Redes complejas como paradigma para estudiar sistemas complejos

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The Group

Some numbers

- 8 Senior scientists;
- 5 Post-docs;
- 7 PhD students;
- former members, students, external members, ...

`http://complex.unizar.es`

Research lines

- Quantum technologies
  - Circuit QED
  - Superconducting quantum circuits
  - Quantum metamaterials
  - Molecular qubits
  - Quantum plasmonics
  - Quantum and complex systems

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- Biological Systems
  - All-atom simulations
  - “Coarse-grained” models of DNA and proteins
  - Mesoscopic models of polymers: stretching and translocation
  - Molecular motors
- Dynamics of non-linear systems
  - Friction at the nanoscale
  - Josephson arrays and superconducting circuits
  - Nonlinearity at the micro and nano scale
- Interdiciplinary Physics and Complex Networks
  - Cognitive dynamics
  - Communication networks
  - Epidemiology
  - Evolutionary biology
  - Network theory
  - Online social systems
  - Socio-economic systems
  - Synchronization phenomena
  - Systems biology

Question:

What is a complex system?

"Answer":

- Complex systems contain many constituents interacting nonlinearly;

Introduction

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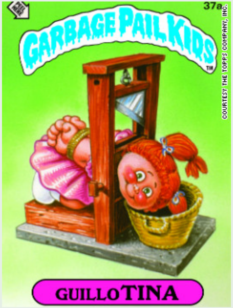
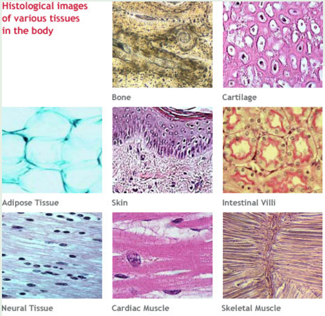
- Complex systems contain many constituents interacting nonlinearly;
- The constituents of a complex system are interdependent;

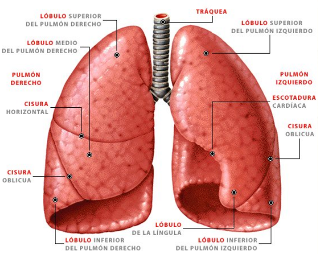


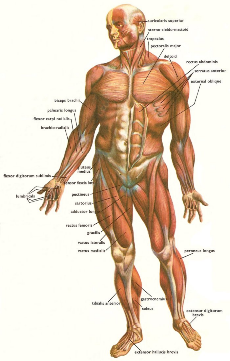
Introduction

"Answer":

- Complex systems contain many constituents interacting nonlinearly;
- The constituents of a complex system are interdependent;
- A complex system possesses a structure spanning several scales;





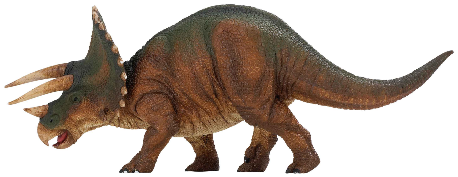


Introduction

“Answer”:

- Complex systems contain many constituents interacting nonlinearly;
- The constituents of a complex system are interdependent;
- A complex system possesses a structure spanning several scales;
- A complex system is capable of emerging behavior.

Studying a complex system by means of complex networks is like paleontology.



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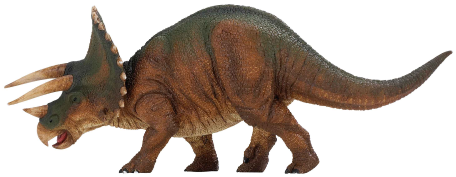
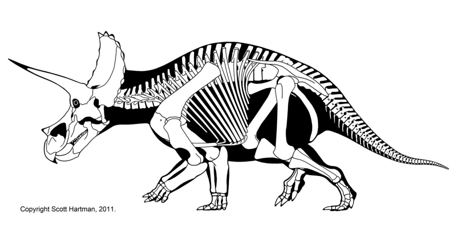
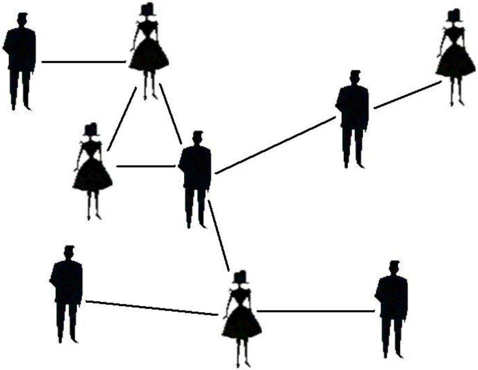




Complex Networks

Definition:

A complex network is a mathematical object composed by two sets:



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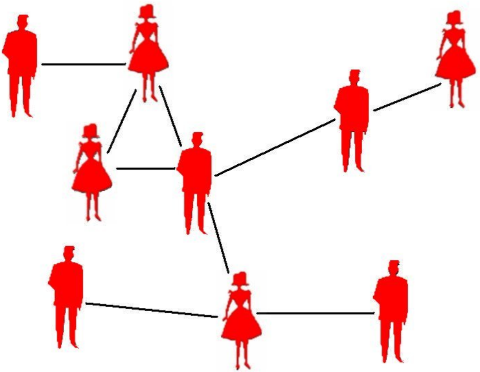

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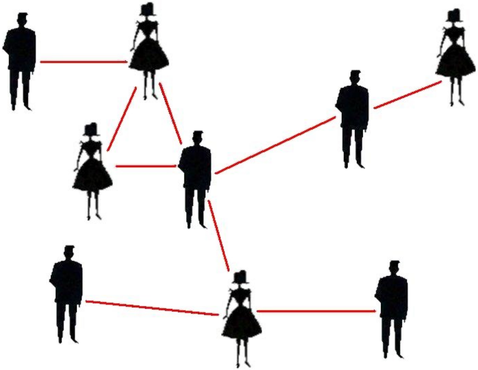
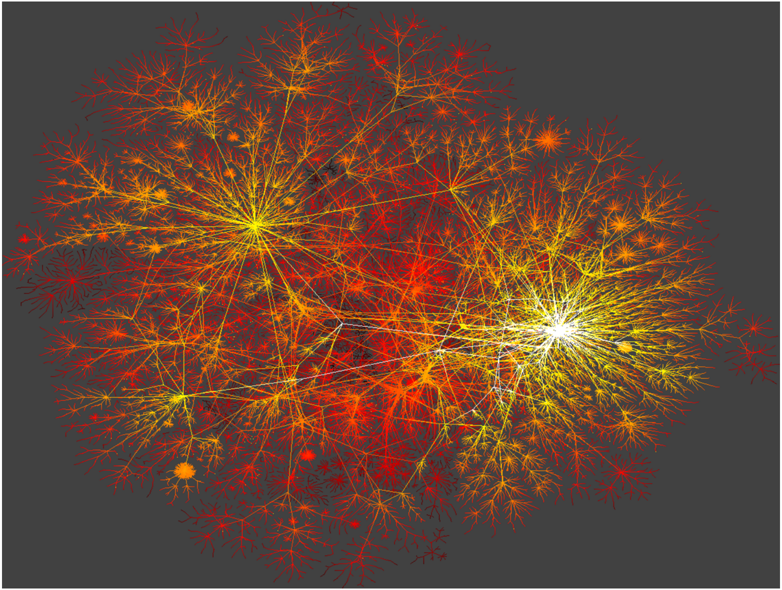


A world made of complex networks



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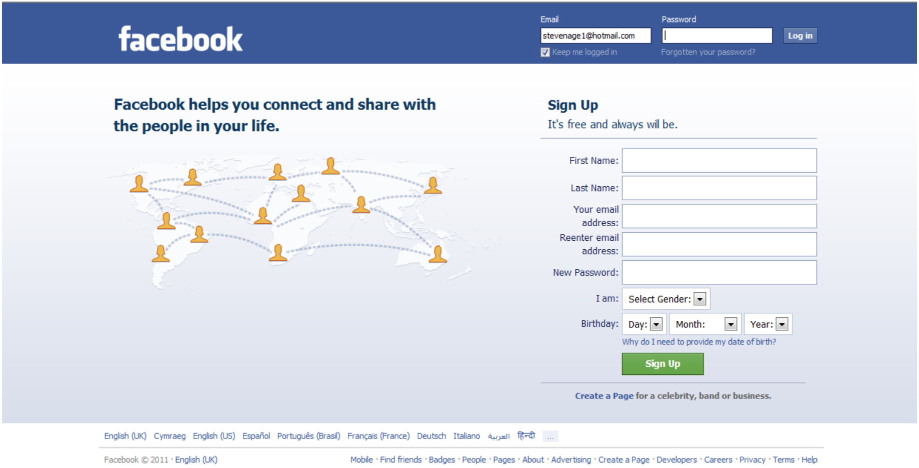

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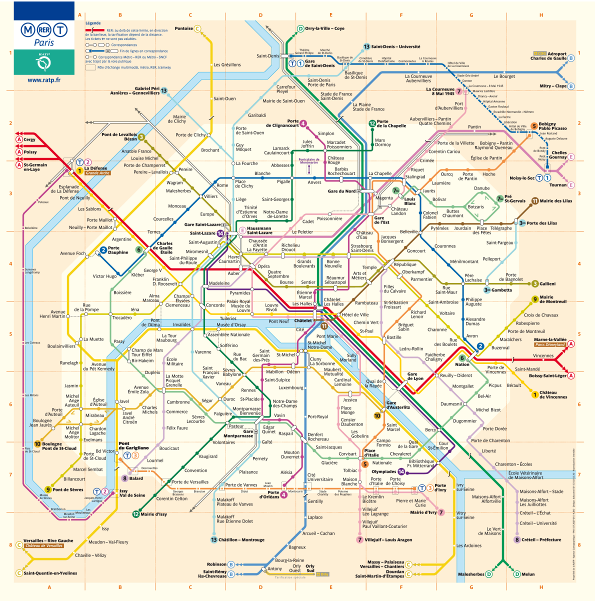
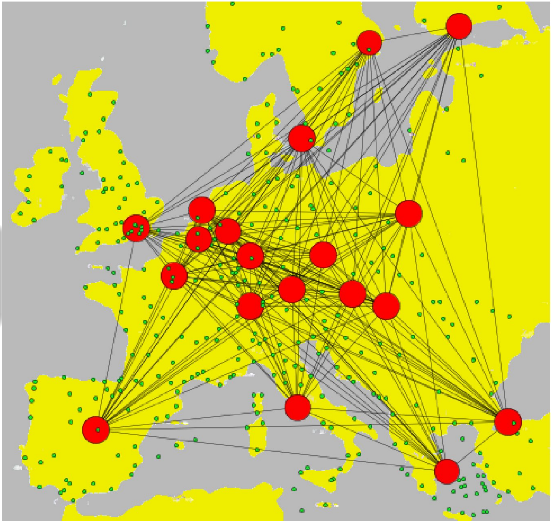


Structure and dynamics on complex networks



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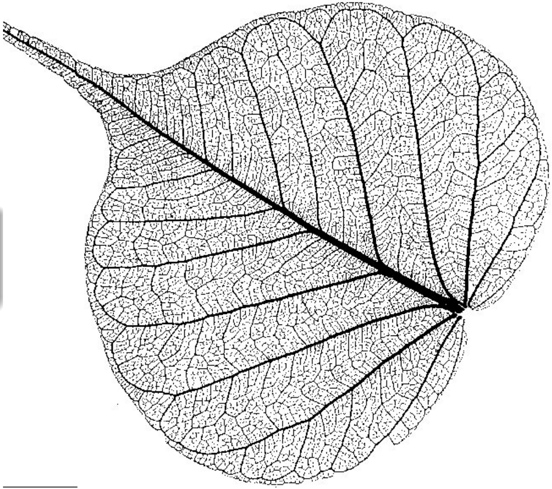

Structure and dynamics on complex networks

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It means studing phenomena acting on a network which varies in time.

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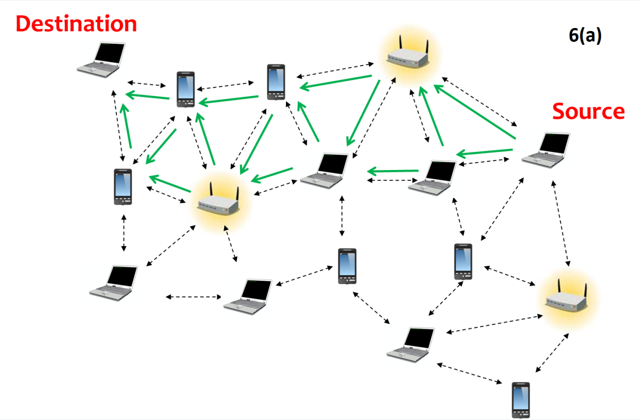
Many different dynamics one single tool

Which dynamics can be model onto a network?



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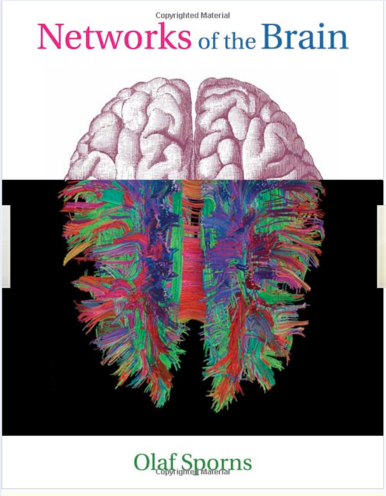


- Traffic/routing;
- Evolution of Cooperation;
- Rumors/Diseases spreading;
- Synchronization;
- Brain;

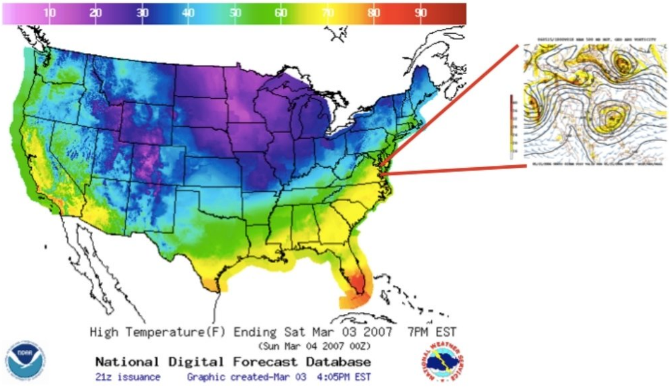
A concrete example: Spreading of Infections

Do we need a Motivation?

A concrete example: Spreading of Infections

We need some kind of Forecast!

Example: weather forecast

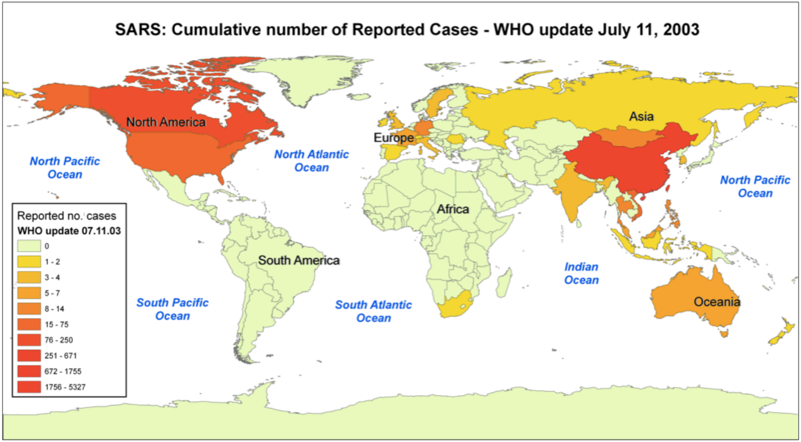


Numerical weather prediction uses mathematical models of the atmosphere to predict the weather. Manipulating the huge datasets with the most powerful supercomputers in the world.

A concrete example: Spreading of Infections

We need some kind of Forecast!

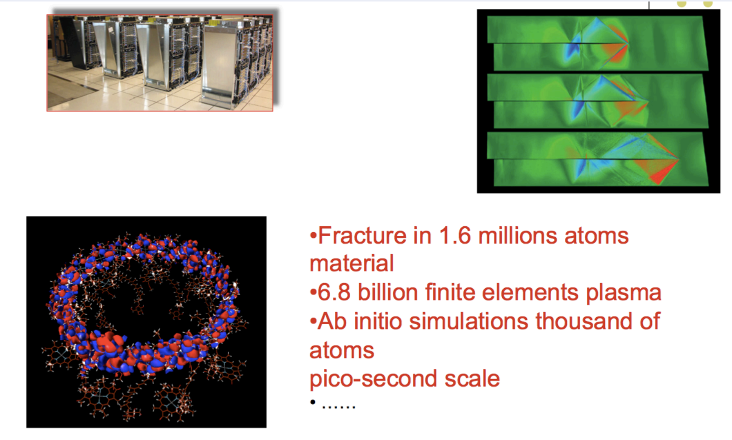
Epidemic Forecast?



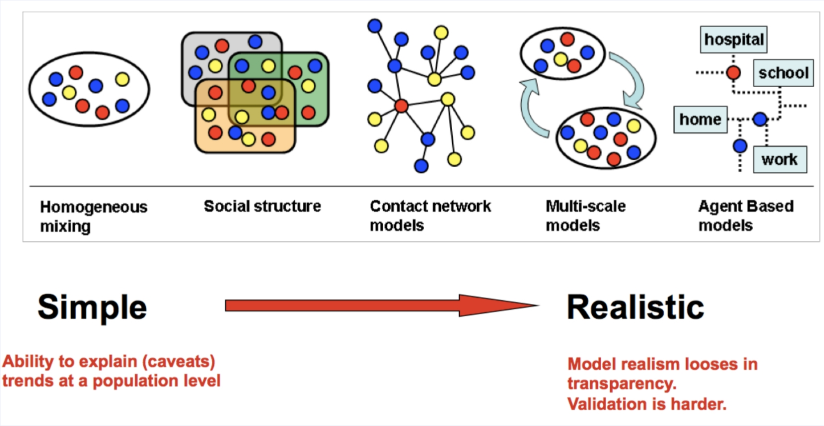
A concrete example: Spreading of Infections

We have enough computational power. . .

Other problems (other than weather forecasting) where computation is massively used:

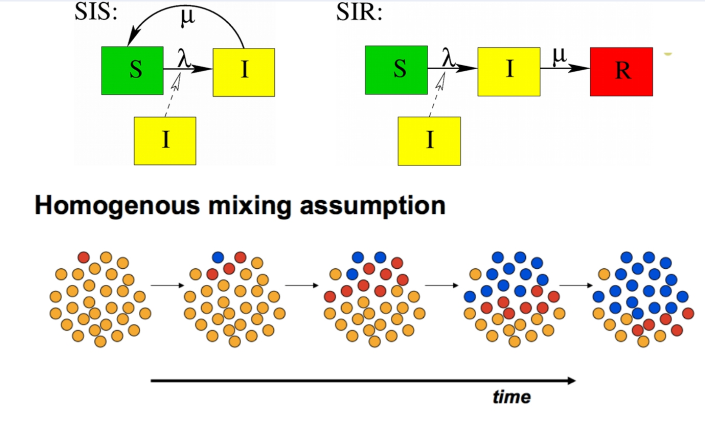


Approaches to Epidemic Modeling: From Simple to Complex



Homogeneous mixing

Compartmental models: Two classical examples



Homogeneous mixing

- Susceptible-Infected-Susceptible (SIS):

$$\begin{aligned} t : S + I &\xrightarrow{\lambda} (t+1) : I + I \\ t : I &\xrightarrow{\mu} (t+1) : S \end{aligned}$$

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- Well-mixed formulation: $s(t) = S(t)/N$ and $i(t) = I(t)/N$

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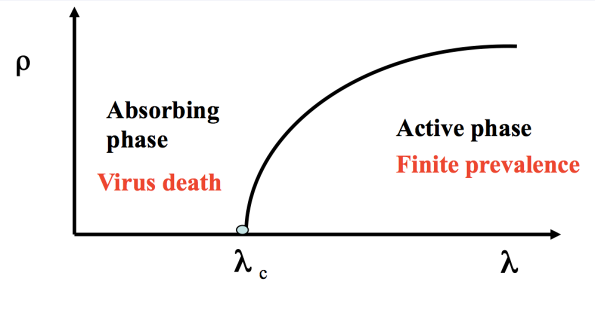

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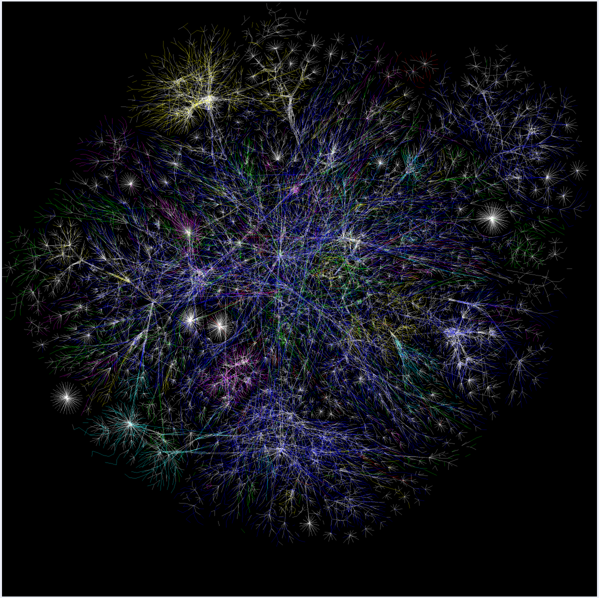
WHY?

Homogeneous mixing



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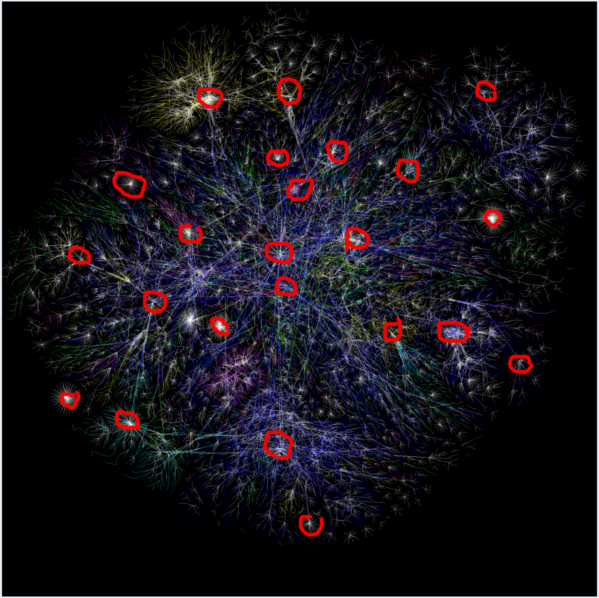

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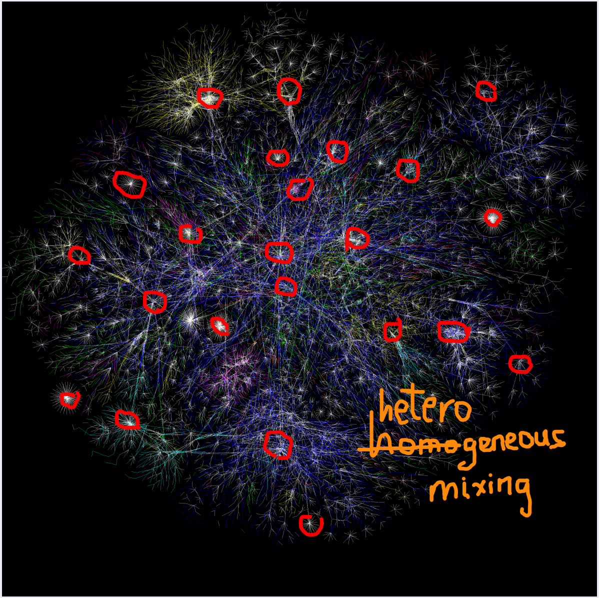
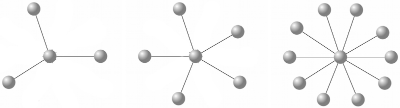


Heterogeneous mixing reveals the **Scale-free property**



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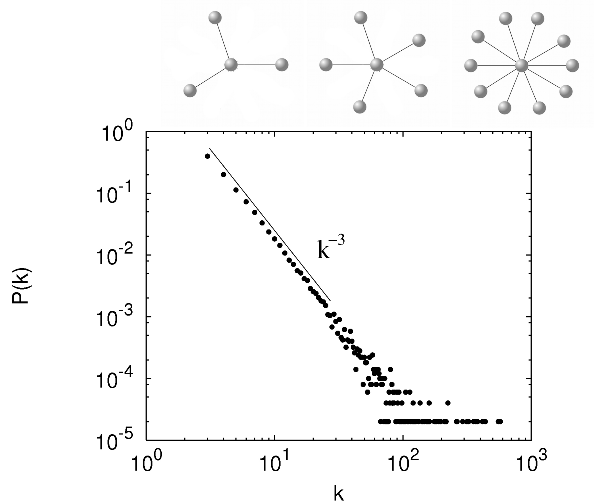

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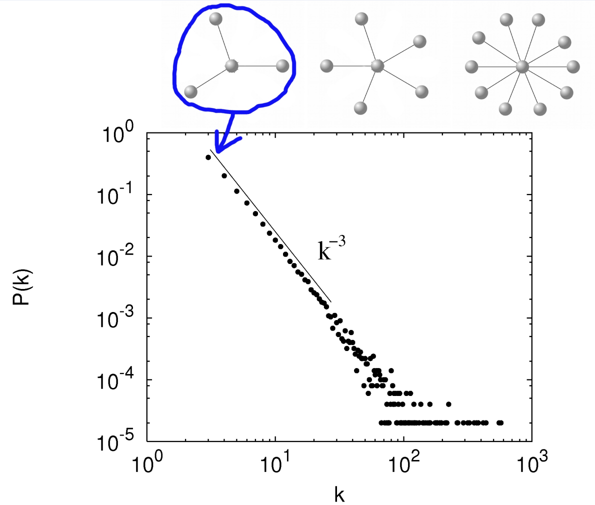

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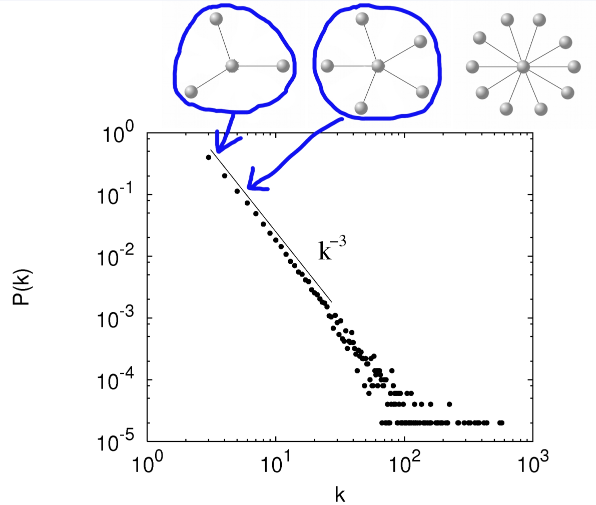

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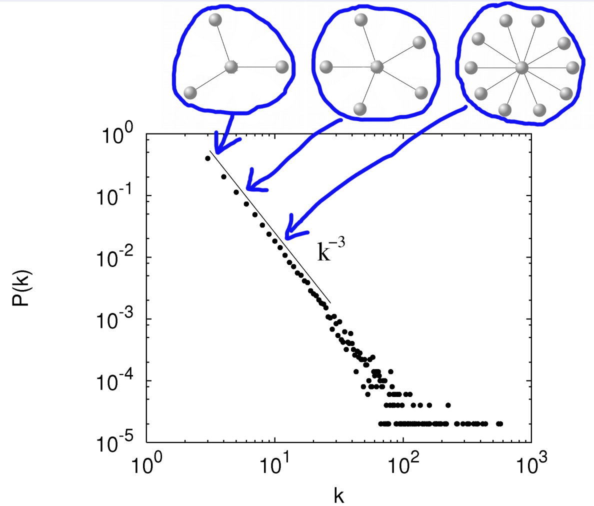

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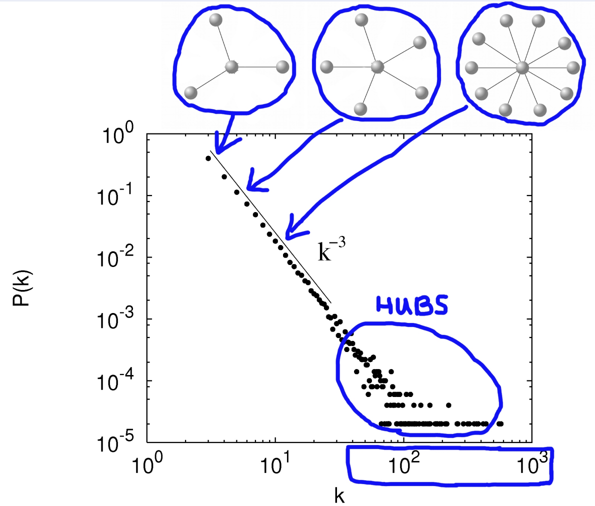

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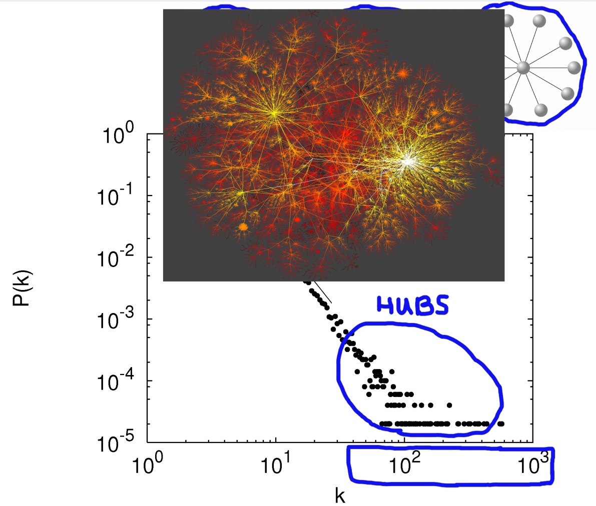

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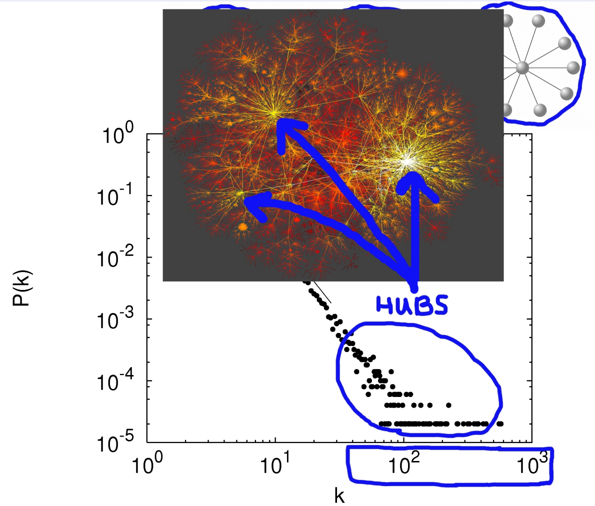

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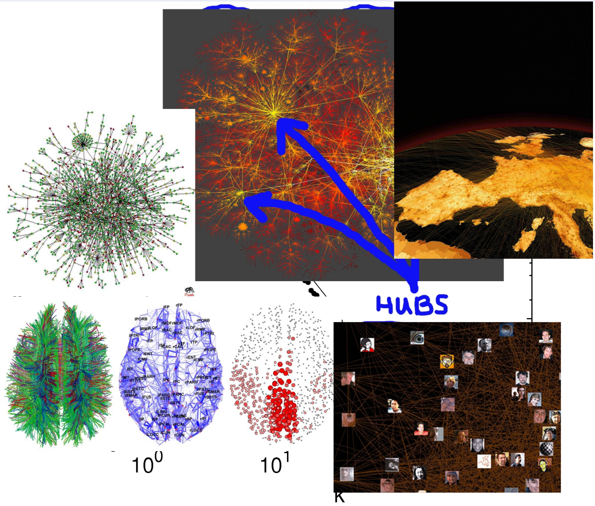

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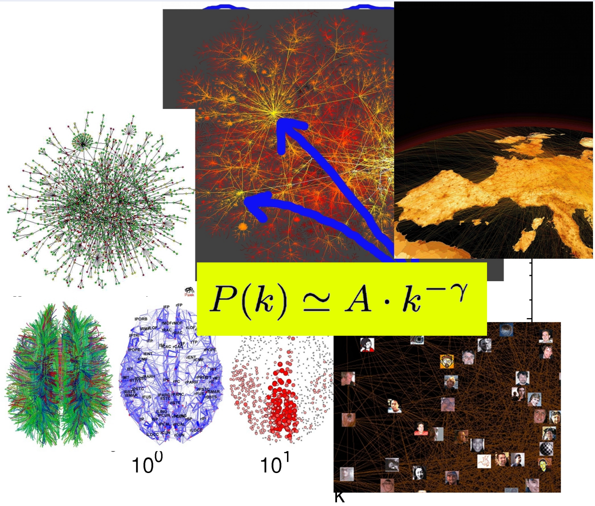
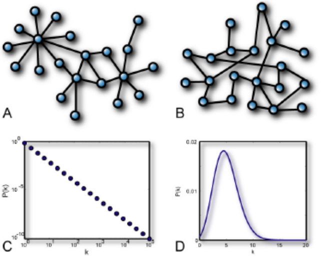


Heterogeneous mixing: The Network approach

- General situation the probability that an individual is connected with k neighbors is given by $P(k) = N_k/N$.



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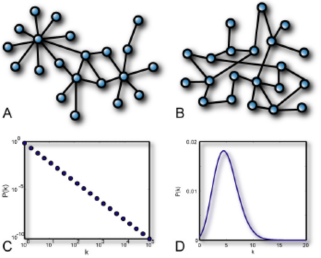
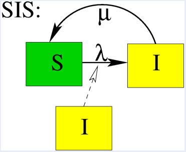


- Can we build a formalism for contact-based epidemic without any homogeneous assumption?
- Can we do this from microscopic principles?

Let us explore the SIS case...

Network approach: General Formalism



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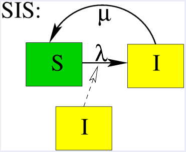

- Consider the probability $p_i(t)$ that node i is infected at time t,

$$\begin{aligned} p_i(t+1) &= (1-p_i(t))(1-q_i(t)) + p_i(t)(1-\mu) \\ &\quad + \mu p_i(t)(1-q_i(t)) \end{aligned}$$

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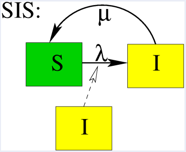

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whith $q_i(t)$: probability of node i not being infected by any neighbor

$$q_i(t) = \prod_{j=1}^{N} (1 - \lambda r_{ji} p_j(t))$$

where r_{ij} measures the contact probability between j and i.

Network approach: General Formalism

- If $\langle k \rangle$ and $\langle k^2 \rangle$ are the first and second moments of $P(k)$, the epidemic threshold is given by:

$$\lambda_c = \frac{\mu \langle k \rangle}{\langle k^2 \rangle} \neq \frac{\mu}{\langle k \rangle} \leftarrow \text{Homog. mixing}$$

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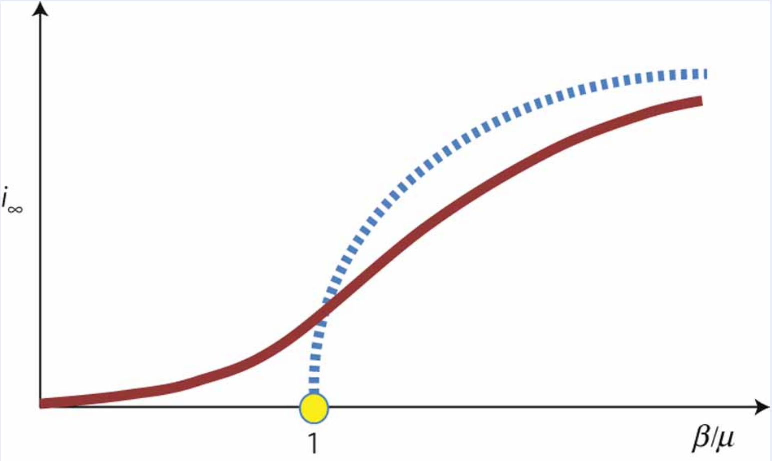
In the internet $P(k) \sim k^{-2.2}$ **then** $\langle k^2 \rangle = \sum_k k^2 P(k) \to \infty$ **and**

$$\lambda_c = \frac{\mu \langle k \rangle}{\langle k^2 \rangle} \to 0!!!!$$

Absence of epidemic threshold



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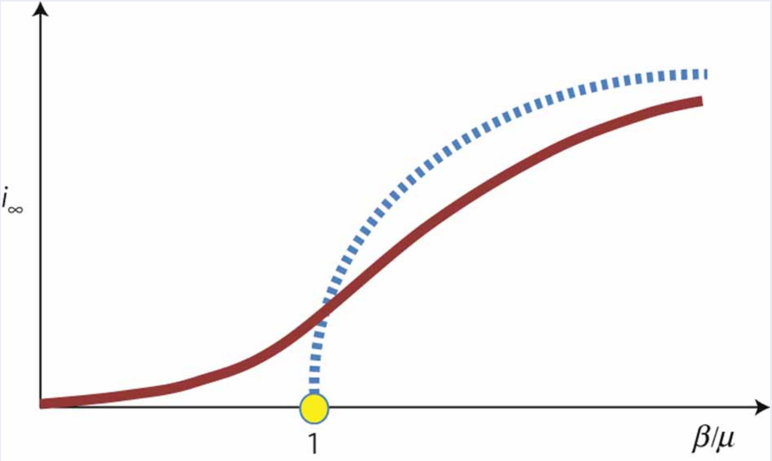
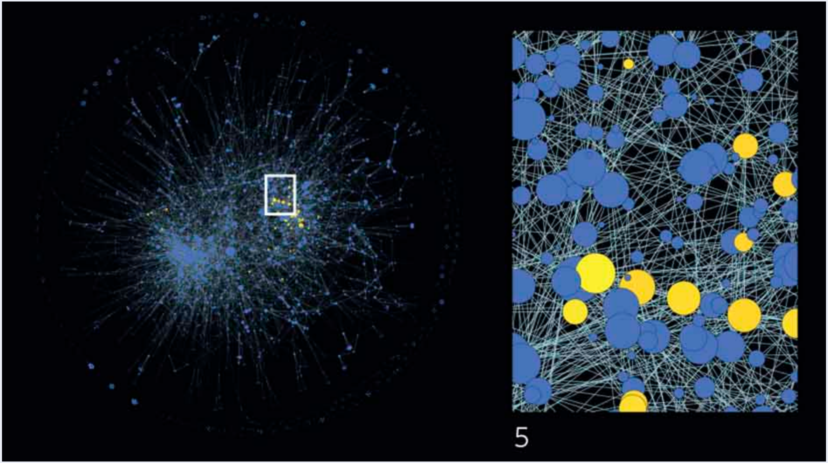


THERE IS NO EPIDEMIC THRESHOLD
WHEN $P(k) \sim k^{-\gamma}$ with $\gamma < 3$:
ANY SMALL AMOUNT OF VIRUS CREATES AN EPIDEMY

Simulation on a scale-free network:



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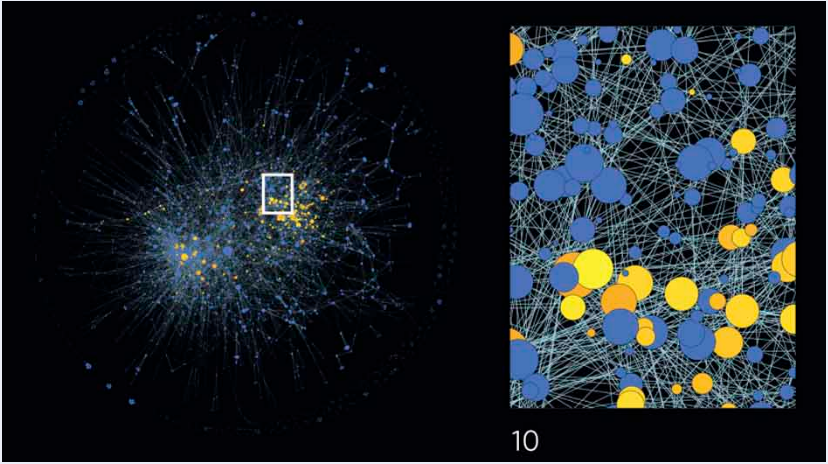

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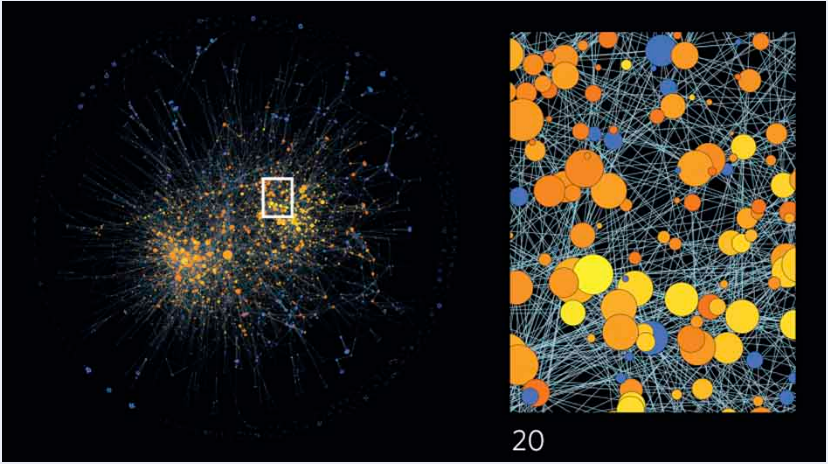
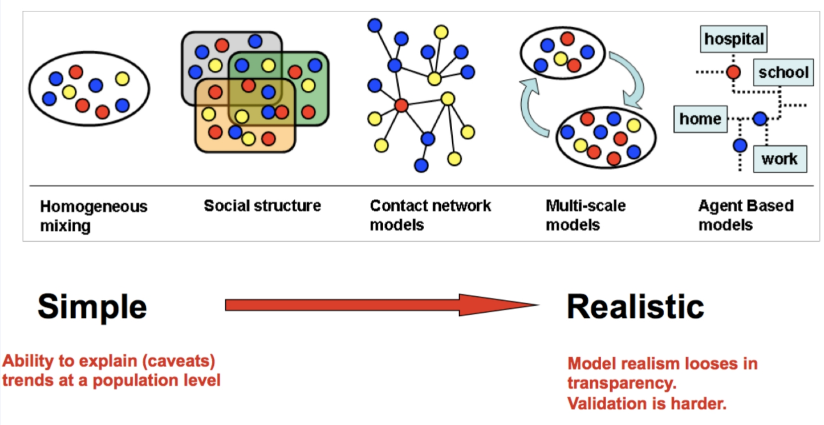


Next step?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school
home
work
Homogeneous mixing
Social structure
Contact network models
Multi-scale models
Agent Based models
Simple
Realistic
Ability to explain (caveats) trends at a population level
Model realism looses in transparency.
Validation is harder.

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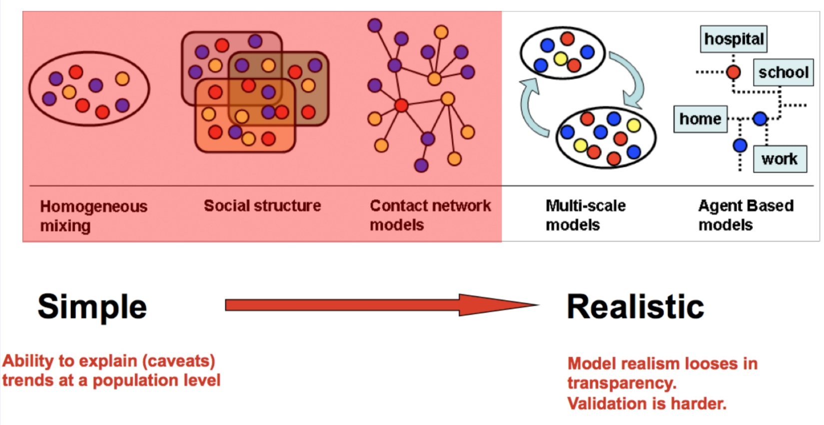
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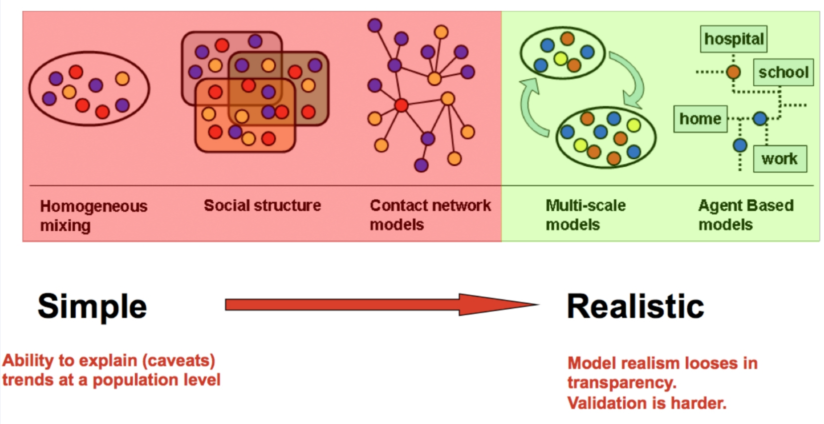


Next Step: Metapopulation modeling

Metapopulation models

Metapopulation models combine different scales whose structural patterns are of importance for the spread of an epidemy...

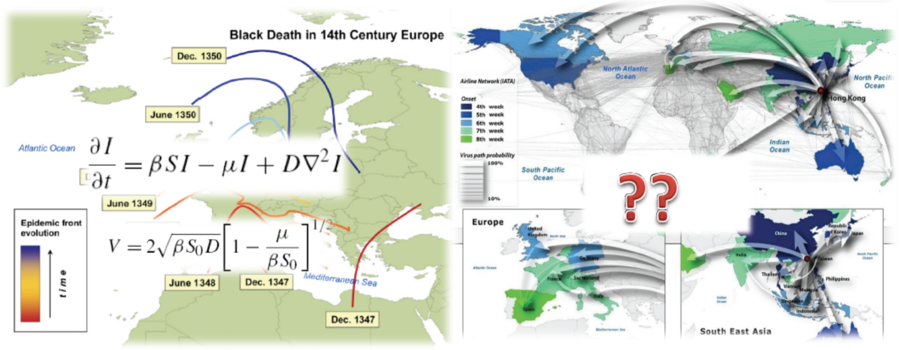
Metapopulation models

Metapopulation models combine different scales whose structural patterns are of importance for the spread of an epidemy...

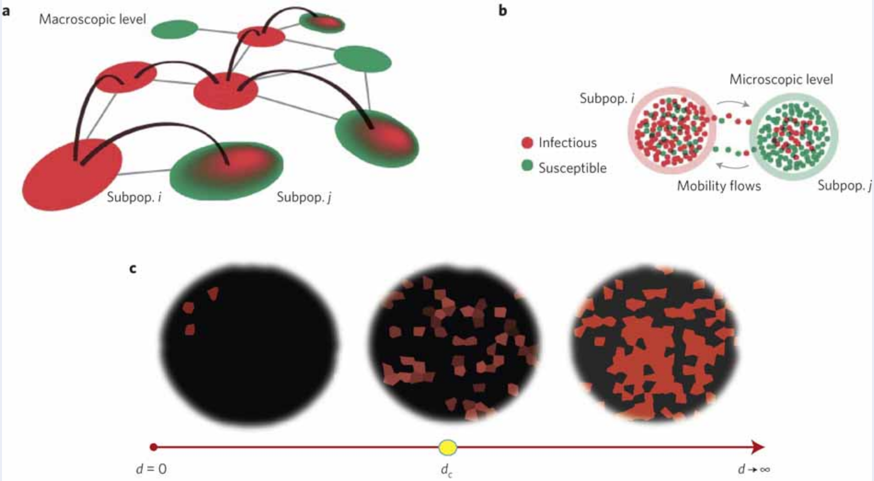
... and transport patterns are now one of the most important features:

Black death in1347: a continuous diffusion process

SARS epidemics: a discrete network driven process



Metapopulation models



Metapopulation models

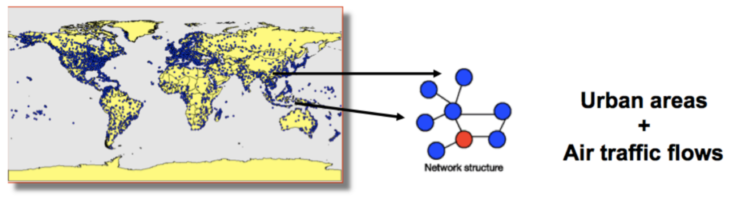
We need data!

- Transportation infrastructures
- Behavioral Networks
- Census data
- Commuting/traveling patterns

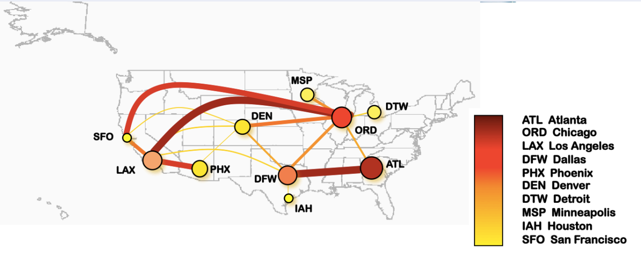
  - Different scales:
    - International
    - Intra-nation (county/city/municipality)
    - Intra-city (workplace/daily commuters/individuals behavior)

Metapopulation models

Two-mode data:

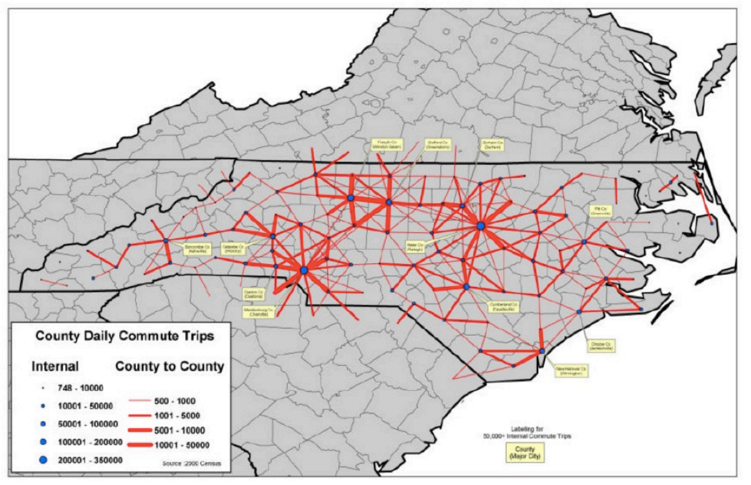


- Air-transportation network:



Metapopulation models

- At a lower level we find the local transportation system:

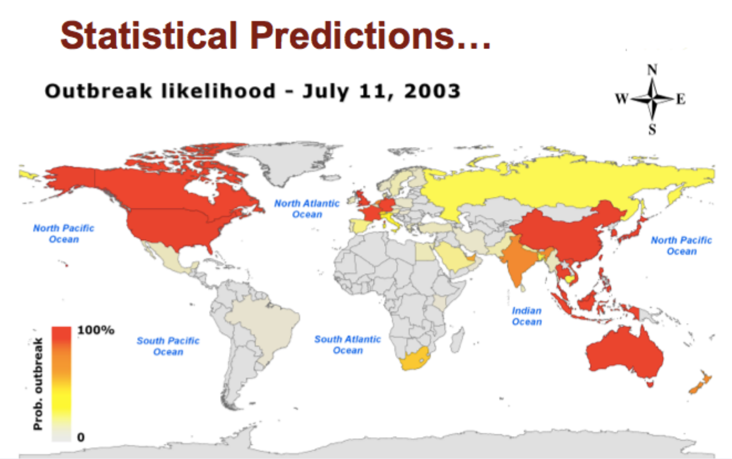


Global spread of infective individuals

- Probability that any individual in the class X travel from $j \to l$
  - Proportional to the traffic flow
  - Inversely proportional to the population

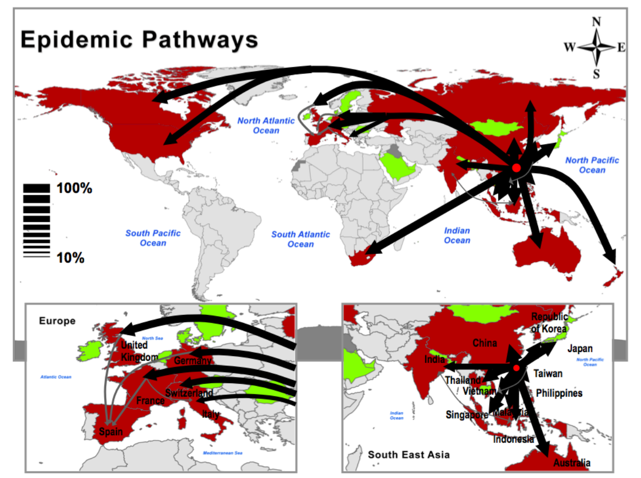
$$\boxed{p_{jl} = \frac{w_{jl}}{N_j} \Delta t}$$

SARS case - Statistical Validation



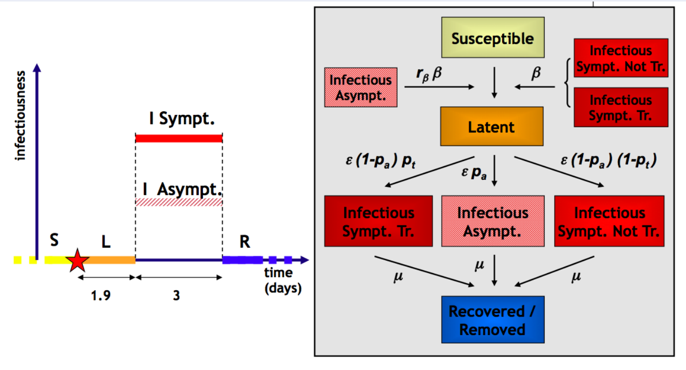
SARS case - Statistical Validation

Reconstruction of epidemic pathways



The case of Avian influenza (H5N1)

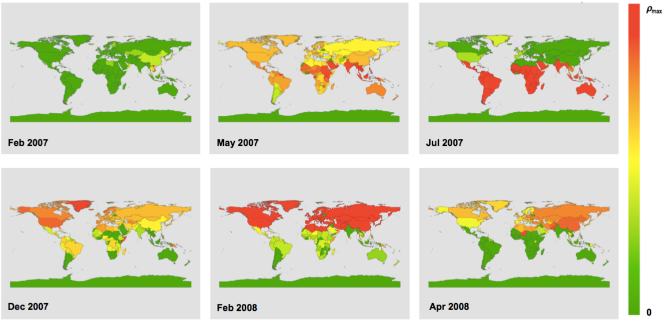
A more refined compartmental model:



Parameters are taken by looking the basic reproductive number of the virus R_0

The case of Avian influenza (H5N1)

Pandemic forecast...



Pandemic with R_0=1.6 starting from Hanoi (Vietnam) in October 2006
Baseline scenario

Future perspectives

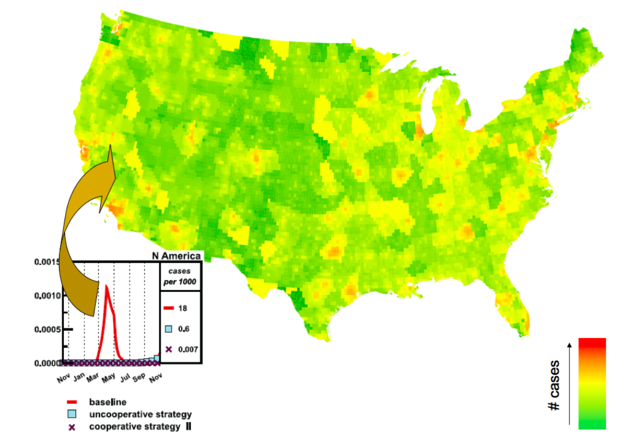
Containment strategies . . .

- Travel restrictions
  - Partial
  - Full (Entire country quarantine)
- What we learn. . .
  - Complex global world calls for a non-local perspective
  - Preparedness is not just a local issue
  - Real sharing of resources discussed by policy makers

. . . Still we lack refined data about world-wide census!

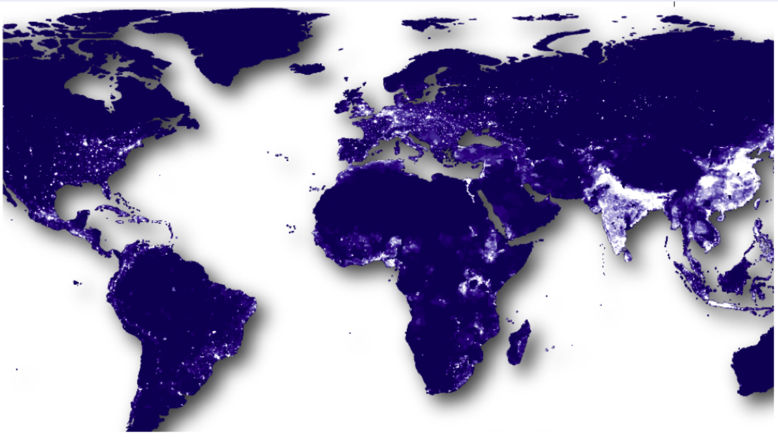
Future perspectives

From reliable regional forecast. . .



Future perspectives

. . . to a world-wide one



Social behavior coupled with epidemic forecasting

- Difference with weather forecast: Information affects epidemics itself!!

Future perspectives

Two social features against prevention:

- Improvements of treatments decrease the risk perception (e.g. AIDS).
- Vaccination have a cost: pain, side effects, ecc (e.g. Measles).

Vaccination is a Prisoner's Dilemma game:

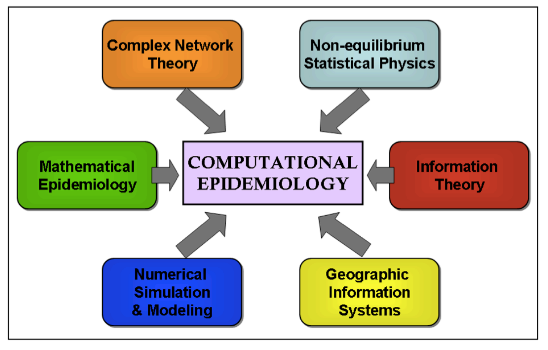
- Global welfare: If all of us take the vaccine then epidemy will die out.
- Own profit: If all my neighbors take the vaccine I am also immune without costs!

Two dynamical processes entangled:

- Evolutionary Game theory (for local decissions)
- Evolution of an epidemic (risk perception)

Future perspectives

Modern Epidemiology represents the success of interdisciplinarity as it involves a broad range of research disciplines:



http://complex.unizar.es/