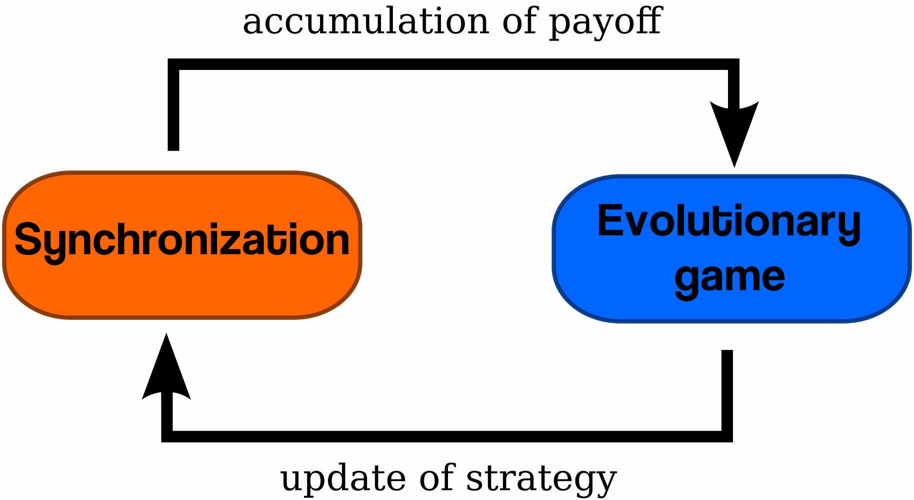
$$r_{L_l} = \frac{1}{k_l} \sum_{j=1}^{N} a_{lj} \frac{\left| e^{i\theta_l} + e^{i\theta_j} \right|}{2}$$

$$r_L \in [0, 1]\,,$$

Cost

$$c_l = \Delta\dot{\theta}_l = \left| \dot{\theta}_l(t) - \dot{\theta}_l(t-1) \right|$$

The Evolutionary Kuramoto's Dilemma



The Evolutionary Kuramoto's Dilemma

Question:

How the underlying topology of the interactions affects the emergence of cooperation/synchronization?

The Evolutionary Kuramoto's Dilemma

Question:

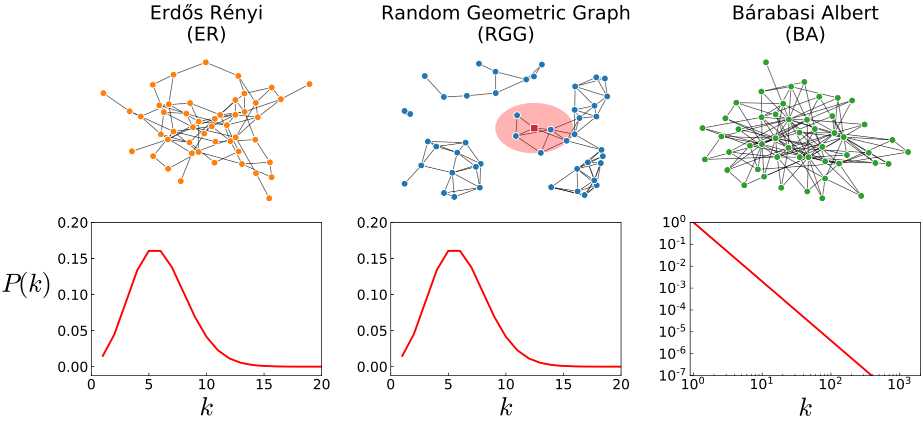
How the underlying topology of the interactions affects the emergence of cooperation/synchronization?

Answer

We consider three different topologies:

- **ER** Erdős-Rényi random graphs
- **RGG** Random Geometric Graph
- **BA** Barabási-Albert scale-free

The Evolutionary Kuramoto's Dilemma

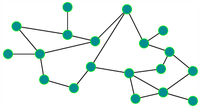


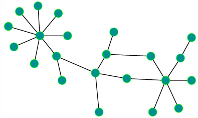
Note:

All nets have $N = 1000$ and $\langle k \rangle = 8$

The Evolutionary Kuramoto's Dilemma

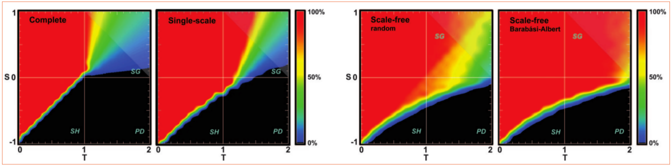
Erdős Rényi

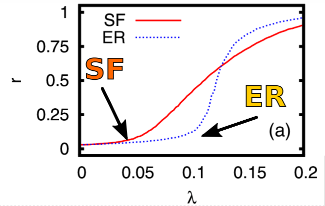




Scale Free

ER SF



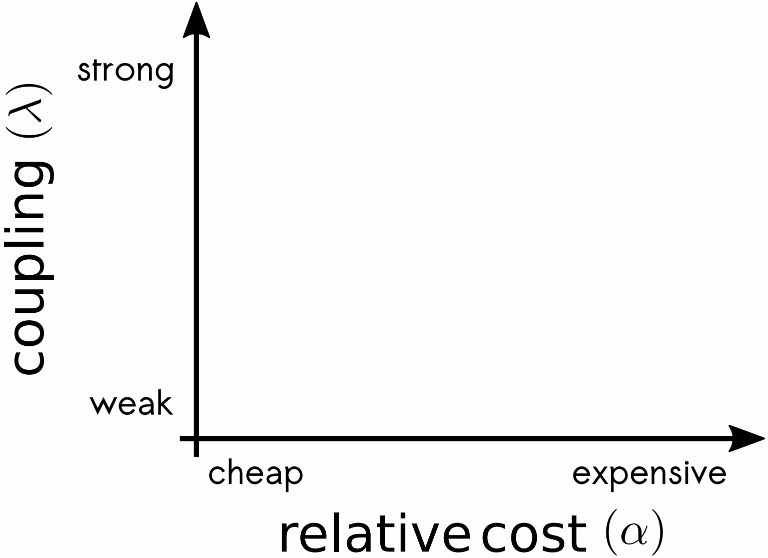


- Santos, F., *et al.* (2006). Proceedings of the National Academy of Sciences, 103, 3490–3494.
- Gómez-Gardeñes, J., *et al.* (2007). Physical Review Letters, 98, 34101.

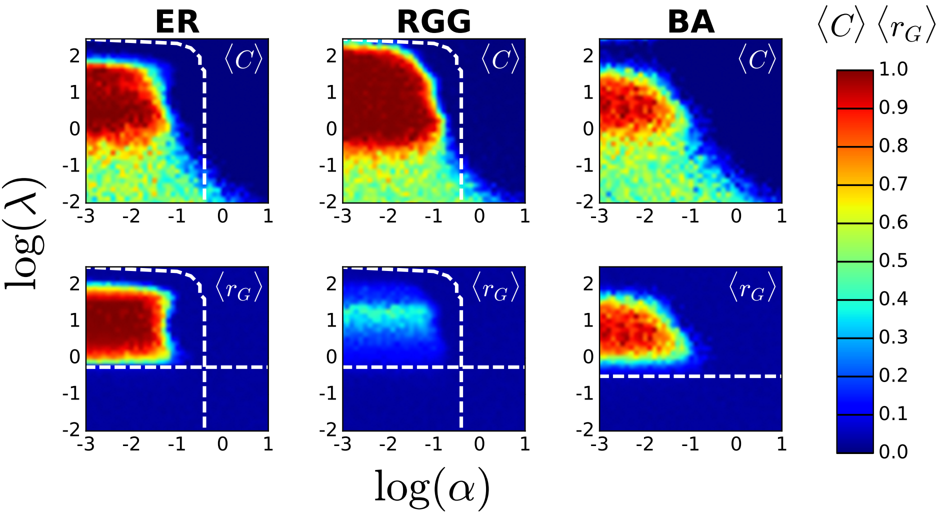

Results

Macroscopic behaviour

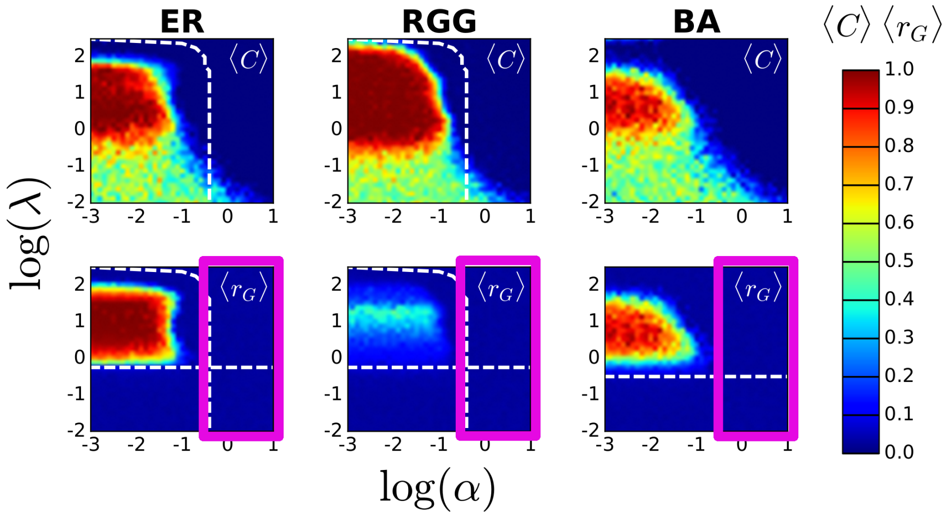
Macroscopic behaviour



Macroscopic behaviour



Macroscopic behaviour



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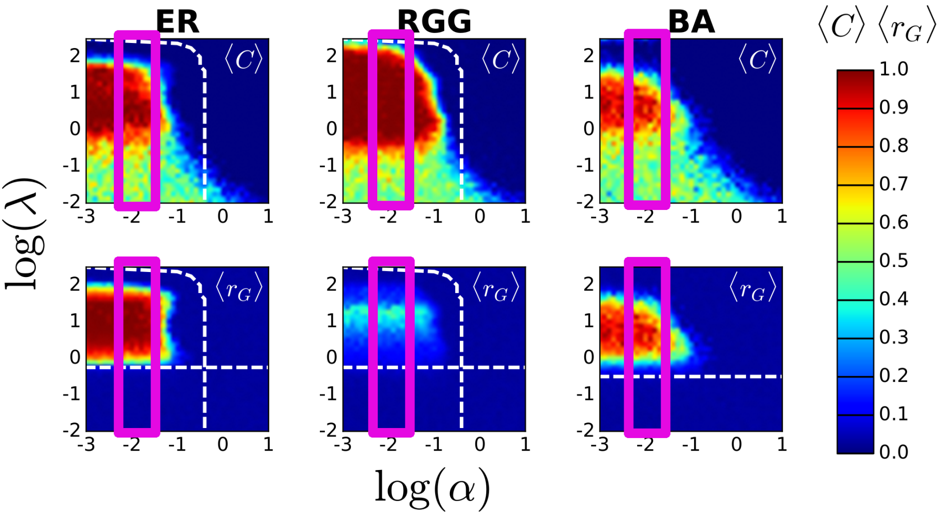
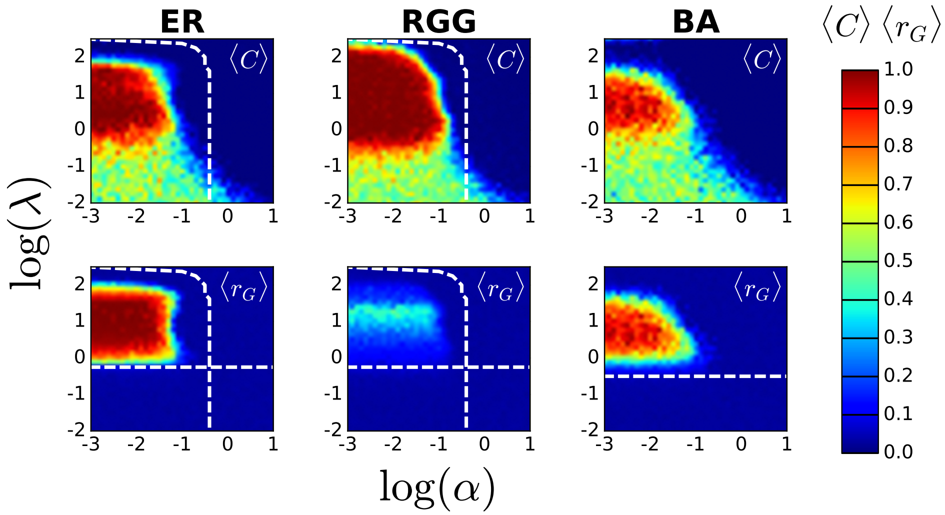


Macroscopic behaviour



Macroscopic behaviour

Lower bound

$$\lambda_c = \lambda_c^{MF} \frac{\langle k \rangle}{\langle k^2 \rangle}$$

- Arenas, A., *et al.* (2008). Physics Reports, **469**, 93–153.
- Ohtsuki, H. *et al.* (2006). Nature, **441**, 502–505.

Macroscopic behaviour

Upper bound

$$\frac{\Delta b}{\Delta c} = \frac{b_{Coop} - b_{Def}}{c} > \langle k \rangle$$

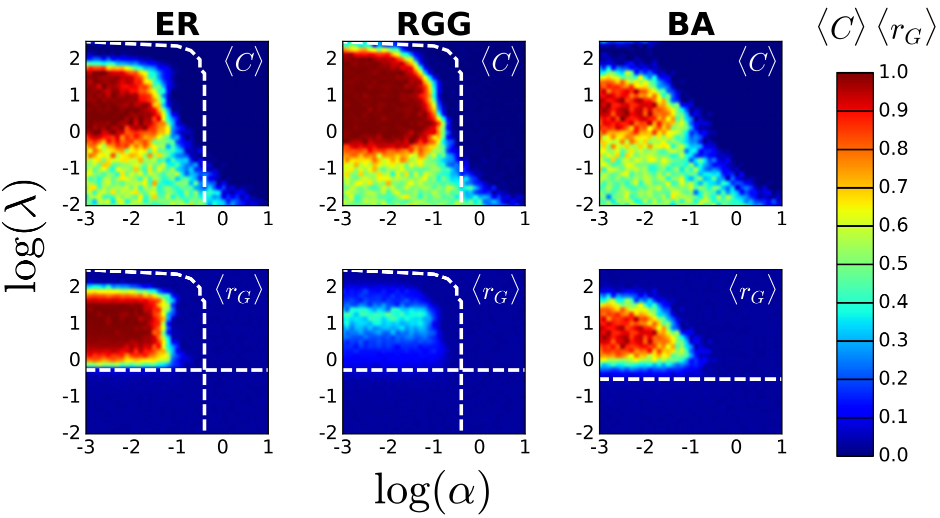
$$\frac{\sqrt{2\left[1 + \sin(\varepsilon\lambda)\right]} - \sqrt{2}}{\varepsilon\lambda\langle k \rangle}\pi > \alpha\,.$$

Lower bound

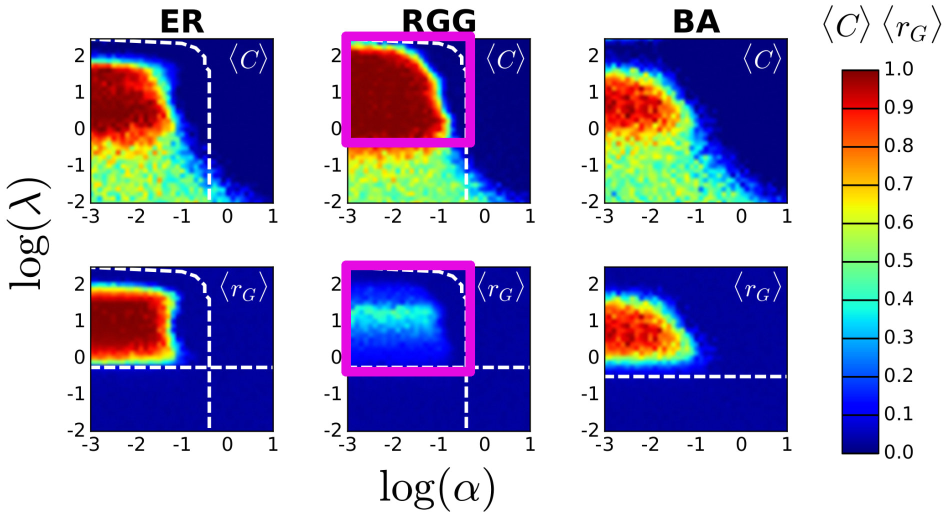
$$\lambda_c = \lambda_c^{MF}\frac{\langle k \rangle}{\langle k^2 \rangle}$$

- Arenas, A., *et al.* (2008). Physics Reports, 469, 93–153.
- Ohtsuki, H. *et al.* (2006). Nature, 441, 502–505.

Macroscopic behaviour

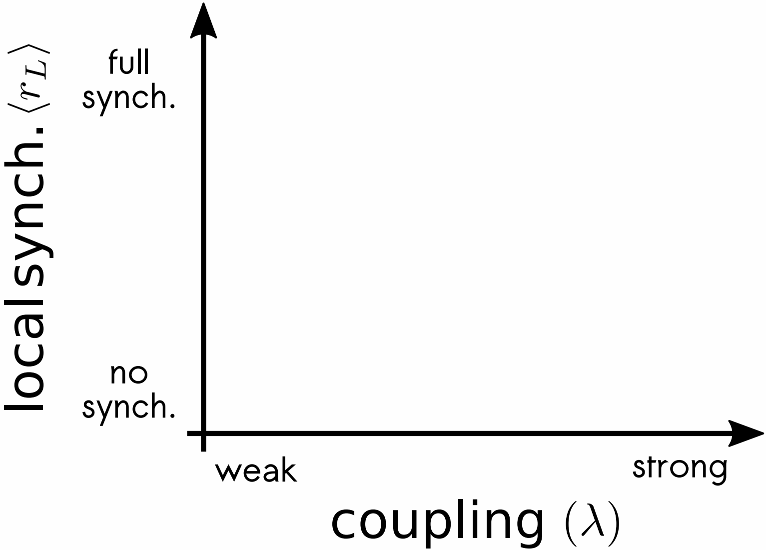


Macroscopic behaviour

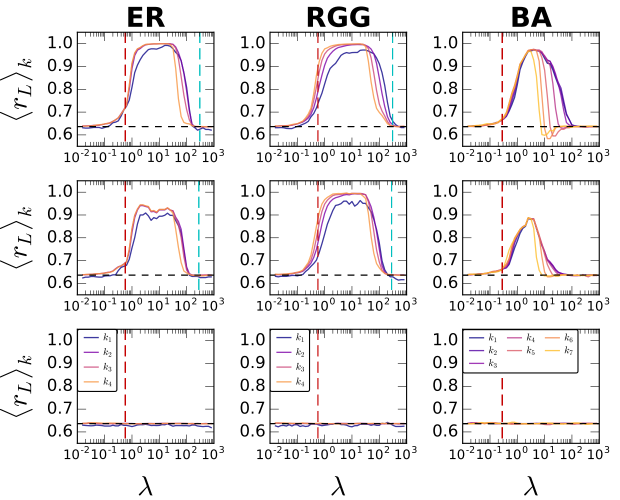


Microscopic behaviour

Microscopic behaviour



Microscopic behaviour



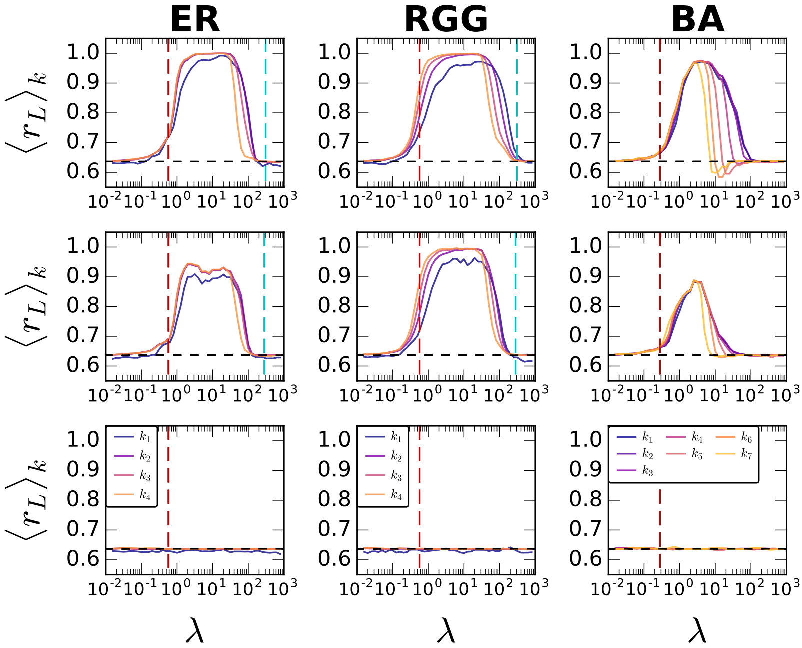
Three regimes of relative cost:

$\alpha = 10^{-3}$ Cheap

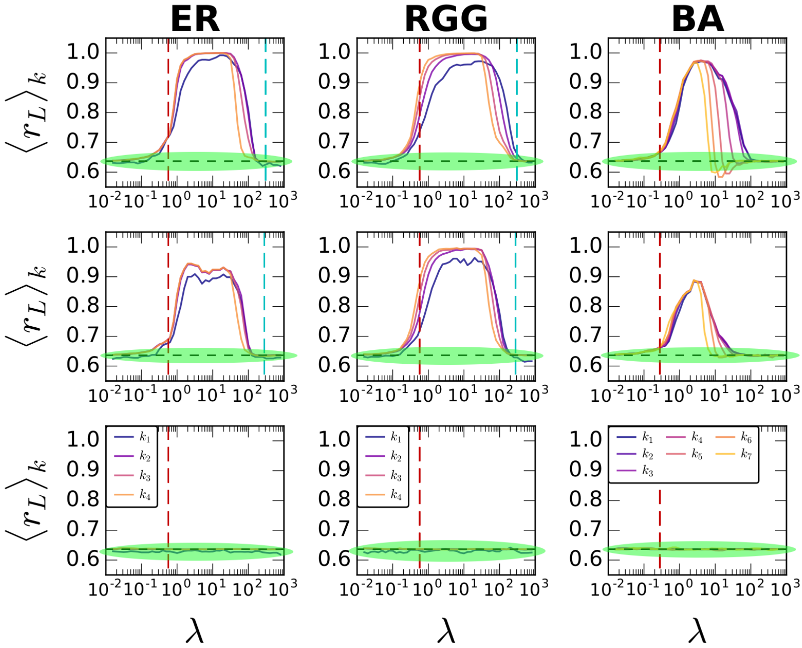
$\alpha = 10^{-1.4}$ Medium

$\alpha = 10^{0}$ Expensive

Microscopic behaviour



Microscopic behaviour



Microscopic behaviour

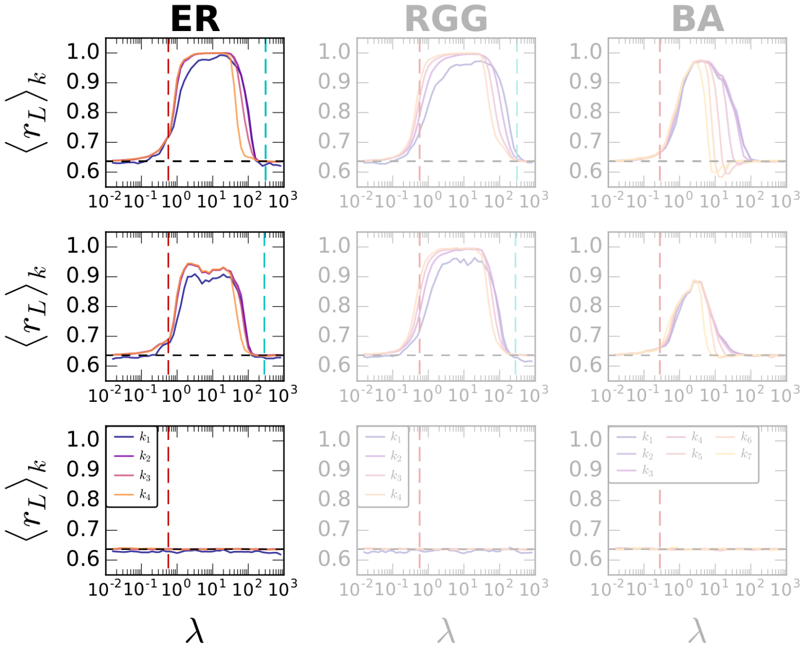
average pairwise order parameter

$$\overline{r_{lm}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\left\|1+e^{i\theta}\right\|}{2}\, d\theta =$$

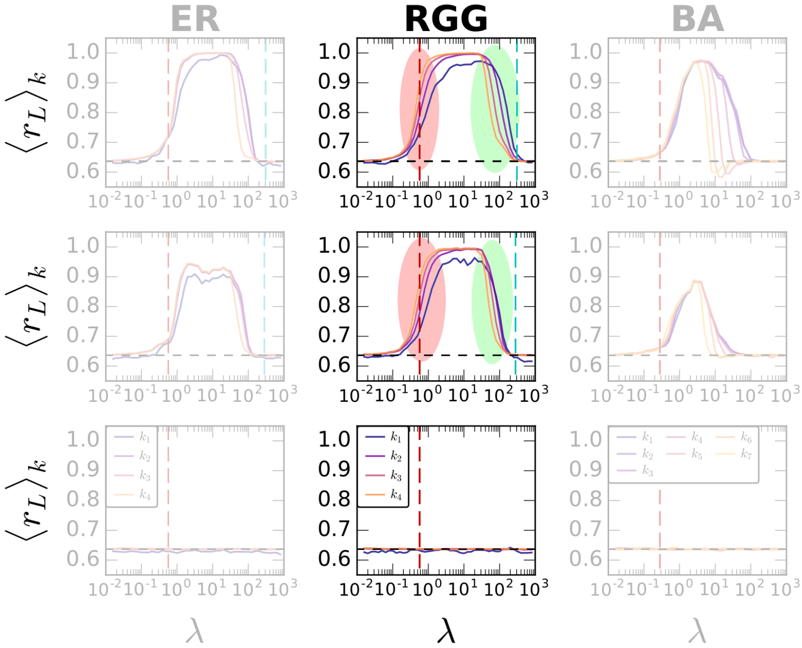
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\|1+\cos\theta + i\sin\theta\|}{2}\, d\theta =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\sqrt{[1+\cos\theta]^2+\sin^2\theta}}{2}\, d\theta = \frac{4}{2\pi} = \frac{2}{\pi} \sim 0.6366\,.$$

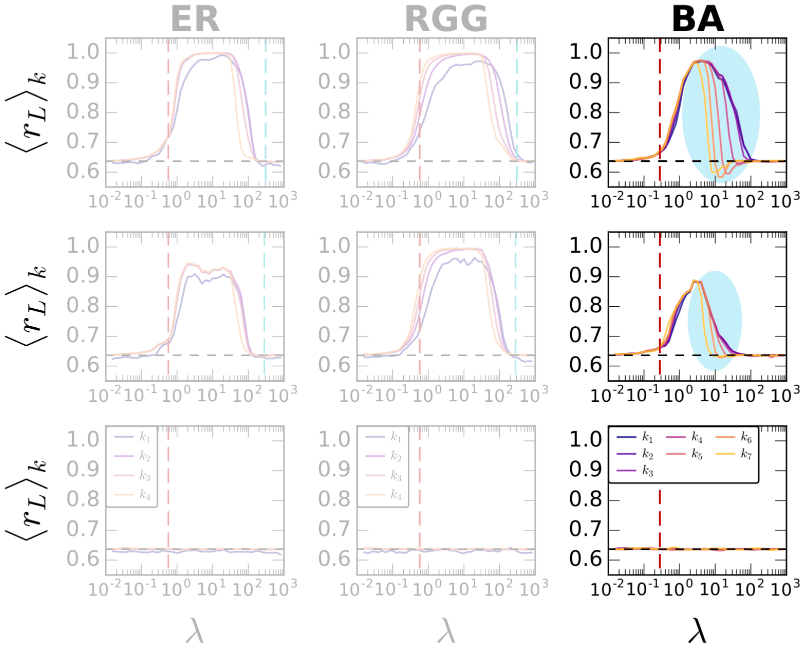
Microscopic behaviour



Microscopic behaviour



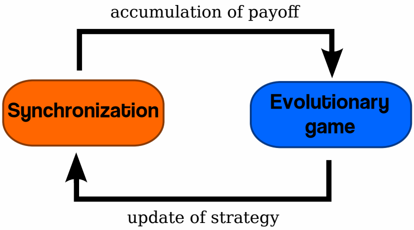
Microscopic behaviour

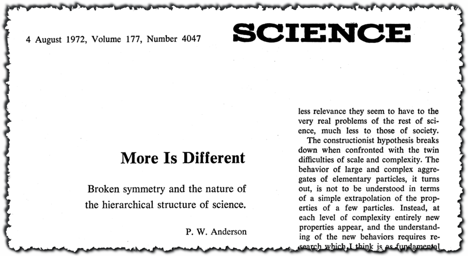



Summing
up ...

Take home messages

Coevolutionary model (Evolutionary Kuramoto's Dilemma) based on synchronization and evolutionary game theory.




4 August 1972, Volume 177, Number 4047

SCIENCE

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

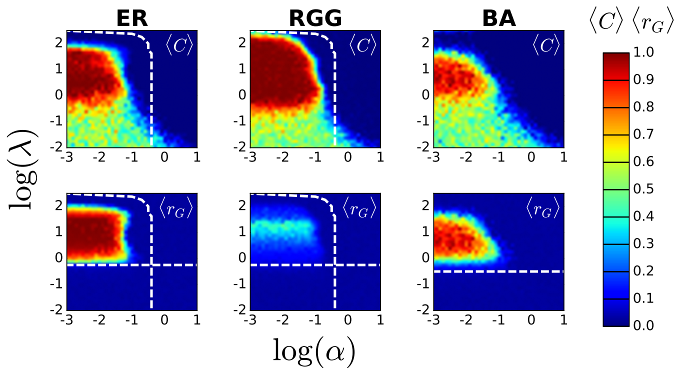
less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which, I think, is as fundamental

• Anderson, P. W. (1972). More Is Different. Science, 177, 393–396.

Take home messages

Role of the underlying topology in the emergence of cooperation/synchronization.



Take home messages

• Sumpter, D. J. T. (2006). The principles of collective animal behaviour. Phil. Trans. Roy. Soc. B: Biological Sciences, 361, 5–22.

Acknowledgements

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Univ. Carlos III Madrid

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Coevolution of Synchronization and Cooperation in Costly Networked Interactions

Alberto Antonioni and Alessio Cardillo
Phys. Rev. Lett. **118**, 238301 – Published 8 June 2017

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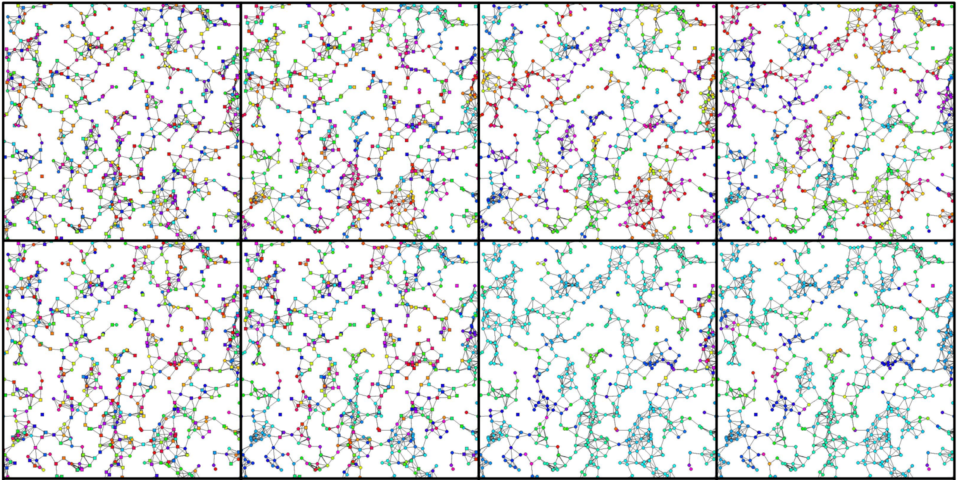
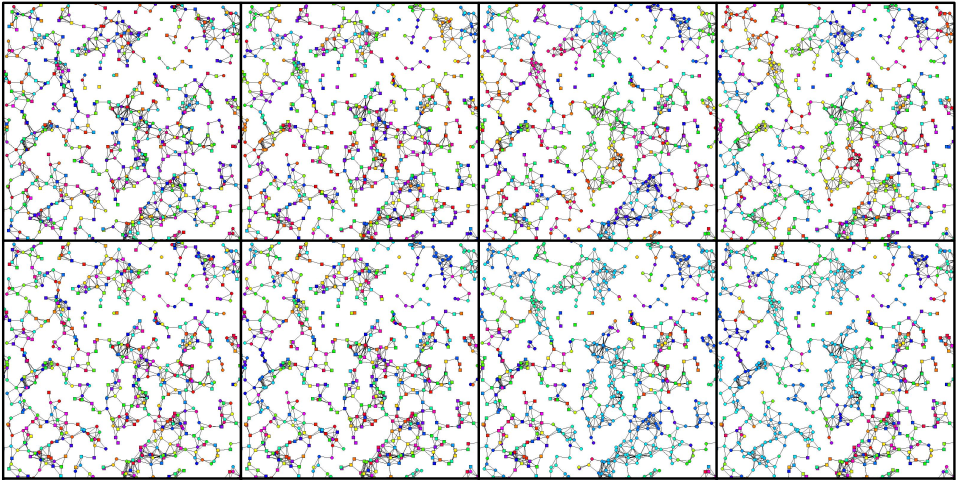
Extra contents

Microscopic behaviour in RGG



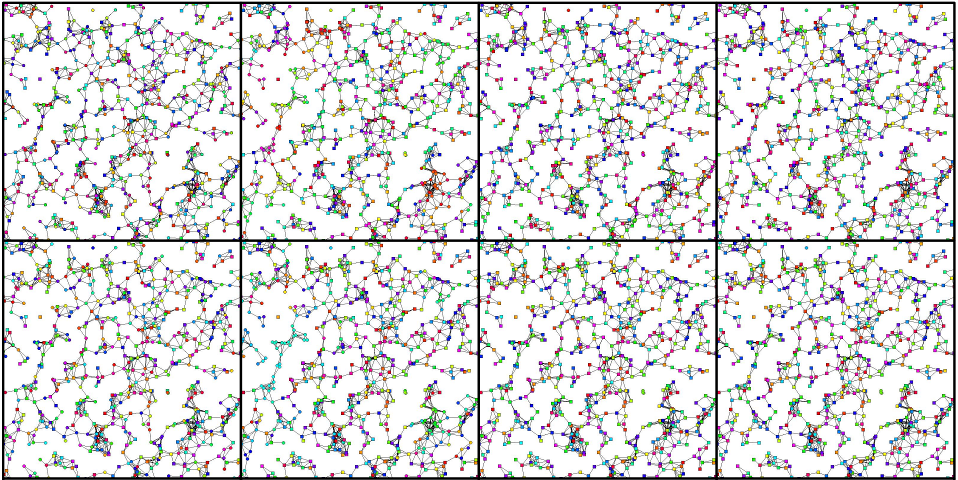
t = 0	t = 200	t = 2000	t = 5000
r_G = 0.011	r_G = 0.054	r_G = 0.114	r_G = 0.070
r_L = 0.640	r_L = 0.787	r_L = 0.913	r_L = 0.918
coop = 0.500	coop = 0.565	coop = 0.937	coop = 0.998

Microscopic behaviour in RGG



t = 0	t = 200	t = 2000	t = 5000
r_G = 0.039	r_G = 0.097	r_G = 0.042	r_G = 0.094
r_L = 0.641	r_L = 0.773	r_L = 0.840	r_L = 0.822
coop = 0.500	coop = 0.479	coop = 0.690	coop = 0.604

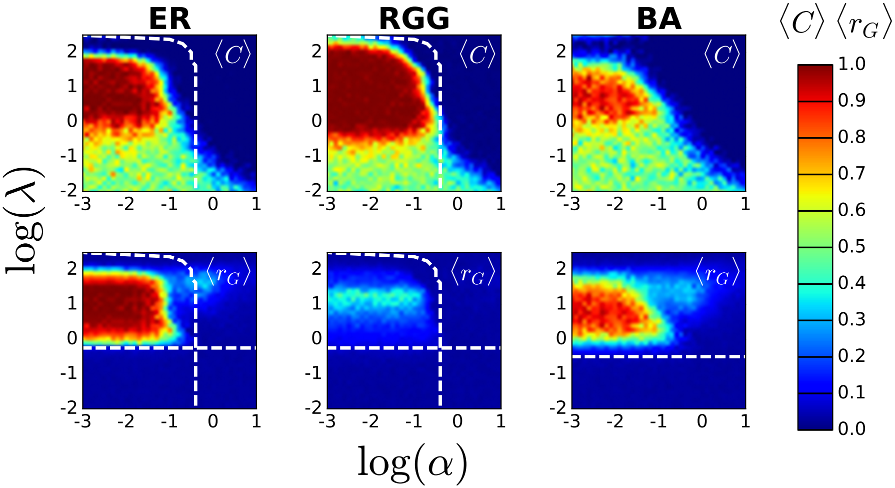
Microscopic behaviour in RGG



t = 0	t = 200	t = 2000	t = 5000
r_G = 0.033	r_G = 0.048	r_G = 0.053	r_G = 0.042
r_L = 0.631	r_L = 0.738	r_L = 0.647	r_L = 0.635
coop = 0.500	coop = 0.322	coop = 0.017	coop = 0.008

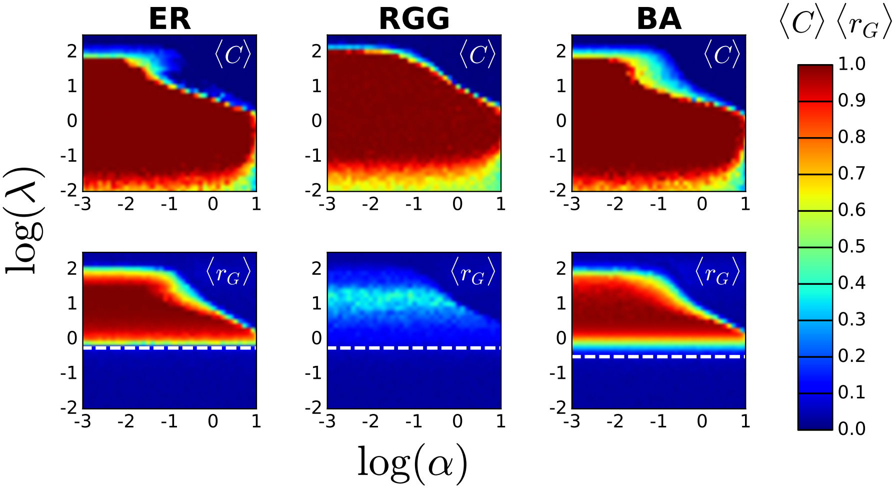
Other update rules

Asynchronous Fermi



Other update rules

Synchronous Imitation of the best



Fermi's Rule

$$P_{l \to m} = \frac{1}{1 + e^{-\beta(p_m - p_l)}} .$$