Social Networks and Cooperation II: going beyond replication

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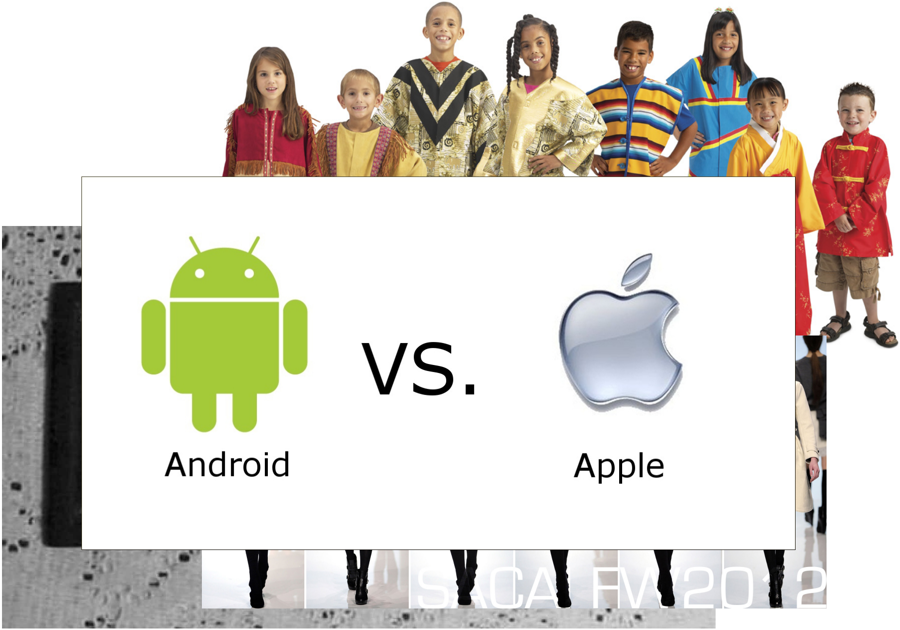
Thursday 1 September 2016, Mediterranean School on Complex Networks, Salina, Italy

DRINKING FOUNTAIN
WHITE
COLORED



VS.
Android
Apple

If we have **competition** between **two ideas**, which one will prevail?

If people tend to become **more alike** in their beliefs, attitudes, and behaviors when they **interact**; why do not all such differences eventually **disappear**?

What force drives the **segregation** of black people in the USA?

Questions

How rumors spread?

What are the mechanisms behind the **adoption** of a new technology?

- ~~Motivation~~.
- ★ Topic 1: Opinion Dynamics
- ★ Topic 2: Cultural Dissemination
- ★ Topic 3: Segregation

short pause (with some questions)

- ★ Topic $4+\frac{1}{2}$: Complex/Social contagions
- Take home messages
- Questions

Part I

Social dynamics

Topic 1

Opinion Dynamics

The Voter Model

Main assumptions

- System of size N.
- Elements are arranged on a Regular Lattice (1D)
- Two distinc species (state variable $s = \pm 1$).

Clifford, P., & Sudbury, A. "*A Model for Spatial Conflict*", Biometrika, **60**, 581 (1973).

The Voter Model

Main assumptions

- System of size N.
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- One interaction per time step.

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The Voter Model

Main assumptions

- System of size N.
- Elements are arranged on a Regular Lattice (1D)
- Two distinc species (state variable $s = \pm 1$).
- One interaction per time step.
- Update of state $s_j = s_i$.

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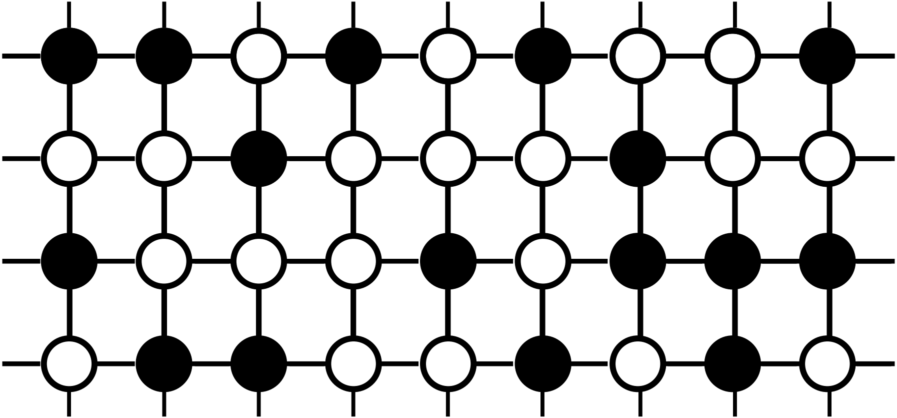
The Voter Model



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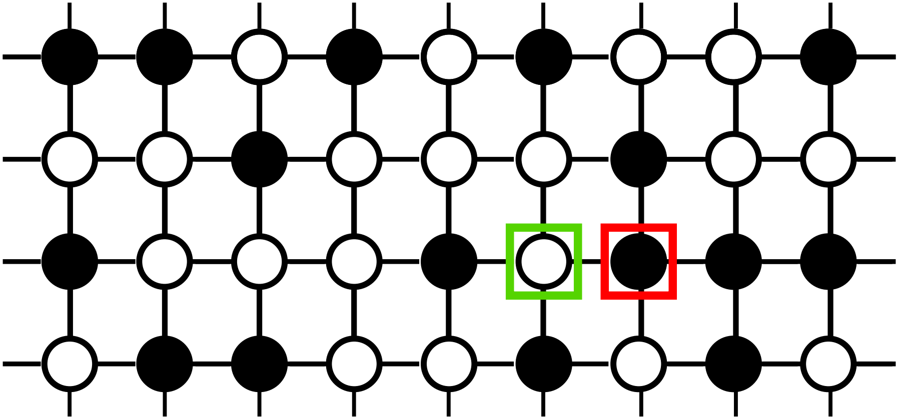
The Voter Model



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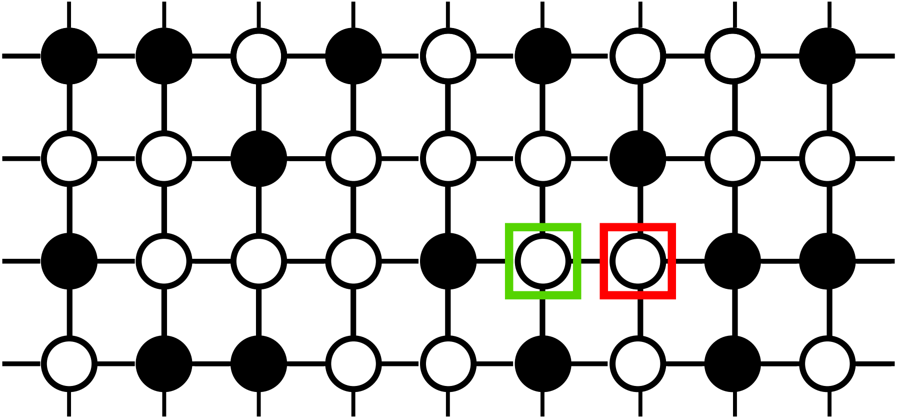

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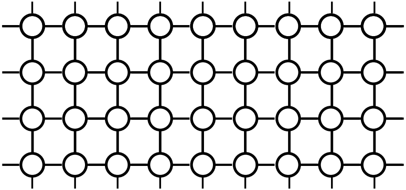
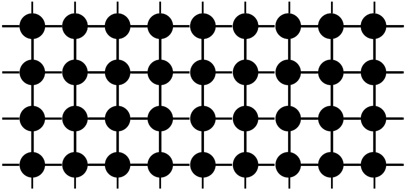
The Voter Model



- Two **adsorbing states**

Castellano, C., Fortunato, S., & Loreto, V. (2009). Rev. Mod. Phys., **81**, 591–646.



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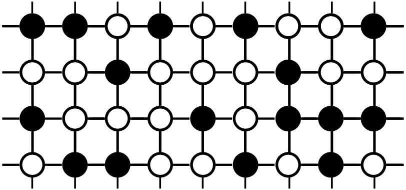
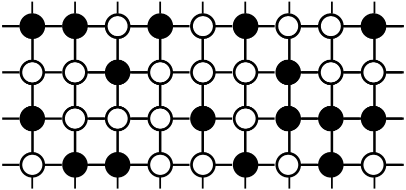


- Two **adsorbing states**
- For $d > 2$ **coexistence**.

Castellano, C., Fortunato, S., & Loreto, V. (2009). Rev. Mod. Phys., **81**, 591–646.

The Voter Model



Relevant quantities

$E_+ \rightarrow$ Exit probability

$T_N \rightarrow$ Time of convergence

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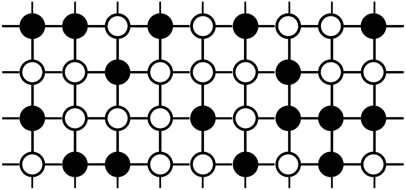
The Voter Model



Relevant quantities

$E_+ \rightarrow$ Exit probability

$T_N \rightarrow$ Time of convergence

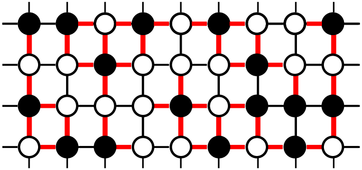
$$T_N = \begin{cases} N^2 & \text{for } d = 1 \\ N \log N & \text{for } d = 2 \\ N & \text{for } d > 2 \end{cases}$$

Castellano, C., Fortunato, S., & Loreto, V. (2009). Rev. Mod. Phys., **81**, 591–646.

Order Parameters

$$\rho = \frac{1}{K}\sum_{i=1}^{N}\sum_{j\in N_i}\frac{1-s_i s_j}{2} = \begin{cases} 0 & \text{Order} \\ \frac{1}{2} & \text{Disorder} \end{cases}$$

$$m_l = \sum \rho_{++} + \rho_{--} = \begin{cases} 1 & \text{Order } + \\ 0 & \text{Disorder} \\ -1 & \text{Order } - \end{cases}$$



Suchecki, K., Eguíluz, V. M., & San Miguel, M. (2005). Phys. Rev. E, **72**, 36132.

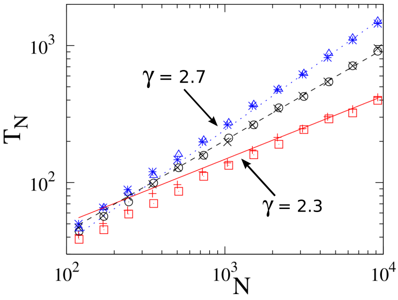
Sood, V., & Redner, S. (2005). Phys. Rev. Lett., **94**, 178701.

Vazquez, F., & Eguíluz, V. M. (2008). New Jour. Phys., **10**, 63011.

Possible developments

- Enlarge the set of values of the state variable $\Longrightarrow$ Use of Potts models.
- Quenched disorder $\Longrightarrow$ Introduction of Zealots.
- Restricted interactions $\Longrightarrow$ Only "moderate" individuals interact,
- Memory effects $\Longrightarrow$ Cumulative interactions.
- Topology of interactions $\Longrightarrow$ COMPLEX NETWORKS

The VM on Complex Networks



Convergence Time

$$T_N \propto N \frac{\langle k \rangle^2}{\langle k^2 \rangle}$$

T_N in Scale-Free ($P(k) \propto k^{-\gamma}$)

$$T_N \sim \begin{cases} N & \text{for} \quad \gamma > 3 \\ \dfrac{N}{\ln N} & \text{for} \quad \gamma = 3 \\ N^{\frac{2\gamma-1}{\gamma-1}} & \text{for} \quad 2 < \gamma < 3 \\ (\ln N)^2 & \text{for} \quad \gamma = 2 \\ \mathcal{O}(1) & \text{for} \quad \gamma < 2) \end{cases}$$

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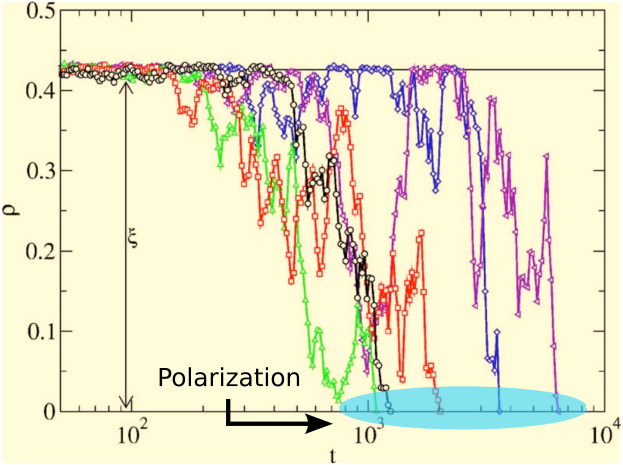

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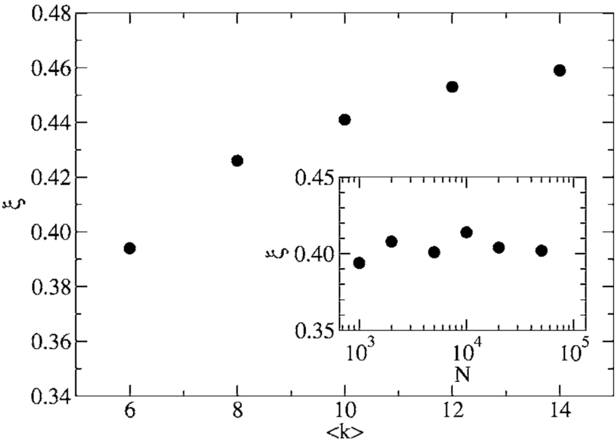

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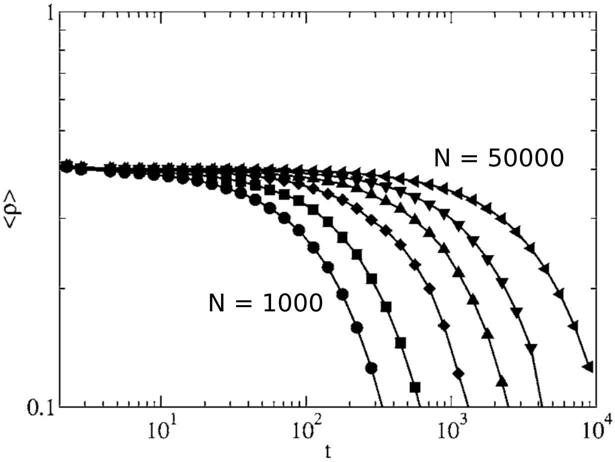

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The VM on Complex Networks

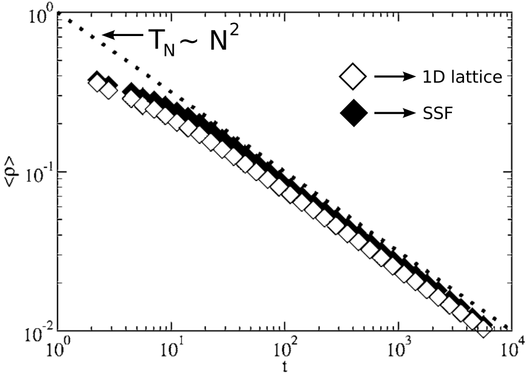
Scale-Free topologies

SSF $\rightarrow$ Structured Scale-Free
SWSF $\rightarrow$ Small-World Scale-Free
RSF $\rightarrow$ Random Scale-Free
SF-BA $\rightarrow$ Barábasi-Albert Scale-Free

Suchecki, K., Eguíluz, V. M., & San Miguel, M. (2005). Phys. Rev. E, **72**, 36132.

The VM on Complex Networks

The presence of structure **SLOW DOWN** polarization ($T_N \sim N^2$)



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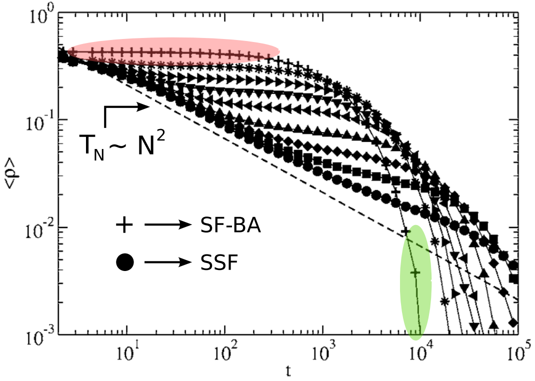
The VM on Complex Networks

The passage SSF → RSF display more disorder but quicker convergence.



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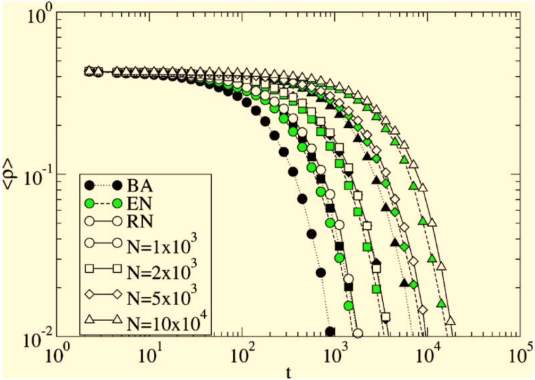
The VM on Complex Networks

In general, the hierarchy between SF, EN and RN is preserved with size.



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The VM on Complex Networks

Application

Application

PRL **112,** 158701 (2014) PHYSICAL REVIEW LETTERS week ending 18 APRIL 2014

Is the Voter Model a Model for Voters?

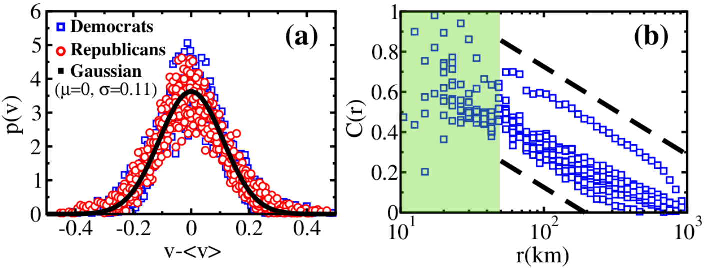
Juan Fernández-Gracia,[1,*] Krzysztof Suchecki,[2] José J. Ramasco,[1] Maxi San Miguel,[1] and Víctor M. Eguíluz[1]

[1]*IFISC (CSIC-UIB), 07122 Palma de Mallorca, Spain*

[2]*Faculty of Physics and Center of Excellence for Complex Systems Research, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland*

(Received 4 September 2013; revised manuscript received 4 February 2014; published 18 April 2014)

The voter model has been studied extensively as a paradigmatic opinion dynamics model. However, its ability to model real opinion dynamics has not been addressed. We introduce a noisy voter model (accounting for social influence) with recurrent mobility of agents (as a proxy for social context), where



U.S. Elections from 1980 to 2012

Fernández-Gracia, J., Suchecki, K., Ramasco, J. J., San Miguel, M., & Eguíluz, V. M. (2014). Phys. Rev. Lett., **112**, 158701.

The model

- Noisy VM + Recurrent Diffusion (home/work)
- 1 Node = 1 US County
- $V_{ij} \rightarrow$ # of voters (up) living in i and working in j.

Noisy VM

$$r^{-}_{ij} = V_{ij}\left[\underbrace{\alpha\frac{N_i - V_i}{N_i}}_{\text{home}} + \underbrace{(1-\alpha)\frac{N'_j - V'_j}{N'_j}}_{\text{work}}\right] + \underbrace{N_{ij}\frac{D}{2}\eta^{-}_{ij}}_{\text{noise}} \; .$$

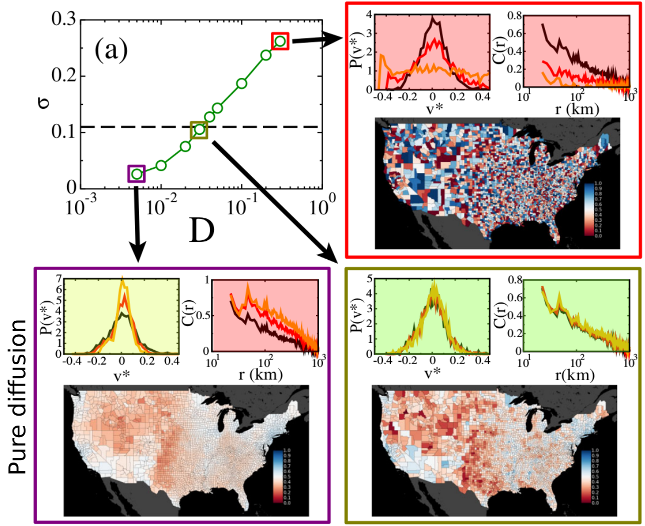
$$r^{+}_{ij} = (N_{ij} - V_{ij})\left[\alpha\frac{V_i}{N_i} + (1-\alpha)\frac{V'_j}{N'_j}\right] + N_{ij}\frac{D}{2}\eta^{+}_{ij} \; .$$

Diffusion

$$\frac{d\,V_{ij}}{dt} = \underbrace{\alpha\sum_l A_{ijl}V_{il}}_{\text{live in } i \text{ work in } l} + \underbrace{(1-\alpha)\sum_l B_{ijl}V_{lj}}_{\text{live in } l \text{ and work in } j} + \underbrace{D\,\eta_{ij}}_{\text{noise}} \; .$$

Fernández-Gracia, J., Suchecki, K., Ramasco, J. J., San Miguel, M., & Eguíluz, V. M. (2014). Phys. Rev. Lett., **112**, 158701.

Application



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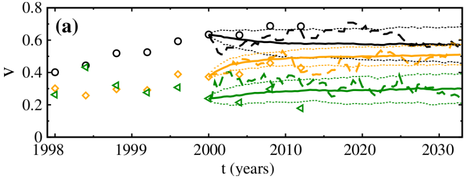
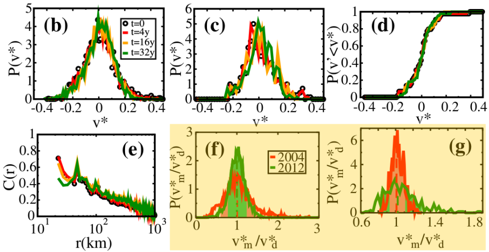
Application

Fraction of Dem. voters



○ → high pop.
◇ → med pop.
◁ → low pop.

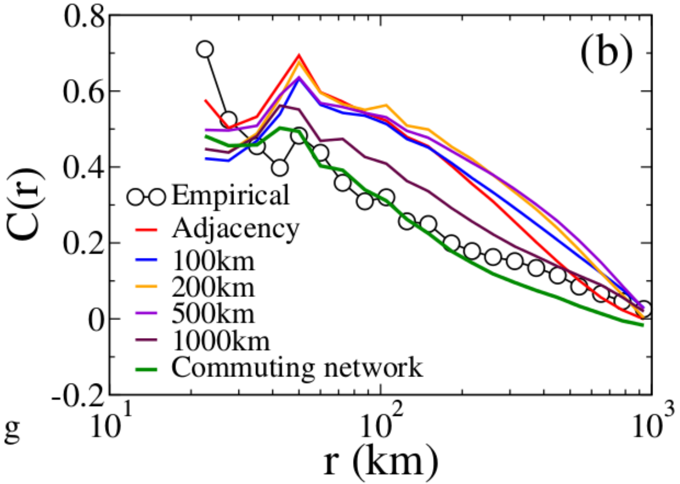
Counties **Congress** **States**



→ Voting fractions

→ Difference Data/Model

Fernández-Gracia, J., Suchecki, K., Ramasco, J. J., San Miguel, M., & Eguíluz, V. M. (2014). Phys. Rev. Lett., **112**, 158701.



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Topic 2

Cultural Dissemination

With the term "**culture**" we indicate the things over which people influence each other.

Axelrod, R. (1997). Journal of Conflict Resolution, **41**, 203–226.

Cultural Dynamics can be seen as a **multidimensional extension** of Opinion Dynamics.

Castellano, C., Fortunato, S., & Loreto, V. (2009). Rev. Mod. Phys., **81**, 591–646.

The Axelrod's model

If people tend to become **more alike** in their beliefs, attitudes, and behaviors when they **interact**, why do not all such differences eventually **disappear**?

Axelrod, R. (1997). Journal of Conflict Resolution, **41**, 203–226.

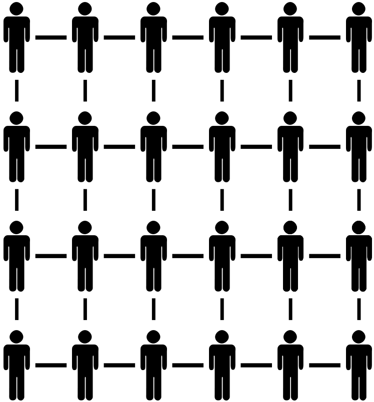
Main assumptions

- **Social Influence** $\rightarrow$ Interactions make agents more similar.
- **Homophily** $\rightarrow$ Interactions are more frequent between agents that are more similar.
- Agents are adaptive and irrational.
- Absence of a central authority.

The Axelrod's model

The model

- N agents on a 2D lattice



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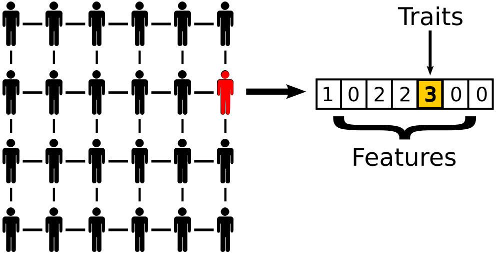
Klemm, K., Eguíluz, V. M., Toral, R., & San Miguel, M. (2003). Phys. Rev. E, **67**, 26120.

The Axelrod's model

The model

- N agents on a 2D lattice
- $F \geq 2$ features with $q > 1$ traits. (cultural complexity)



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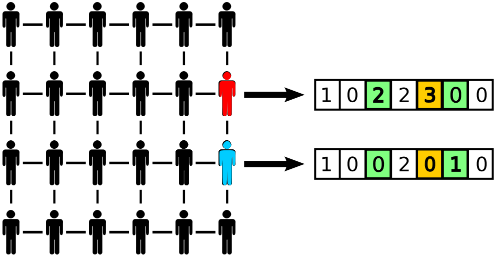
The model

- N agents on a 2D lattice
- $F \geq 2$ features with $q > 1$ traits. (cultural complexity)
- Compute the overlap

$$\omega(i,j) = \frac{1}{F}\sum_{f=1}^{F} \delta_{\sigma_{if},\sigma_{jf}}\,.$$



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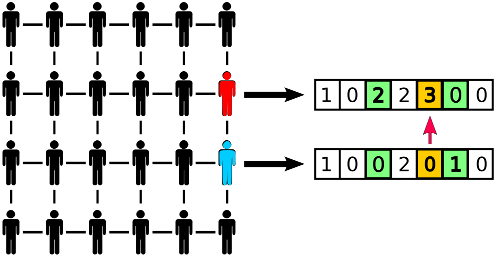
The Axelrod's model

The model

- N agents on a 2D lattice
- $F \geq 2$ features with $q > 1$ traits. (cultural complexity)
- Compute the overlap

$$\omega(i,j) = \frac{1}{F} \sum_{f=1}^{F} \delta_{\sigma_{if},\sigma_{jf}} \, .$$

- Select one (different) feature a random and adopt it.



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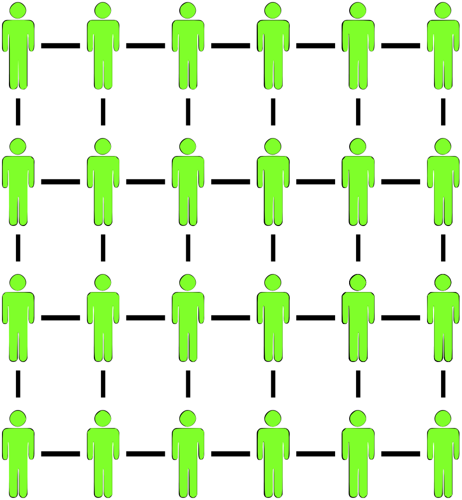
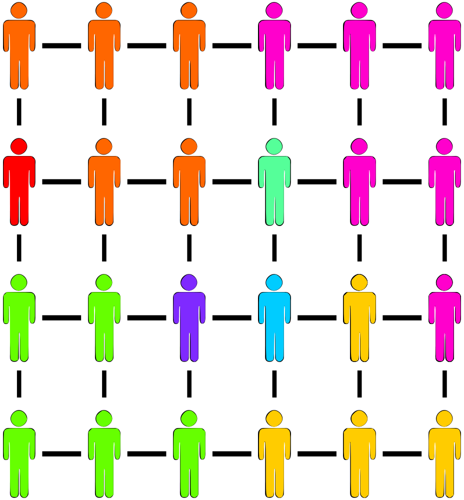
The Axelrod's model

- Two adsorbing states



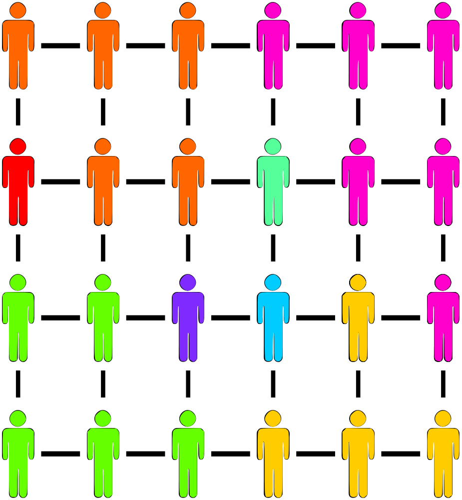
The Axelrod's model

- Two adsorbing states



The Axelrod's model

- Two adsorbing states
- The frozen disorder state depends on F and q



Order Parameters

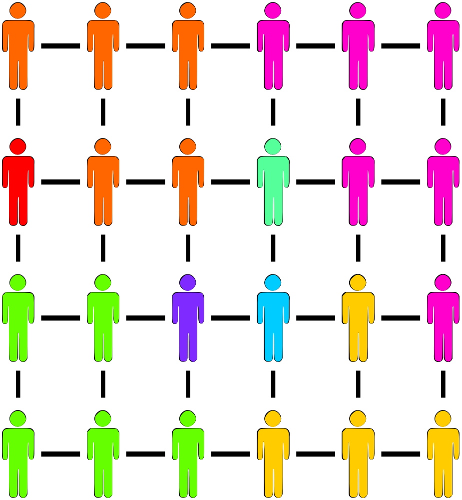
$$s = \frac{\langle S_m \rangle}{N} \longrightarrow \text{Giant comp.}$$

$$g = \frac{\langle N_g \rangle}{N} \longrightarrow \text{Nr. comp.}$$

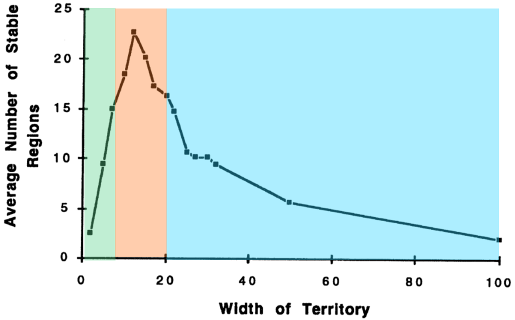
$$n_a \longrightarrow \text{Nr. active bonds}$$

$$\text{Order} = \begin{cases} s & \simeq 1 \\ g & \simeq \frac{1}{N} \end{cases}$$

$$\text{Disorder} = \begin{cases} s & \simeq \frac{1}{N} \\ g & \simeq \varepsilon \leq 1 \end{cases}$$



The Axelrod's model



Main Results in Lattices

- The Nr. of cultural domains depends on the size of the lattice *N*.

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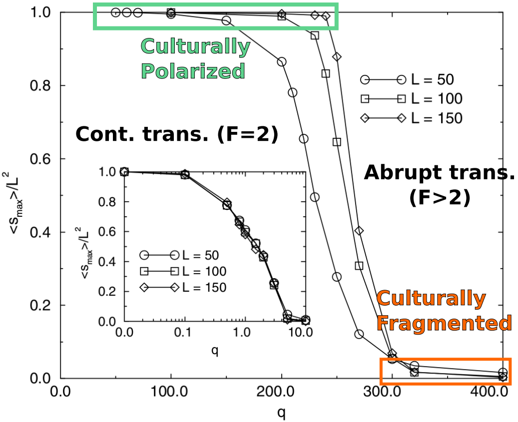
Castellano, C., Marsili, M., & Vespignani, A. (2000). Phys. Rev. Lett., **85**, 3536.



Main Results in Lattices

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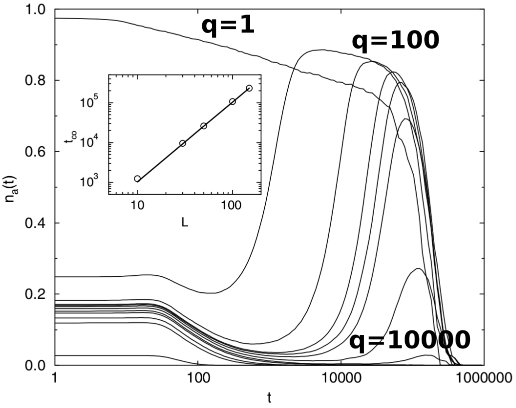
The Axelrod's model



Main Results in Lattices

- The Nr. of cultural domains depends on the size of the lattice *N*.
- The order of transition changes for $F > 2$.
- $n_a(t)$ depends on q.

Axelrod, R. (1997). Journal of Conflict Resolution, **41**, 203–226.

Castellano, C., Marsili, M., & Vespignani, A. (2000). Phys. Rev. Lett., **85**, 3536.

The Axelrod's model

Summing up . . .

1. g grows with q (TRIVIAL)
2. g decreases when $\langle k \rangle$ increases (TRIVIAL)
3. g decreases when F increases (NON TRIVIAL)
4. g decreases when N increases (NON TRIVIAL).

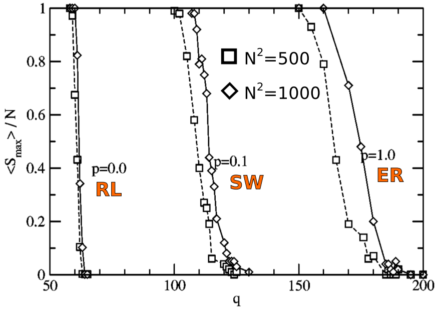
Why people tend to be different?

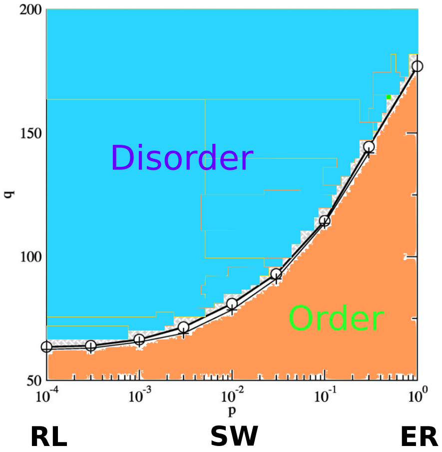
- Social Differentiation
- Fads and Fashions
- Extremisms
- Drift
- Geographic Isolation
- Specialization
- Changing environment or technology

Possible developments

- Spontaneous mutation of a trait $\Longrightarrow$ Cultural Drift.
- Dominant traits $\Longrightarrow$ Cultural Subgroups.
- Threshold on similarity $\Longrightarrow$ Confidence Bound (Deffuant),
- External field $\Longrightarrow$ Mass Media.
- Embedding in metric space $\Longrightarrow$ Metric difference among traits.
- Non-random traits $\Longrightarrow$ Geographical effects.
- Some sites less likely to change status $\Longrightarrow$ Social Status.
- Adoption of new traits $\Longrightarrow$ Technological changes.
- Mobility
- Differentiation upon interaction $\Longrightarrow$ Cultural Divergence.
- Topology of interactions $\Longrightarrow$ COMPLEX NETWORKS

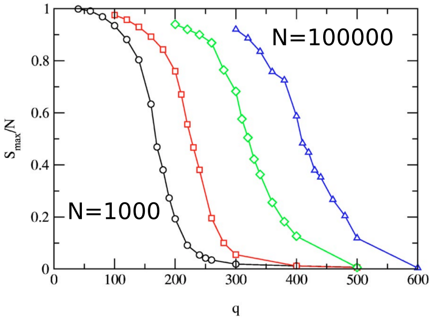
Random Networks (SW/ER)

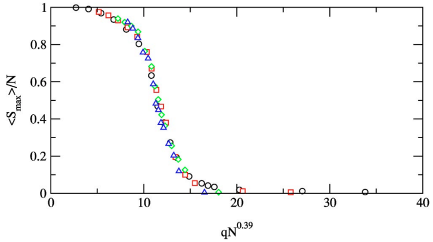




Klemm, K., Eguíluz, V. M., Toral, R., & San Miguel, M. (2003). Phys. Rev. E, **67**, 26120.

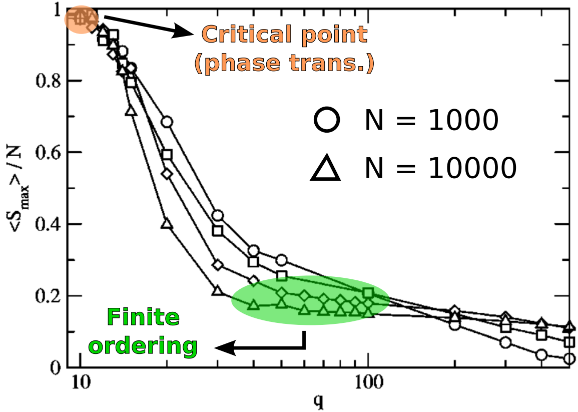
Random Scale-Free Networks (RSF)





Klemm, K., Eguíluz, V. M., Toral, R., & San Miguel, M. (2003). Phys. Rev. E, **67**, 26120.

Structured Scale-Free Networks (SSF)

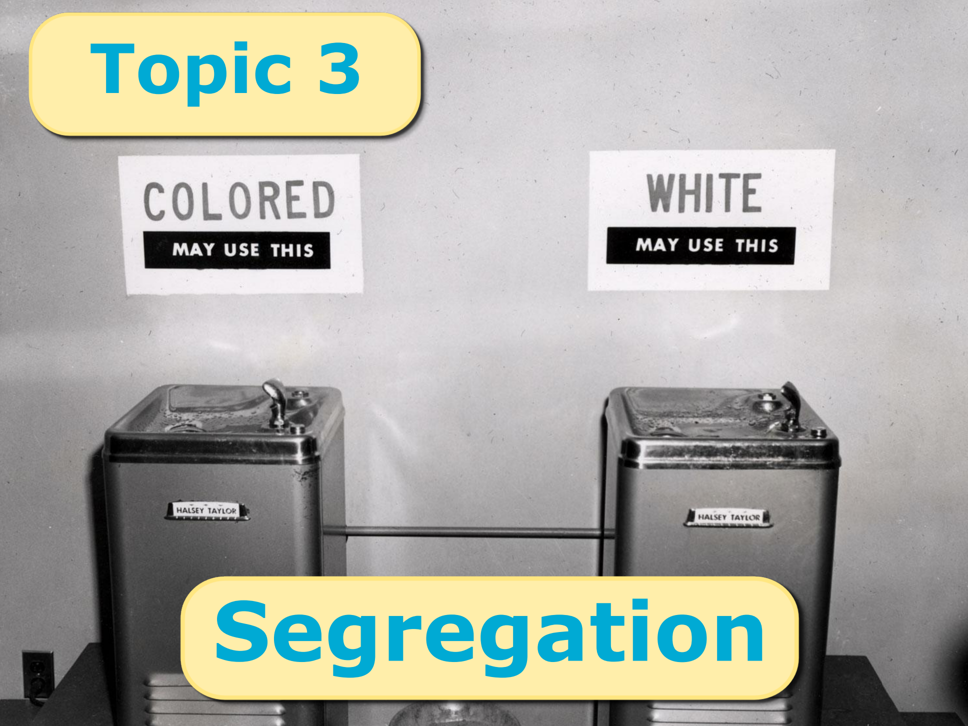


Klemm, K., Eguíluz, V. M., Toral, R., & San Miguel, M. (2003). Phys. Rev. E, **67**, 26120.

K. Klemm & V.M. Eguíluz, Phys. Rev. E **65**, 036123 (2002).

The AM on Networks

Topic 3



Segregation

The Schelling's model

"People get ***separated*** *along many lines and in many ways. There is* ***segregation*** *by sex, age, income, language, religion, color, taste, comparative advantage, historical location."*

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Schelling's model

- Organized action: legal–illegal, open–covert (*i.e.* civil rights)
- Economic: rich–poor, more edu.–less edu. (*i.e.* social equity)
- Individual decision

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Schelling's model

Schelling's Model is about: *"the kinds of **segregation** –or **separation**, or **sorting** – that can result from **discriminatory individual behavior**. By 'discriminatory' I mean reflecting an awareness, conscious or unconscious, [. . .] that influences decisions on where to live, whom to sit by, what occupation to join or to avoid, whom to play with or whom to talk to."*

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Schelling's model

The model

- Two species on a 1D lattice with N elements

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Schelling's model

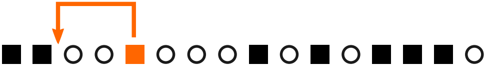
The model

- Two species on a 1D lattice with N elements
- Each agent has an aspiration level

$$f_i = \frac{N_i}{N_{\text{tot}}}$$

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Schelling's model



The model

- Two species on a 1D lattice with N elements
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$$f_i = \frac{N_i}{N_{\mathrm{tot}}}$$

- Unsatisfied agents migrate

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Schelling's model

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- Unsatisfied agents migrate
- One iteration per time until stationary state

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Schelling's model

■ ■ ■ ○ ○ ○ ○ ○ ■ ■ ○ ○ ○ ■ ■ ■

○ ○ ○ ○ ○ ○ ○ ○ ■ ■ ■ ■ ■ ■ ■ ■

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Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

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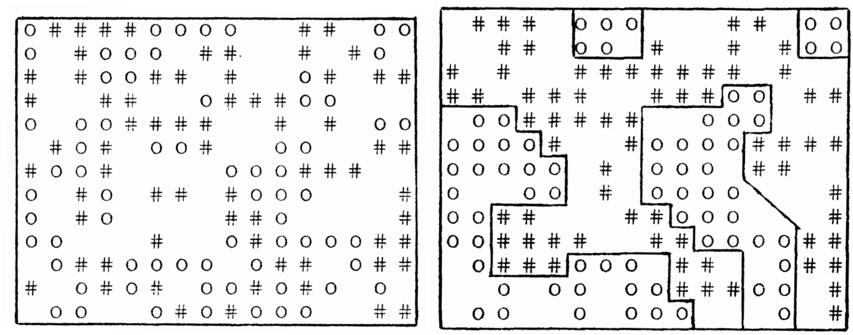


Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

Free parameters

- Size of neighbourhood
- Initial mixing
- Motion rules
- Aspiration level

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

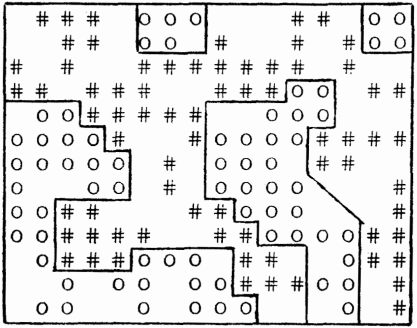
Free parameters

- Size of neighbourhood
- Initial mixing
- Motion rules
- Aspiration level
    - Increase of unsatisfied agents
    - Higher number of migrations
    - Enlargement of regions of segregation
    - If $f_1 + f_2 > 1 \rightarrow$ **NO COEXISTENCE**

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

Main Result

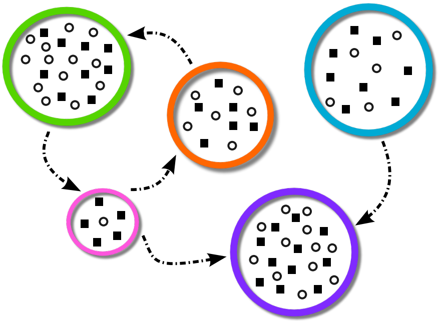
Agents do not need to be completely intollerant to observe segregation



Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

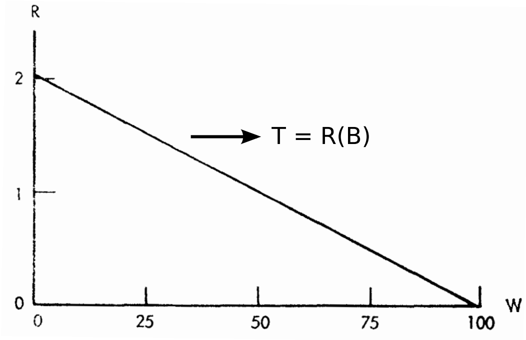
The Bounded Neighbourhood

- The neighbourhood = physical location
- All-to-all inside the neighbourhood
- Migration among neighbourhoods



Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

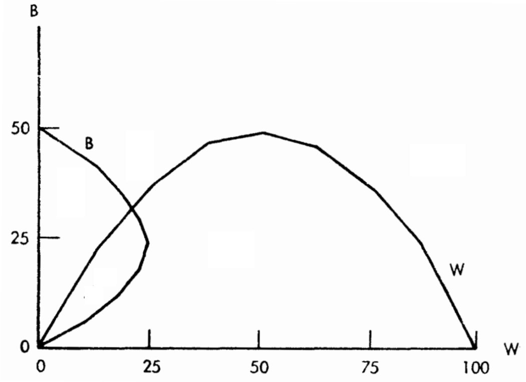
The Bounded Neighbourhood



$T = R(X)$ with $X \in \{B, W\}$

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

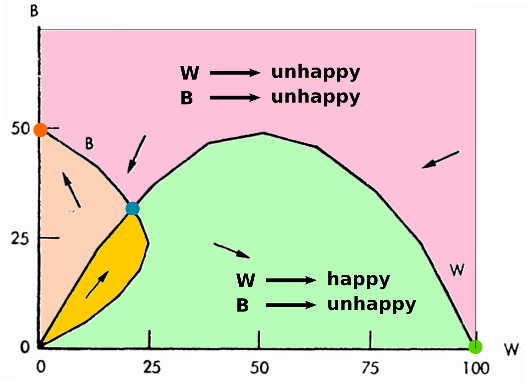
The Bounded Neighbourhood



$$B(W) = R_w(W)\, W$$
$$W(B) = R_b(B)\, B$$

Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Bounded Neighbourhood



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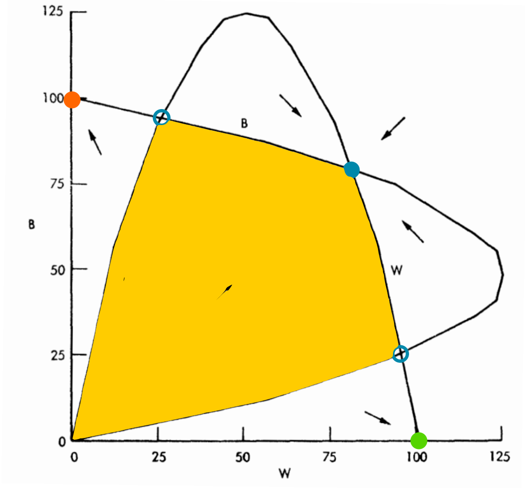
Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

The Bounded Neighbourhood



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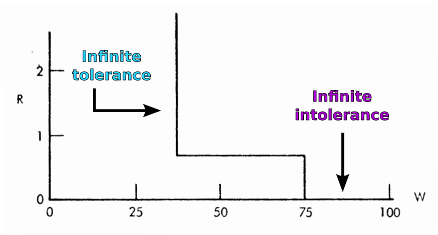
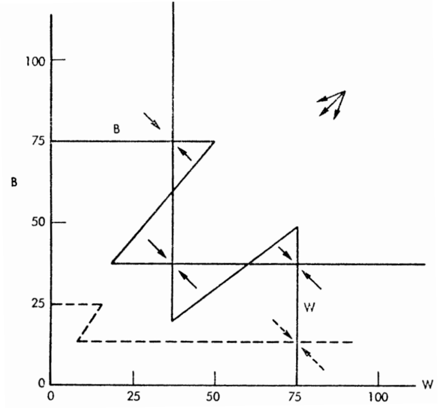
The Bounded Neighbourhood





Schelling, T. C. (1971). Jour. Math. Sociol., **1**, 143–186.

Further developments

- Account for different aspiration levels.
- Alter the initial mixing (simulate geographical constraints).
- Change the movement rule.
- Topology of interactions $\Longrightarrow$ COMPLEX NETWORKS

An example of AM & SM

PHYSICAL REVIEW E **80**, 046123 (2009)

Residential segregation and cultural dissemination: An Axelrod-Schelling model

C. Gracia-Lázaro,[1] L. F. Lafuerza,[2] L. M. Floría,[3,1,*] and Y. Moreno[3,4,†]

[1]*Departamento de Física de la Materia Condensada, Universidad de Zaragoza, Zaragoza E-50009, Spain*

[2]*Instituto de Física Interdisciplinar y Sistemas Complejos (IFISC), CSIC–UIB, E-07122 Palma de Mallorca, Spain*

[3]*Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, Zaragoza 50009, Spain*

[4]*Departamento de Física Teórica, Universidad de Zaragoza, Zaragoza E-50009, Spain*

(Received 14 July 2009; revised manuscript received 29 September 2009; published 26 October 2009)

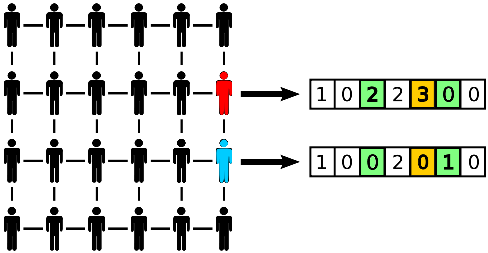
In the Axelrod's model of cultural dissemination, we consider the mobility of cultural agents through the introduction of a density of empty sites and the possibility that agents in a dissimilar neighborhood can move to them if their mean cultural similarity with the neighborhood is below some threshold. While for low values of the density of empty sites, the mobility enhances the convergence to a global culture, for high enough values of it, the dynamics can lead to the coexistence of disconnected domains of different cultures. In this regime, the

An example of AM & SM

What happens to the Axelrod's model if we introduce migration of agents based on a Schelling's like mechanism?

Gracia-Lázaro, C., Lafuerza, L. F., Floría, L. M., & Moreno, Y. (2009). Phys. Rev. E, **80**, 46123.

An example of AM & SM



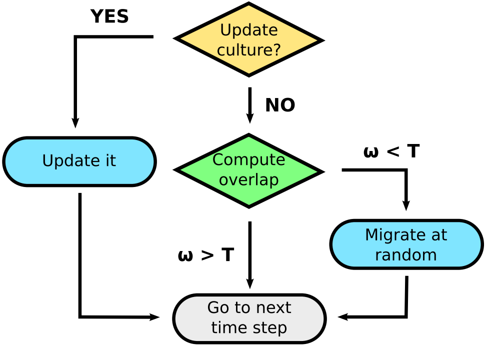
The model

$$\omega_{ij} = \frac{1}{F} \sum_{f=1}^{F} \delta_{\sigma_{if},\sigma_{jf}}$$

$$\overline{\omega_i} = \frac{1}{k_i} \sum_{j=1}^{N} a_{ij}\, \omega_{ij} \,.$$

Gracia-Lázaro, C., Lafuerza, L. F., Floría, L. M., & Moreno, Y. (2009). Phys. Rev. E, **80**, 46123.

An example of AM & SM



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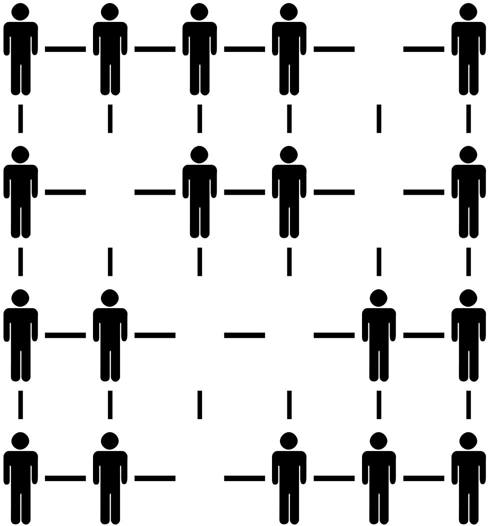
An example of AM & SM

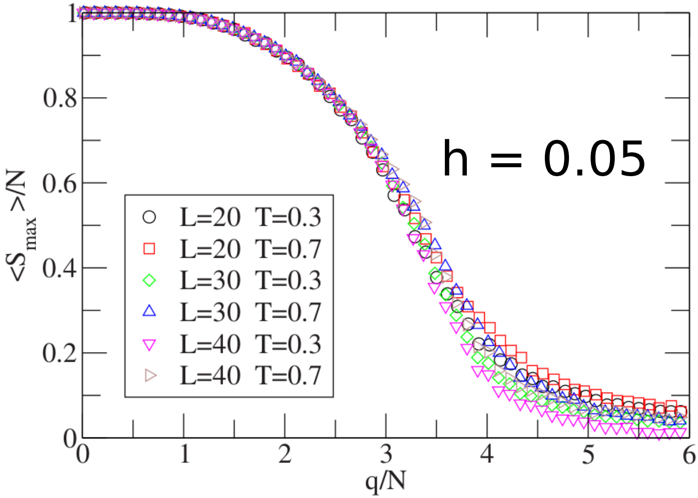


The model

- Fraction of empty sites h.

Gracia-Lázaro, C., Lafuerza, L. F., Floría, L. M., & Moreno, Y. (2009). Phys. Rev. E, **80**, 46123.

An example of AM & SM



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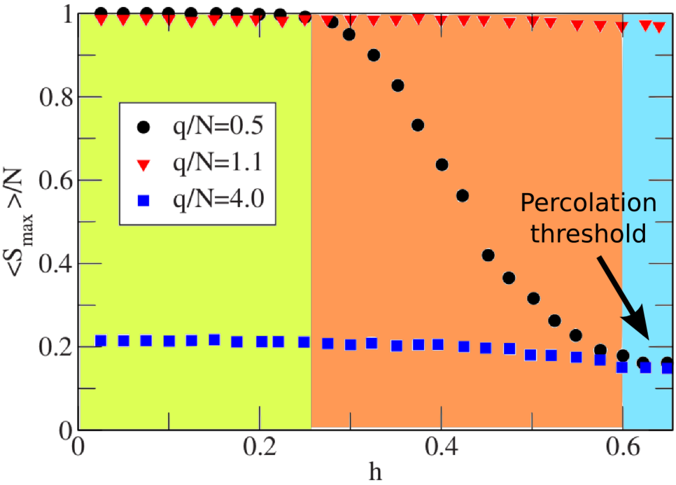

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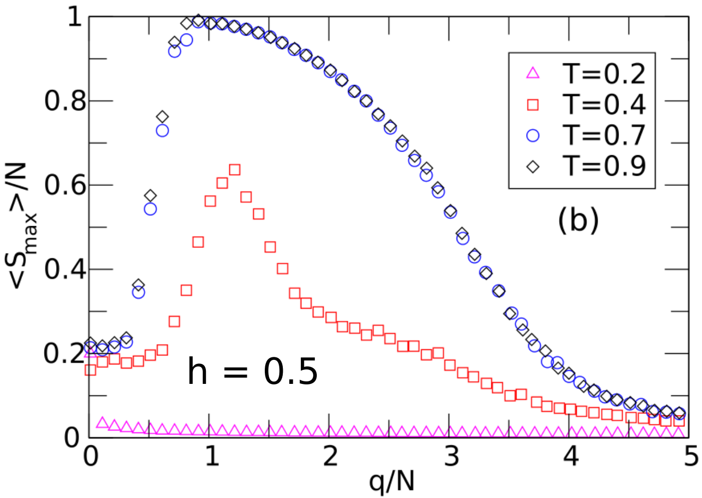


Gracia-Lázaro, C., Lafuerza, L. F., Floría, L. M., & Moreno, Y. (2009). Phys. Rev. E, **80**, 46123.

Networks from a SH process

PNAS

Emergence of segregation in evolving social networks

Adam Douglas Henry[a,1], Paweł Prałat[b], and Cun-Quan Zhang[b]

[a]Division of Public Administration, West Virginia University, Morgantown, WV 26506; and [b]Department of Mathematics, West Virginia University, Morgantown, WV 26506

Edited by William A. V. Clark, University of California, Los Angeles, CA, and approved March 16, 2011 (received for review October 7, 2010)

In many social networks, there is a high correlation between the similarity of actors and the existence of relationships between them. This paper introduces a model of network evolution where actors are assumed to have a small aversion from being connected to others who are dissimilar to themselves, and yet no actor strictly prefers a segregated network. This model is motivated by Schelling's [Schelling TC (1969) Models of segregation. *Am Econ Rev* 59:488–493] classic model of residential segregation, and we show that Schelling's results also apply to the structure of networks;

model of residential segregation (4, 5) for insights into why such networks emerge and persist. Schelling's model focused on the role of individual preferences in shaping emergent segregation patterns and demonstrated that seemingly mild preferences against being a local minority can produce stark and counterintuitive patterns of global segregation. This model challenged the common view that discrimination—defined as a strict preference for homogenous communities, potentially coupled with formal regulations that inhibit integration—is a necessary condition

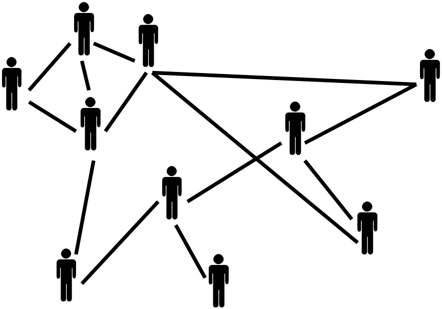
Networks from a SH process

What happens to the structure of a network if we **rewire** the links using a Schelling mechanism?

Henry, A. D., Pralat, P., & Zhang, C.-Q. (2011). Proc. Nat. Acad. Sci. (USA), **108**, 8605–8610.

Networks from a SH process

- Network of N agents



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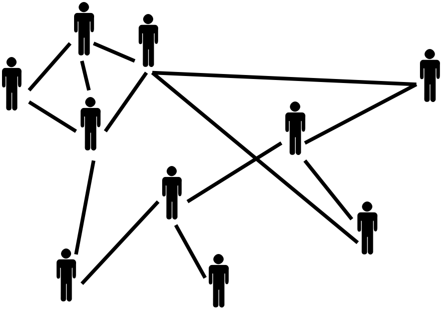
Networks from a SH process

- Network of N agents
- Agents with r features ($f_i \equiv [-1, 1] \Leftrightarrow q = 2$)



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Networks from a SH process

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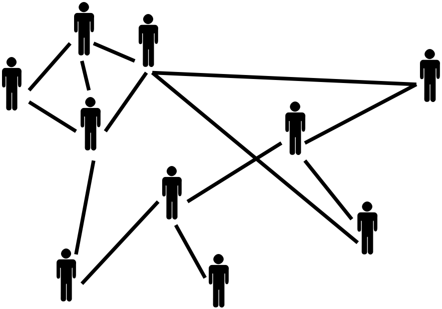

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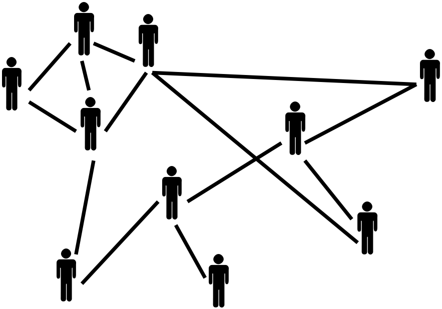

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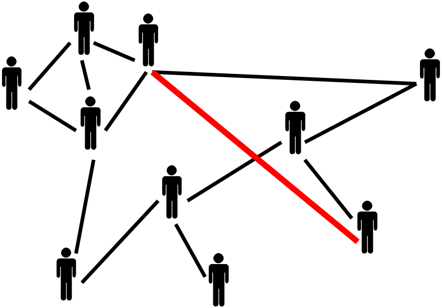
- A broken link is rewired at random. $P = d(\omega(u), \omega(v))\, p$



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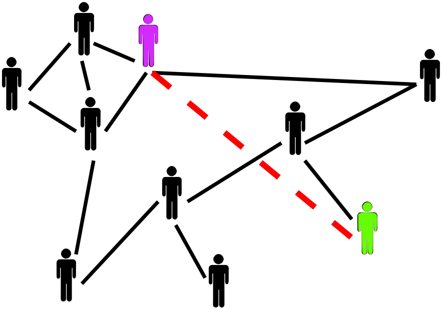

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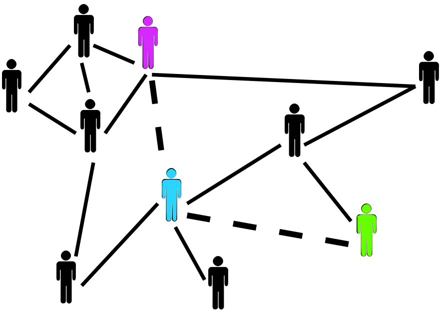

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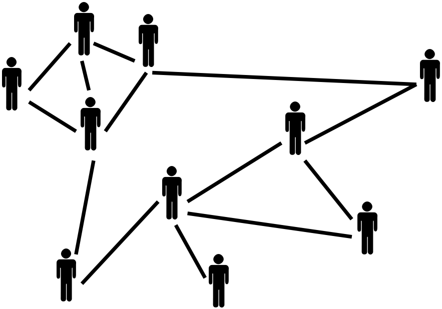

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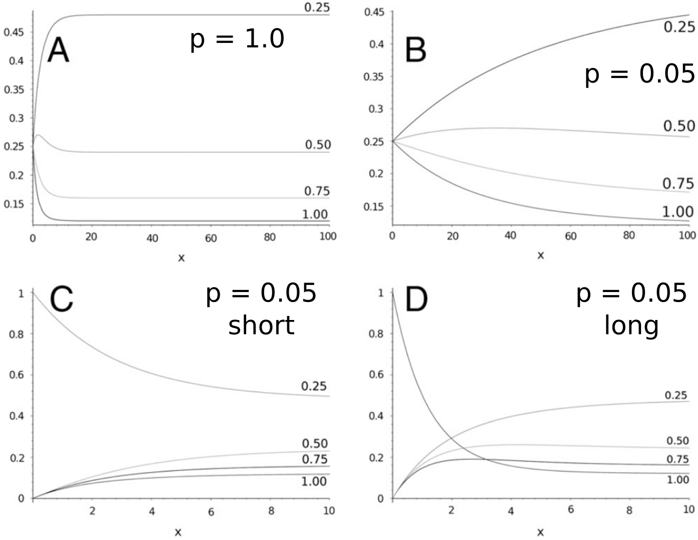
Networks from a SH process



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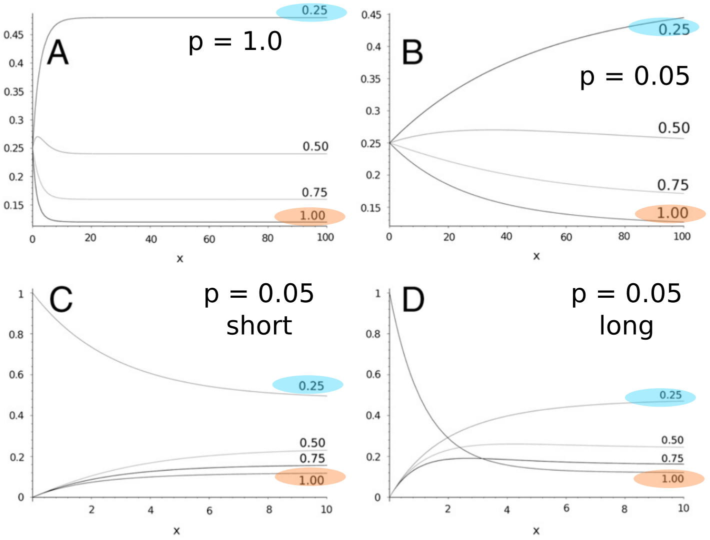
Networks from a SH process



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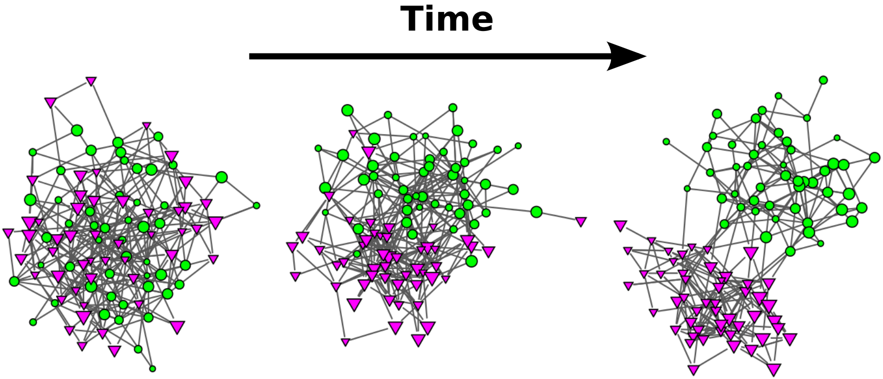


Henry, A. D., Pralat, P., & Zhang, C.-Q. (2011). Proc. Nat. Acad. Sci. (USA), **108**, 8605–8610.


Pause!!!

- ~~Motivation.~~
- ★ ~~Topic 1: Opinion Dynamics~~
- ★ ~~Topic 2: Cultural Dissemination~~
- ★ ~~Topic 3: Segregation~~

~~short pause (with some questions)~~

- ★ Topic $4+\frac{1}{2}$: Complex/Social contagions
- Take home messages
- Questions

Part II

Contagions

Social/Complex Contagion

Topic 4+$^1/_2$

Social/Complex Contagions

What is a "social" contagion?

- Emergence of **social norms/movements**.
- Spreading of **rumors/urban legends**.
- **Adoption** of technological innovations.

Multiple channels

- Strategic Complementarity

Multiple channels

- Strategic Complementarity
- Credibility

Multiple channels

- Strategic Complementarity
- Credibility
- Legitimacy

Social/Complex Contagions

Multiple channels

- Strategic Complementarity
- Credibility
- Legitimacy
- Emotional Contagion

Social/Complex Contagions

Social networks are the pathways along which "social contagions" propagates.

Centola, D., & Macy, M. (2007). Amer. Jour. Sociol., **113**, 702–734.

The Maki Thompson model

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

The Maki Thompson model

PHYSICAL REVIEW E **69**, 066130 (2004)

Dynamics of rumor spreading in complex networks

Yamir Moreno,[1,2] Maziar Nekovee,[3] and Amalio F. Pacheco[1,2]
[1]*Departmento de Física Teórica, Universidad de Zaragoza, Zaragoza 50009, Spain*
[2]*Instituto de Biocomputación y Física de Sistemas Complejos, Universidad de Zaragoza, Zaragoza 50009, Spain*
[3]*Complexity Research Group, Polaris 134 BT Exact, Martlesham, Suffolk IP5 3Re, United Kingdom*
(Received 4 December 2003; published 17 June 2004)

We derive the mean-field equations characterizing the dynamics of a rumor process that takes place on top of complex heterogeneous networks. These equations are solved numerically by means of a stochastic approach. First, we present analytical and Monte Carlo calculations for homogeneous networks and compare the

The Maki Thompson model

The model

- **Ignorant** (I)
- **Spreader** (S)
- **Stifler** (R)

Moreno, Y., Nekovee, M., & Pacheco, A. F. (2004). Phys. Rev. E, **69**, 66130.

The Maki Thompson model

The model

$$I + S \xrightarrow{\lambda} 2S$$

$$S + R \xrightarrow{\alpha} 2R$$

$$S + S \xrightarrow{\alpha} S + R$$

Moreno, Y., Nekovee, M., & Pacheco, A. F. (2004). Phys. Rev. E, **69**, 66130.

The Maki Thompson model

HMF approximation

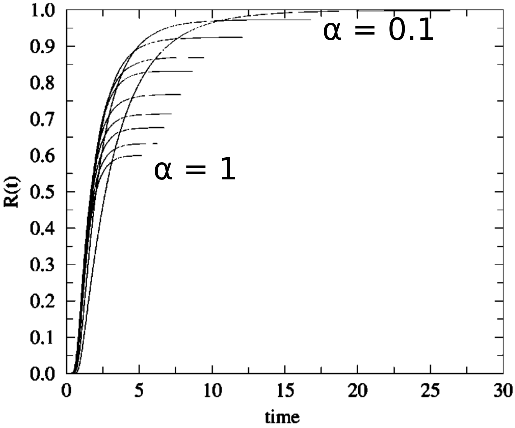
$$\frac{d\, i_k(t)}{dt} = -\lambda\, k\, i_k(t) \sum_{k'} \frac{k' P(k') s_{k'}(t)}{\langle k \rangle}\,,$$

$$\frac{d\, s_k(t)}{dt} = \lambda\, k\, i_k(t) \sum_{k'} \frac{k' P(k') s_{k'}(t)}{\langle k \rangle} - \alpha\, k\, s_k(t) \sum_{k'} \frac{k' P(k') \left[s_{k'}(t) + r_{k'}(t)\right]}{\langle k \rangle}\,,$$

$$\frac{d\, r_k(t)}{dt} = \alpha\, k\, s_k(t) \sum_{k}^{\prime} \frac{k' P(k') \left[s_{k'}(t) + r_{k'}(t)\right]}{\langle k \rangle}\,.$$

Moreno, Y., Nekovee, M., & Pacheco, A. F. (2004). Phys. Rev. E, **69**, 66130.

The Maki Thompson model



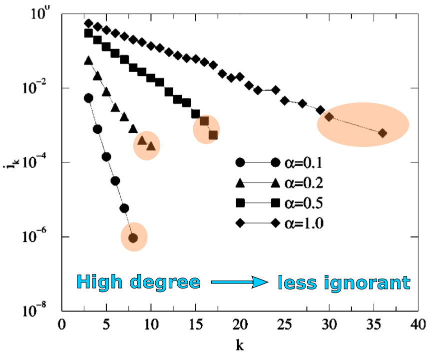
Relevant quantities

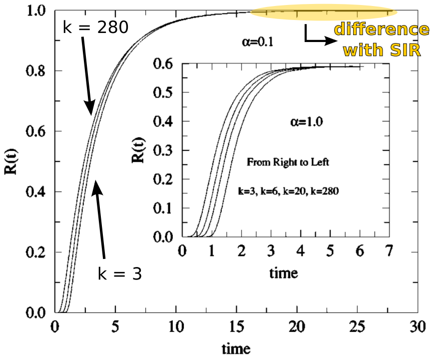
Reliability → # of Stiflers R

Efficiency → cost/benefit

Moreno, Y., Nekovee, M., & Pacheco, A. F. (2004). Phys. Rev. E, **69**, 66130.

The Maki Thompson model





Moreno, Y., Nekovee, M., & Pacheco, A. F. (2004). Phys. Rev. E, **69**, 66130.

Explosive Contagions

Some spreadings takes place very abruptly, such as . . .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Explosive Contagion in Networks

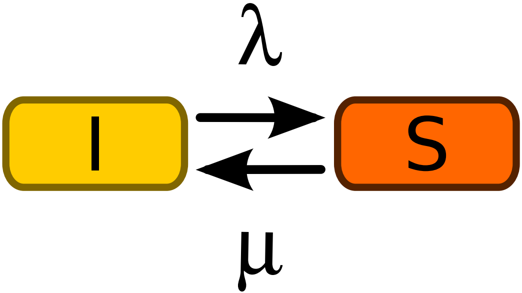
J. Gómez-Gardeñes[1,2], L. Lotero[3,6], S. N. Taraskin[4] & F. J. Pérez-Reche[5]

The spread of social phenomena such as behaviors, ideas or products is an ubiquitous but remarkably complex phenomenon. A successful avenue to study the spread of social phenomena relies on epidemic models by establishing analogies between the transmission of social phenomena and infectious

Explosive Contagions

The model

- Standard SIS model
- $\lambda_{i \to j} = \alpha \exp(\beta\, n^*(i))$.
- Homogeneous networks (ER – RR)



Gómez-Gardeñes, J., Lotero, L., Taraskin, S. N., & Pérez-Reche, F. J. (2016). Sci. Rep., **6**, 19767.

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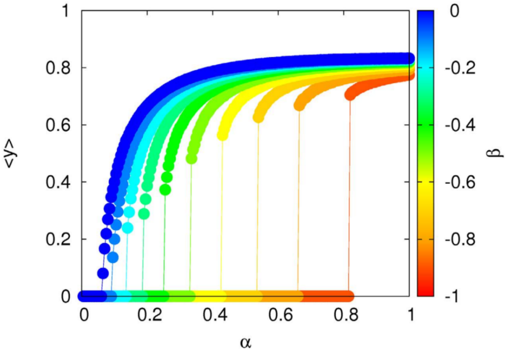

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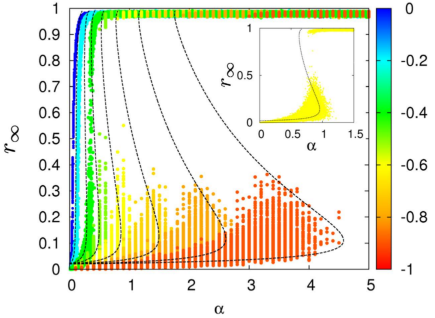
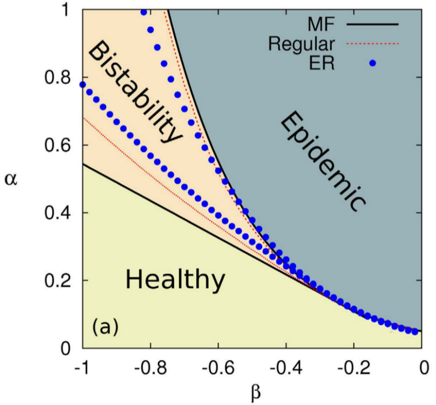
Explosive Contagions





Gómez-Gardeñes, J., Lotero, L., Taraskin, S. N., & Pérez-Reche, F. J. (2016). Sci. Rep., **6**, 19767.

The strength of "weak" ties

Dual weak/strong meaning

- Pairwise relations (weight)
- Entire structure (length)

Granovetter, M. S. (1973). Amer. Jour. Sociol., **78**, 1360.

Centola, D., & Macy, M. (2007). Amer. Jour. Sociol., **113**, 702–734.

The strength of "weak" ties

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  Weak = weak interaction
  Strong = strong interaction

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Granovetter, M. S. (1973). Amer. Jour. Sociol., **78**, 1360.

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The strength of "weak" ties

Dual weak/strong meaning

- Pairwise relations (weight)

  Weak = weak interaction

  Strong = strong interaction

- Entire structure (length)

  Weak = connecting near elements

  Strong = connecting distant elements

Links that are **weak relationally** are usually **strong structurally**.

Granovetter, M. S. (1973). Amer. Jour. Sociol., **78**, 1360.

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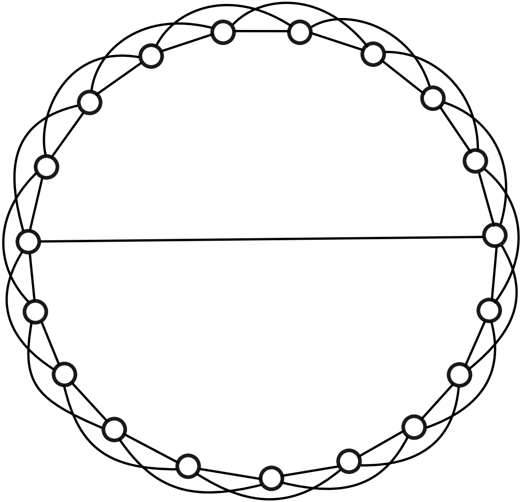
What is the **effect** of weak ties on **complex contagions**?

Weak ties & contagions



- Small-world network

Centola, D., & Macy, M. (2007). Amer. Jour. Sociol., **113**, 702–734.

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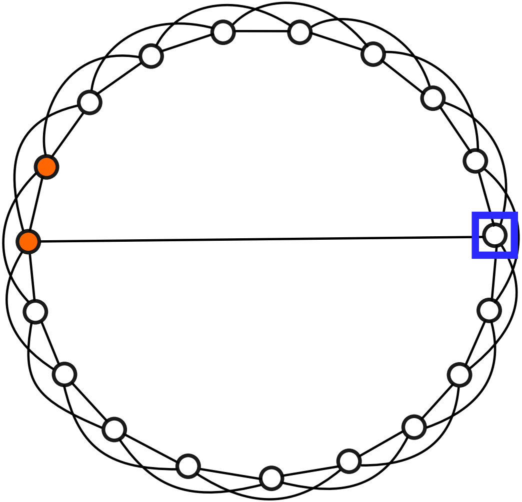
Weak ties & contagions



- Small-world network
- **Simple** contagion

Centola, D., & Macy, M. (2007). Amer. Jour. Sociol., **113**, 702–734.

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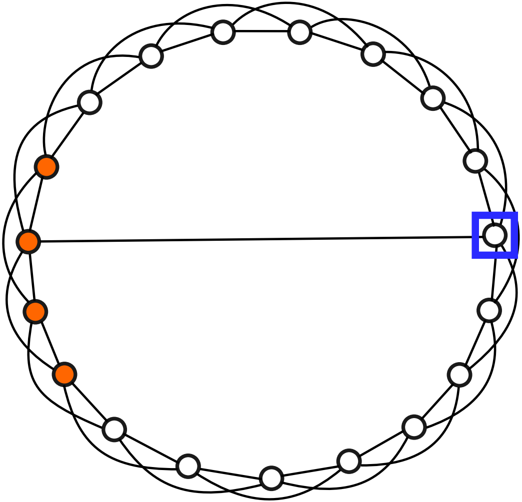
Weak ties & contagions



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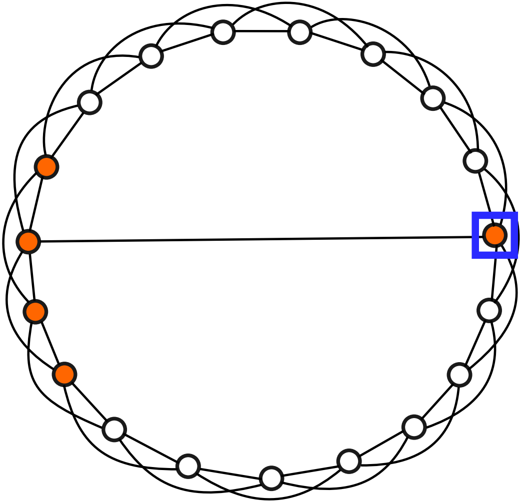
Weak ties & contagions



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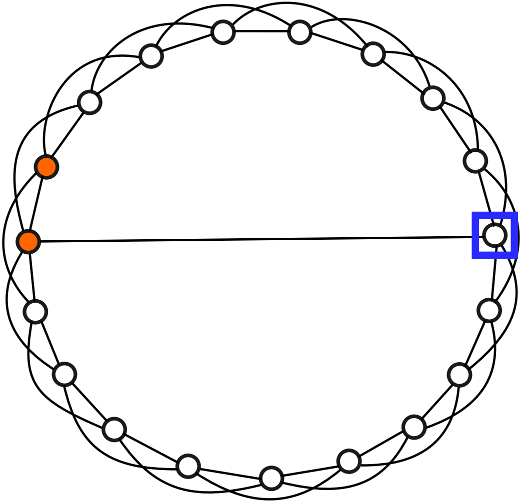
Weak ties & contagions



- Small-world network
- **Simple** contagion
- **Complex** contagion
  Threshold = 2 neigh.

Centola, D., & Macy, M. (2007). Amer. Jour. Sociol., **113**, 702–734.

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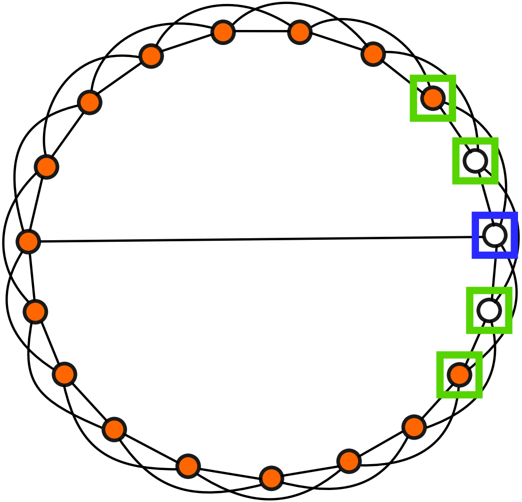
Weak ties & contagions



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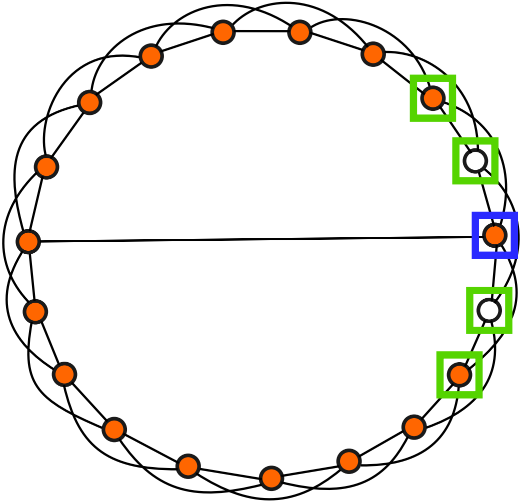
Weak ties & contagions



- Small-world network
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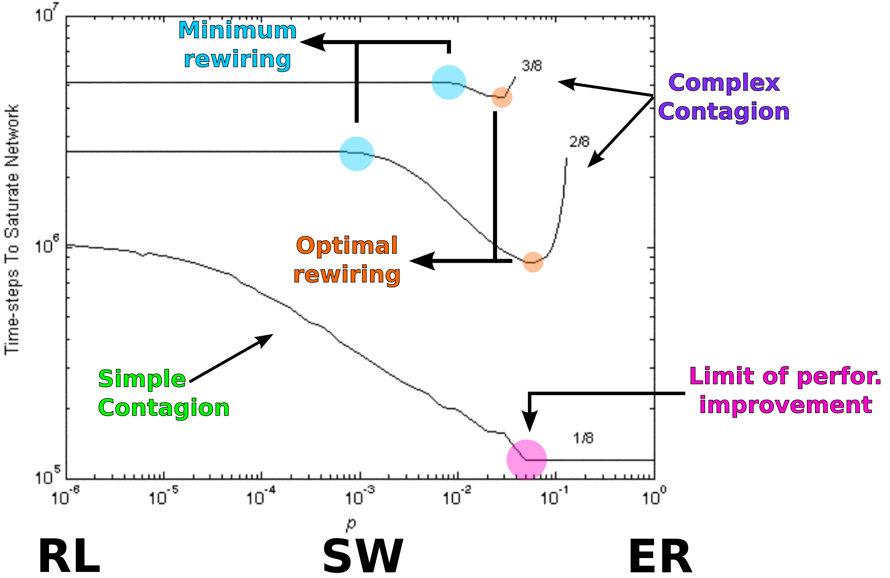
Weak ties & contagions



Our results show that it can be very dangerous to generalize from the spread of information and disease to whatever is to be diffused

Centola, D., & Macy, M. (2007). Amer. Jour. Sociol., **113**, 702–734.

The adoption of Skype

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The adoption of Skype

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Cite this article: Karsai M, Iñiguez G, Kaski K, Kertész J. 2014 Complex contagion process in spreading of online innovation. *J. R. Soc.*

Complex contagion process in spreading of online innovation

Márton Karsai[1,2,3,4], Gerardo Iñiguez[2], Kimmo Kaski[2,5] and János Kertész[2,6,7]

[1]Laboratory for the Modeling of Biological and Socio-technical Systems, Northeastern University, Boston, MA 02115, USA
[2]Department of Biomedical Engineering and Computational Science (BECS), Aalto University School of Science, 00076 Aalto, Finland
[3]Software Technology and Applications Competence Center (STACC), 51003 Tartu, Estonia
[4]Laboratoire de l'Informatique du Parallélisme, INRIA-UMR 5668, IXXI, ENS de Lyon, 69364 Lyon, France
[5]CABDyN Complexity Centre, Säid Business School, University of Oxford, Oxford OX1 1HP, UK
[6]Center for Network Science, Central European University, 1051 Budapest, Hungary
[7]Institute of Physics, Budapest University of Technology and Economics, 1111 Budapest, Hungary

Diffusion of innovation can be interpreted as a social spreading phenom-

The adoption of Skype

The idea

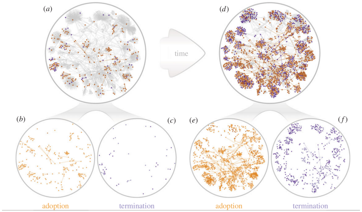
Use VoIP services to "*decode the underlying social structure by acting as proxies for the network of real social ties between individuals, and also provide accurate records of the users' adoption behaviour*".

Karsai, M., Iniguez, G., Kaski, K., & Kertesz, J. (2014). Jour. Roy. Soc. Inter., **11**, 20140694.

The adoption of Skype

Data

- From Sep. 2003 to Mar. 2011
- User Adoption/Termination time
- Users contact lists

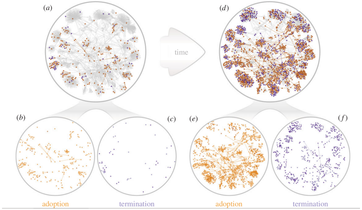


Karsai, M., Iniguez, G., Kaski, K., & Kertesz, J. (2014). Jour. Roy. Soc. Inter., **11**, 20140694.

The adoption of Skype

Adoption/termination processes

- **Spontaneous**
- **Peer Pressured** (Complex contagion)



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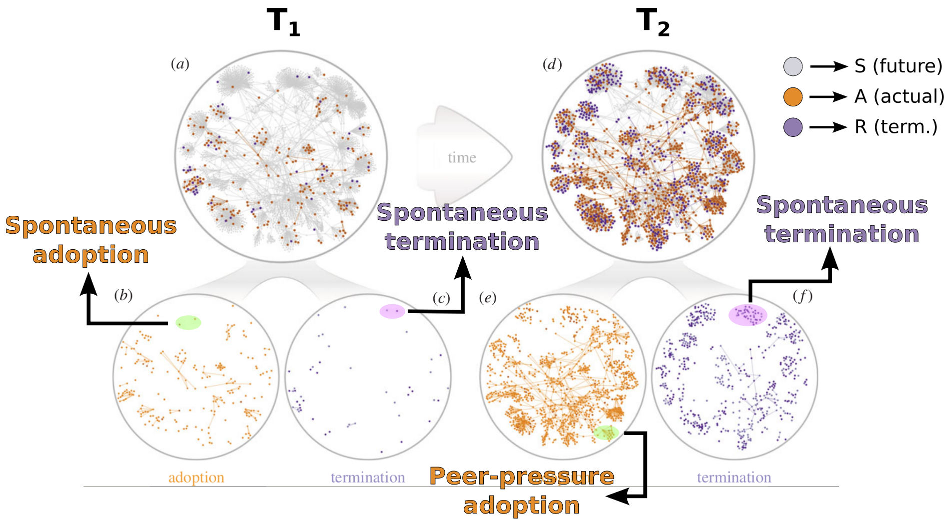
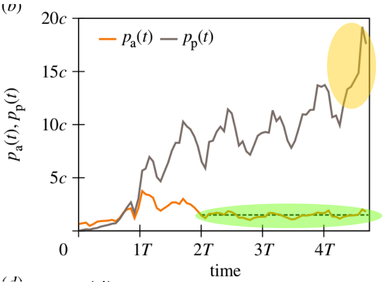
The adoption of Skype

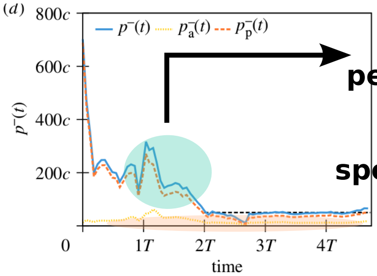


The adoption of Skype



Increasing peer adoption

Constant spont. adoption



Decreasing peer termination

Constant spont. termination

Karsai, M., Iñiguez, G., Kaski, K., & Kertesz, J. (2014). Jour. Roy. Soc. Inter., **11**, 20140694.

The adoption of Skype

Compartmental model

Susceptible (S)

Adopter (A)

Removed (R)

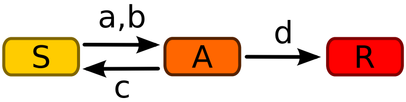
Individual behaviours

(a) → Spontaneous adoption

(b) → Peer-pressure adoption

(c) → Temporary termination

(d) → Permanent termination



Karsai, M., Iniguez, G., Kaski, K., & Kertesz, J. (2014). Jour. Roy. Soc. Inter., **11**, 20140694.

HMF approximation

$$\frac{ds}{dt} = -\Big[\underbrace{p_a}_{a} + \underbrace{p_{pk}(1-p_a)a}_{b}\Big]s + \underbrace{p_s(1-p_r)\,a}_{c},$$

$$\frac{da}{dt} = \Big[\underbrace{p_a}_{a} + \underbrace{p_{pk}(1-p_a)a}_{b}\Big]s - \Big[\underbrace{p_r}_{d} + \underbrace{p_s(1-p_r)}_{c}\Big]a,$$

$$\frac{dr}{dt} = \underbrace{p_r\,a}_{d}.$$

Karsai, M., Iniguez, G., Kaski, K., & Kertesz, J. (2014). Jour. Roy. Soc. Inter., **11**, 20140694.

Rates of adoption/termination

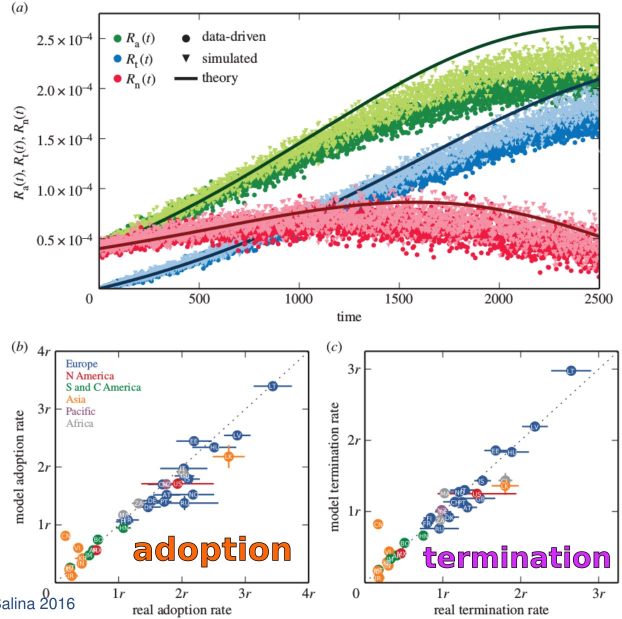
$$R_A(t) = \Big[p_a + p_{pk}(1 - p_a)a\Big]s$$

$$R_T(t) = \Big[p_r + p_s(1 - p_r)\Big]a$$

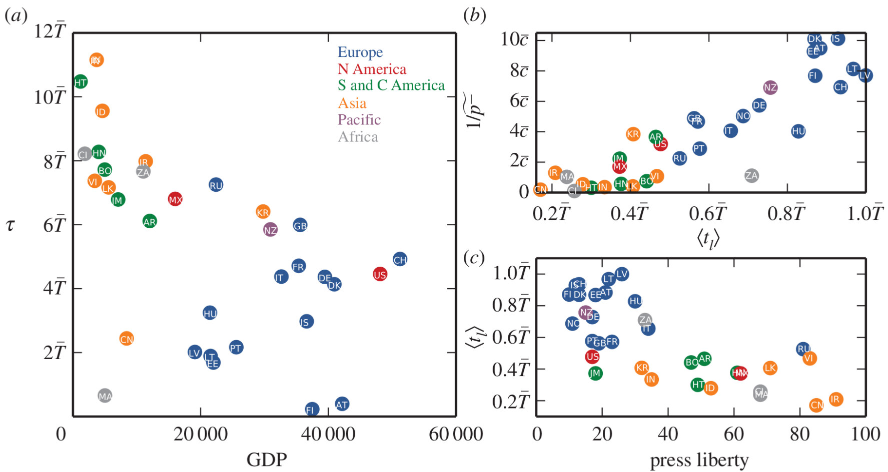
$$R_N(t) = R_A(t) - R_T(t)$$

Karsai, M., Iniguez, G., Kaski, K., & Kertesz, J. (2014). Jour. Roy. Soc. Inter., **11**, 20140694.

The adoption of Skype



The adoption of Skype

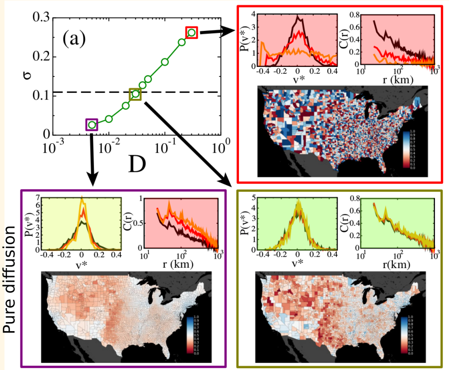


Karsai, M., Iniguez, G., Kaski, K., & Kertesz, J. (2014). Jour. Roy. Soc. Inter., **11**, 20140694.


Finish Line
Ahead

Take home messages

- Competition between two opinions based on the Voter Model.



Take home messages

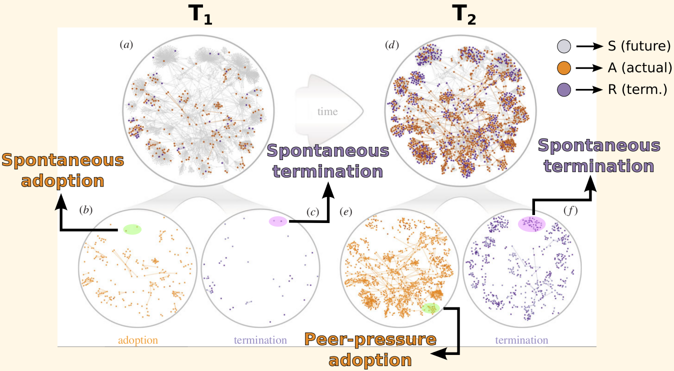
- Emergence of cultural domains as a result of interactions in the Axelrod Model.

Take home messages

- Segregation as the byproduct of relocation associated to different aspiration levels Schelling Model.

Take home messages

- Complex contagion as a framework to study diffusion in Social Systems.



Take home messages

- Competition between two opinions based on the Voter Model.
- Emergence of cultural domains as a result of interactions in the Axelrod Model.
- Segregation as the byproduct of relocation associated to different aspiration levels Schelling Model.
- Complex contagion as a framework to study diffusion in Social Systems.

Contacts

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http://bifi.es/∼cardillo/

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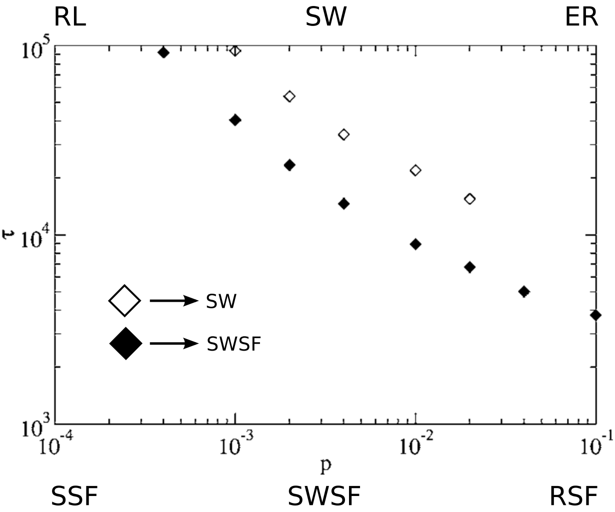
Part III

Appendix

VM on complex networks



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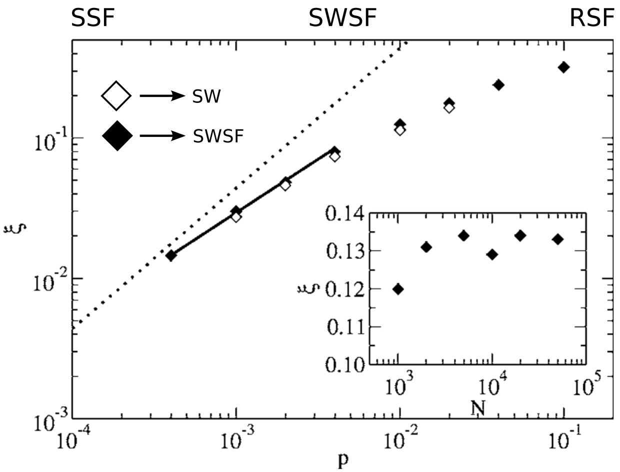
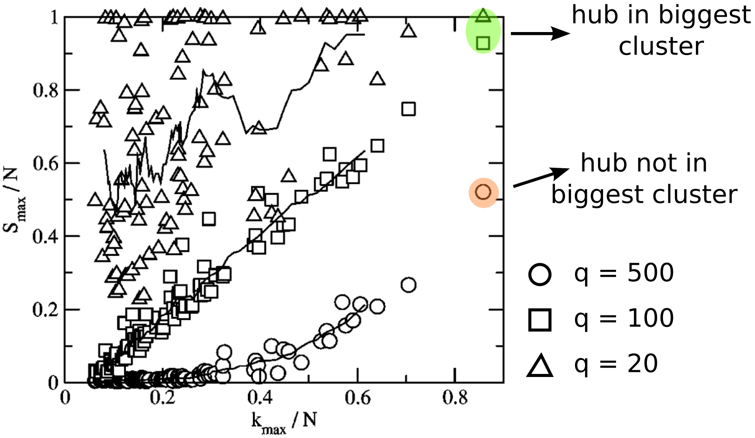
VM on complex networks



Suchecki, K., Eguíluz, V. M., & San Miguel, M. (2005). Phys. Rev. E, **72**, 36132.

AX model on complex networks



Klemm, K., Eguíluz, V. M., Toral, R., & San Miguel, M. (2003). Phys. Rev. E, **67**, 26120.

MK on homogeneous networks

$$\frac{d\,i(t)}{dt} = -\lambda \langle k \rangle\, i(t)\, s(t)\,,$$
$$\frac{d\,s(t)}{dt} = \lambda \langle k \rangle\, i(t)\, s(t) - \alpha \langle k \rangle\, s(t)\,[s(t) + r(t)]\,,$$
$$\frac{d\,r(t)}{dt} = \alpha \langle k \rangle\, s(t)\,[s(t) + r(t)]\,.$$

Solution

$$r_\infty = 1 - e^{-\frac{\lambda}{\alpha} r_\infty}\,.$$

$$\frac{\lambda}{\alpha} > 0 \longrightarrow$$ **NO THRESHOLD**

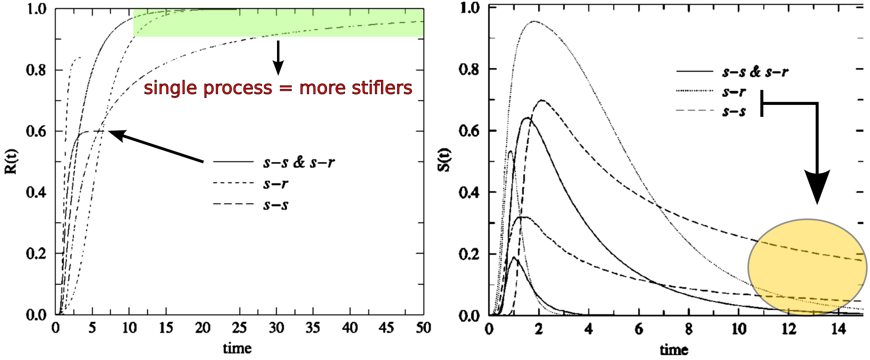
Moreno, Y., Nekovee, M., & Pacheco, A. F. (2004). Phys. Rev. E, **69**, 66130.

Annichilation in MT

- $S + R \xrightarrow{\alpha} 2R$
- $S + S \xrightarrow{\alpha} S + R$
- both

Moreno, Y., Nekovee, M., & Pacheco, A. F. (2004). Phys. Rev. E, **69**, 66130.

Annichilation in MT



Moreno, Y., Nekovee, M., & Pacheco, A. F. (2004). Phys. Rev. E, **69**, 66130.

Skype rates of adoption/termination

$$\begin{cases} p_a(t) &= \dfrac{\#ad(t+\Delta t|SF=0)}{I-N_a(t)} \\ p_p(t) &= \dfrac{\#ad(t+\Delta t|SF\neq 0)}{I-N_a(t)} \end{cases} \Rightarrow \text{adoption}$$

$$\begin{cases} p_a^-(t) &= \dfrac{\#tr(t+\Delta t|SF=0)}{N_a(t)} \\ p_p^-(t) &= \dfrac{\#tr(t+\Delta t|SF\neq 0)}{N_a(t)} \end{cases} \Rightarrow \text{termination}$$

Karsai, M., Iniguez, G., Kaski, K., & Kertesz, J. (2014). Jour. Roy. Soc. Inter., **11**, 20140694.