



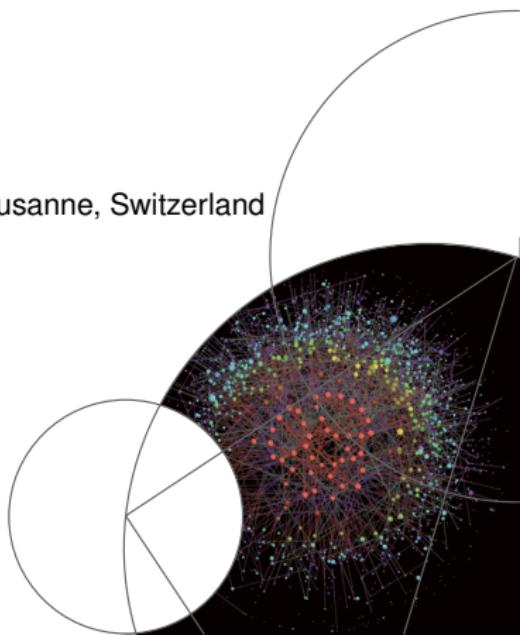
Enhanced extraction of weighted networks backbones

Alessio Cardillo

Laboratory for Statistical Biophysics (LBS)

École Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland

<http://bifi.es/~cardillo/>

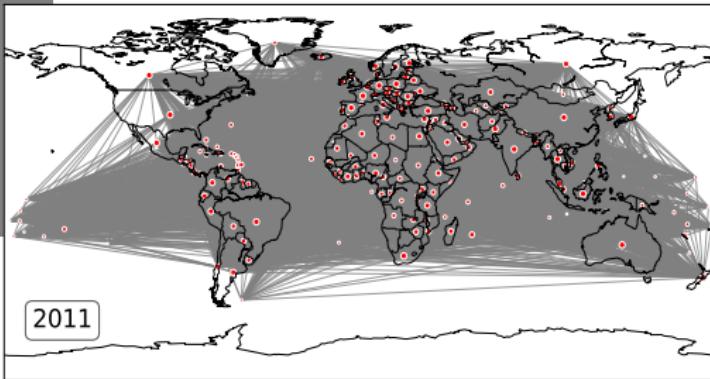
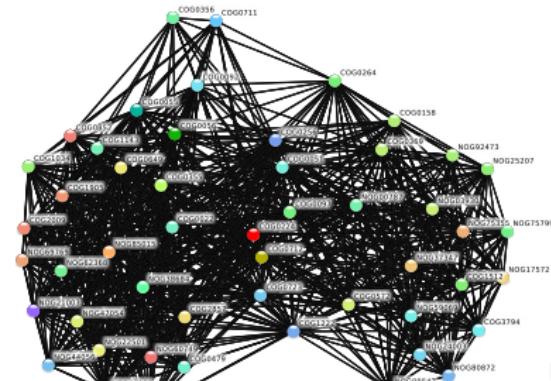
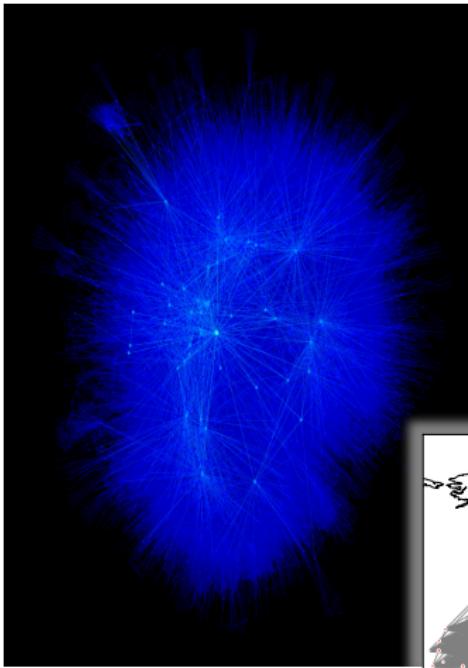




Motivation



Motivation



Motivation

Question:

What can we **learn** from a complex system
whose **network representation** is **noisy** and/or
extremely **dense**?

IAEWm



KEEP CALM

AND

DO

FILTERING

Outline

- Motivation.
- ★ Taxonomy of filtering.
- ★ The ECM-filter
- ★ Results
 - Take home messages
 - Questions

IAEWm

Section 1

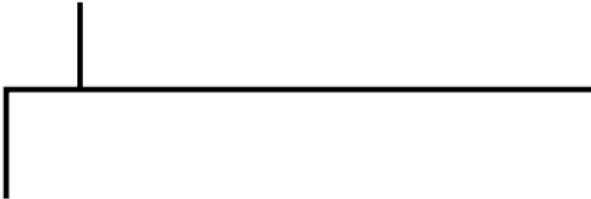
Taxonomy of filtering

KUKUAEWm

Network Filtering

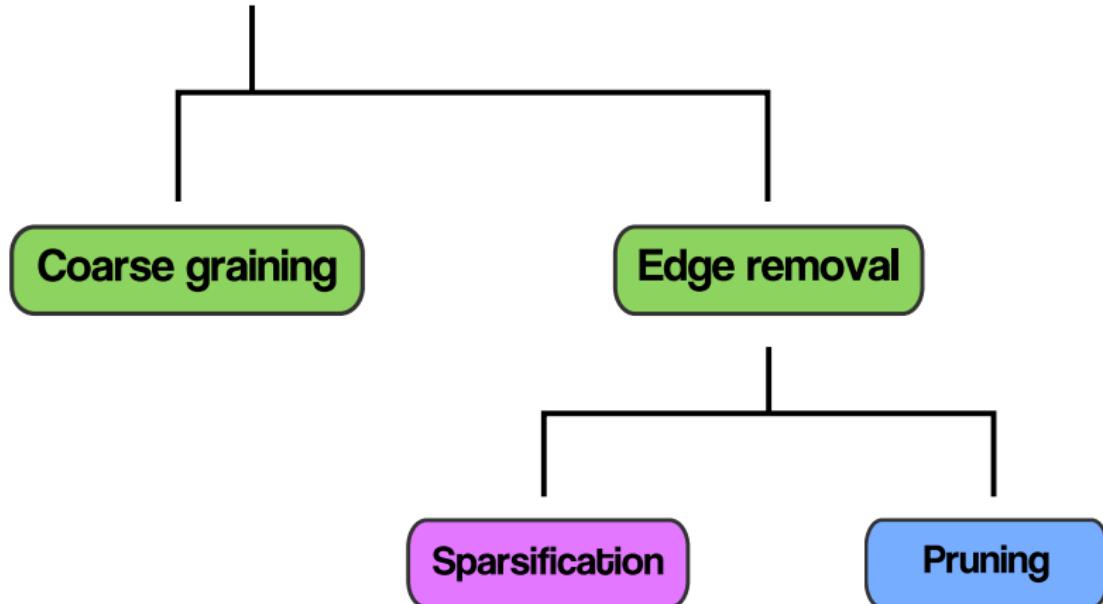
KUKUAEWm

Network Filtering



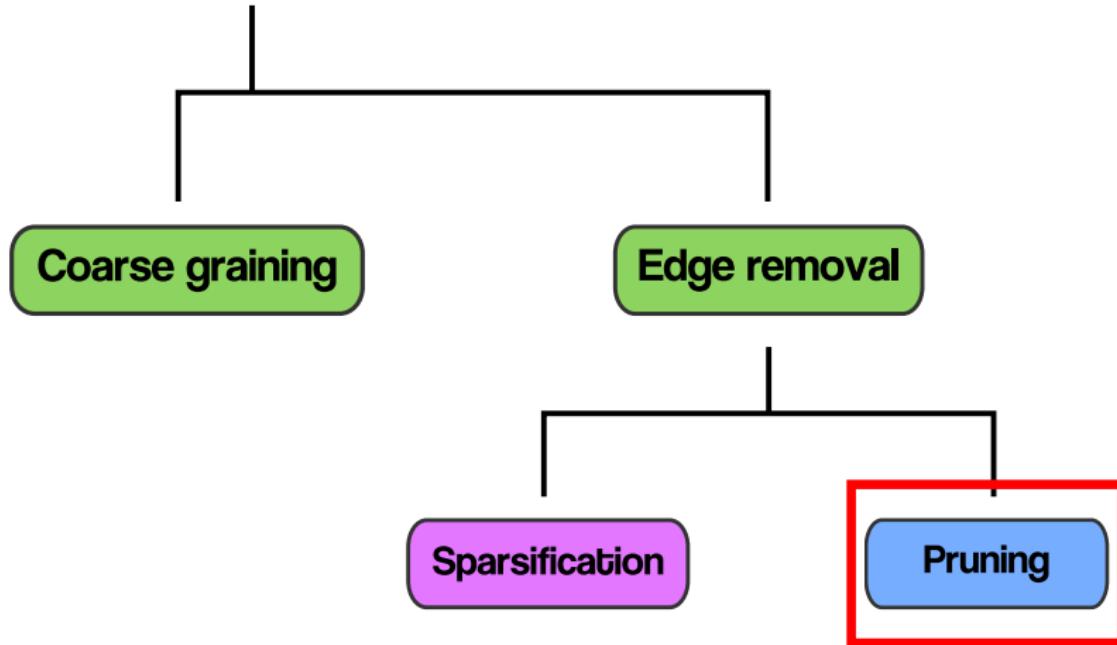
KUKUAEWm

Network Filtering



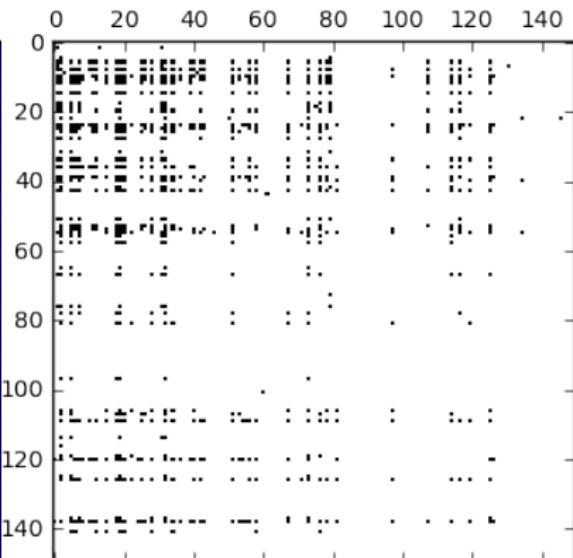
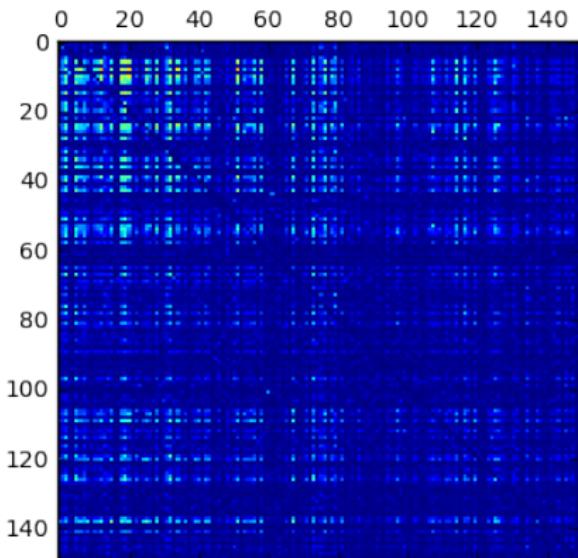
KUKUAEWm

Network Filtering



KUKUAEWm

Thresholding

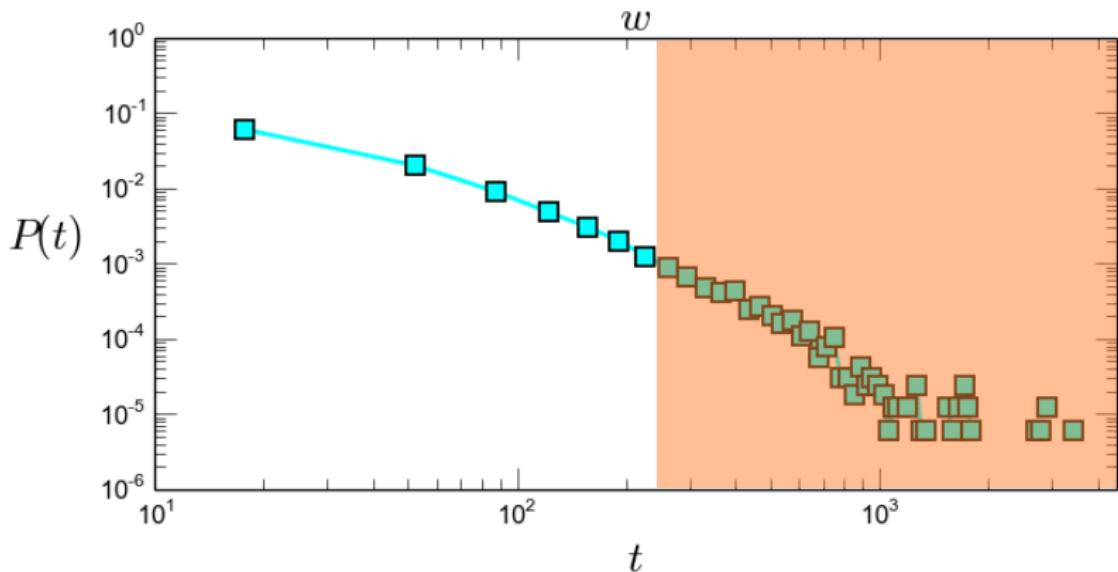


Thresholding

The screenshot shows a research article from PLOS Computational Biology. The title of the article is "A Topological Criterion for Filtering Information in Complex Brain Networks". The authors listed are Fabrizio De Vico Fallani, Vito Latora, and Mario Chavez. The article is identified as a "RESEARCH ARTICLE" and is marked as "OPEN ACCESS" and "PEER-REVIEWED". The version shown is "Version 2", published on January 11, 2017. The DOI provided is <http://dx.doi.org/10.1371/journal.pcbi.1005305>. Below the main title, there is a navigation bar with five tabs: Article (highlighted in green), Authors, Metrics, Comments, and Related Content.

- De Vico Fallani F., et al. *A Topological Criterion for Filtering Information in Complex Brain Networks*. PLoS Comp. Bio. **13** e1005305 (2017).

Thresholding



- Granovetter, M. S. *The Strength of Weak Ties*. Am. Jour. Soc., **78**, 1360 (1973).

Disparity Filter

Institution: EPFL
 Proceedings of the National Academy of Sciences of the United States of America

CURRENT ISSUE // ARCHIVE // NEWS & MULTIMEDIA // FOR AUTHORS // ABOUT PNAS // COLLECTED ARTICLES // BROWSE BY TOPIC // EARLY EDITION

[Home](#) > Current Issue > vol. 106 no. 16 > M. Ángeles Serrano, 6483–6488, doi: 10.1073/pnas.0808904106

 CrossMark click for updates

Extracting the multiscale backbone of complex weighted networks

M. Ángeles Serrano^{a,1}, Marián Boguña^b and Alessandro Vespignani^{c,d}

Author Affiliations *

Edited by Peter J. Bickel, University of California, Berkeley, CA, and approved March 2, 2009 (received for review September 9, 2008)

[Abstract](#) [Full Text](#) [Authors & Info](#) [Figures](#) [SI](#) [Metrics](#) [Related Content](#) [+SI](#)

Abstract

This Issue

April 21, 2009
 vol. 106 no. 16
 Masthead (PDF)
 Table of Contents

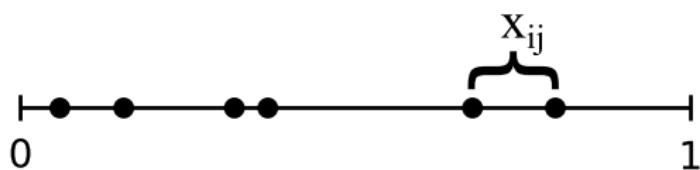
[PREV ARTICLE](#) [NEXT ARTICLE](#)



Don't Miss

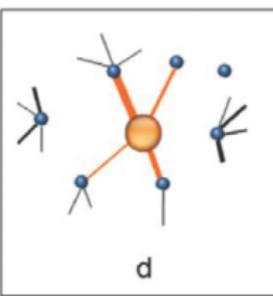
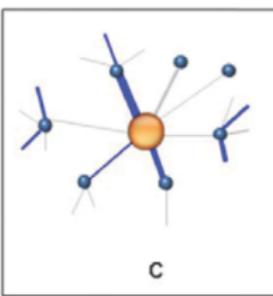
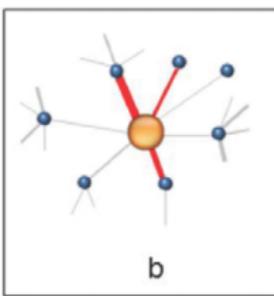
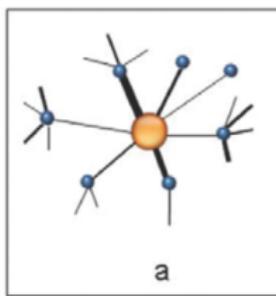
- Serrano M.A., et al. *Extracting the multiscale backbone of complex weighted networks*. Proc. Natl. Acad. Sci. (USA) **106** 6483 (2009).

Disparity Filter



disparity

$$x_{ij} = \frac{w_{ij}}{\sum_j w_{ij}} \text{ or } \frac{w_{ij}}{\sum_i w_{ij}}$$



- Serrano M.A., et al. *Extracting the multiscale backbone of complex weighted networks*. Proc. Natl. Acad. Sci. (USA) **106** 6483 (2009).

Disparity Filter

Pros:

- Easy to implement
- Computationally fast

Cons:

- Not Maxent
- No unique p -value
- Bias towards heavier connections

• Serrano M.A., et al. *Extracting the multiscale backbone of complex weighted networks*. Proc. Natl. Acad. Sci. (USA) **106** 6483 (2009).

GloSS Filter

The screenshot shows a web page for a scientific publication. At the top, it says "PHYSICAL REVIEW E" and "statistical, nonlinear, and soft matter physics". Below that is a navigation bar with links for "Highlights", "Recent", "Accepted", "Authors", "Referees", "Search", "About", and a feed icon. The main title of the article is "Information filtering in complex weighted networks" by Filippo Radicchi, José J. Ramasco, and Santo Fortunato. It was published in Phys. Rev. E 83, 046101 on April 1, 2011. Below the title are buttons for "Article", "References", "Citing Articles (8)", "PDF", "HTML", and "Export Citation". The abstract section starts with "ABSTRACT" and describes how networks can be used to represent systems in nature, society, and technology, mentioning weighted edges and the redundancy of network information.

PHYSICAL REVIEW E
statistical, nonlinear, and soft matter physics

Highlights Recent Accepted Authors Referees Search About 🔍

Information filtering in complex weighted networks

Filippo Radicchi, José J. Ramasco, and Santo Fortunato
Phys. Rev. E 83, 046101 – Published 1 April 2011

Article References Citing Articles (8) PDF HTML Export Citation

ABSTRACT

Many systems in nature, society, and technology can be described as networks, where the vertices are the system's elements, and edges between vertices indicate the interactions between the corresponding elements. Edges may be weighted if the interaction strength is measurable. However, the full network information is often redundant because tools and techniques from network analysis

- Radicchi, F., et al. *Information filtering in complex weighted networks*. Physical Review E, **83** 046101. (2011).

GloSS Filter

Pros:

- Comparison between real network and a null-model
- Unique p -value assigned to edges

Cons:

- Not Maxent
- Heavy constraints (topology + weight distro.)
- Very aggressive (limitations on p -value)

- Radicchi, F., et al. *Information filtering in complex weighted networks*. Physical Review E, **83** 046101. (2011).

Dianati Filter

PHYSICAL REVIEW E

covering statistical, nonlinear, biological, and soft matter physics

Highlights Recent Accepted Authors Referees Search Press About 

Unwinding the hairball graph: Pruning algorithms for weighted complex networks

Navid Dianati

Phys. Rev. E **93**, 012304 – Published 11 January 2016

Article

References

No Citing Articles

PDF

HTML

Export Citation



ABSTRACT

Empirical networks of weighted dyadic relations often contain "noisy" edges that alter the global characteristics of the network and obfuscate the most important structures therein. Graph pruning is the process of identifying the most significant edges according to a generative null model and

- Dianati, N. “*Unwinding the hairball graph: Pruning algorithms for weighted complex networks*”. Phys. Rev. E, **93**, 012304 (2016).

Dianati Filter

Pros:

- Comparison between real network and a null-model based on Maxent
- Two versions: global and local

Cons:

- Maxent based on conservation of $\{s_i\}$

• Dianati, N. “*Unwinding the hairball graph: Pruning algorithms for weighted complex networks*”. Phys. Rev. E, **93**, 012304 (2016).

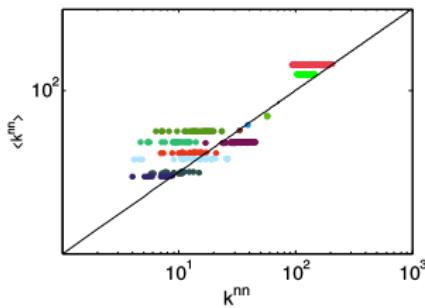
Section 2

The ECM-filter

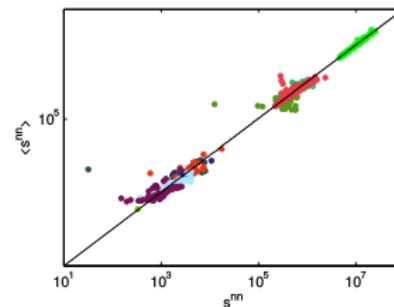
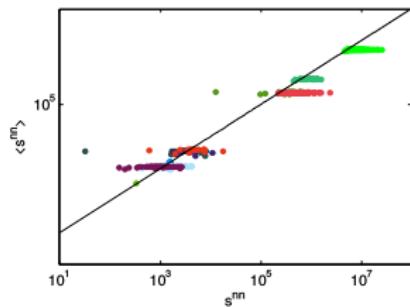
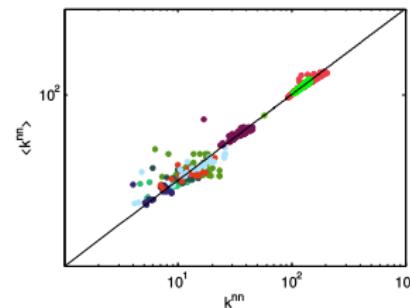
KUKUAEWm

ECM-filter

degree OR strength



degree AND strength



KUKUAEWm

Mastrandrea, R., et al. Enhanced reconstruction of weighted networks from strengths and degrees. New Jour. Phys., **16**, 043022. (2014).

ECM-filter

Main Features

- ① Based on the comparison between the observed network and a null model one.

- Mastrandrea, R., et al. *Enhanced reconstruction of weighted networks from strengths and degrees*. New Jour. Phys., **16**, 043022. (2014).

ECM-filter

Main Features

- ① Based on the comparison between the observed network and a null model one.
- ② Null model: maximum-entropy canonical ensemble of networks satisfying given constraints.

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ECM-filter

Main Features

- ① Based on the comparison between the observed network and a null model one.
- ② Null model: maximum-entropy canonical ensemble of networks satisfying given constraints.
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- Mastrandrea, R., et al. *Enhanced reconstruction of weighted networks from strengths and degrees*. New Jour. Phys., **16**, 043022. (2014).

ECM-filter

Main Features

- ① Based on the comparison between the observed network and a null model one.
- ② Null model: maximum-entropy canonical ensemble of networks satisfying given constraints.
- ③ Constraints: $\{k_i\}$ and $\{s_i\}$ preserved (on average).
- ④ Two versions: **local** (focus on links) and **global** (focus on entire network).

- Mastrandrea, R., et al. *Enhanced reconstruction of weighted networks from strengths and degrees*. New Jour. Phys., **16**, 043022. (2014).

IAEWm

ECM-filter

Local filter

- ① Generate the null model ensemble and compute:

$$q_{ij}(w) \equiv \frac{(x_i x_j)^{\Theta(w_{ij})} (y_i y_j)^{w_{ij}} (1 - y_i y_j)}{1 - y_i y_j + x_i x_j y_i y_j}.$$

IAEWm

ECM-filter

Local filter

- ① Generate the null model ensemble and compute:

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- ② Compute the probability:

$$p_{ij}(w^*) = 1 - \sum_{w=0}^{w^*-1} q_{ij}(w).$$

IAEWm

ECM-filter

Local filter

- ① Generate the null model ensemble and compute:

$$q_{ij}(w) \equiv \frac{(x_i x_j)^{\Theta(w_{ij})} (y_i y_j)^{w_{ij}} (1 - y_i y_j)}{1 - y_i y_j + x_i x_j y_i y_j}.$$

- ② Compute the probability:

$$p_{ij}(w^*) = 1 - \sum_{w=0}^{w^*-1} q_{ij}(w).$$

- ③ Associate for each link a p -value, γ , such that $P(w_{ij} > w_{ij}^*)$.

ECM-filter

Local filter

- ① Generate the null model ensemble and compute:

$$q_{ij}(w) \equiv \frac{(x_i x_j)^{\Theta(w_{ij})} (y_i y_j)^{w_{ij}} (1 - y_i y_j)}{1 - y_i y_j + x_i x_j y_i y_j}.$$

- ② Compute the probability:

$$p_{ij}(w^*) = 1 - \sum_{w=0}^{w^*-1} q_{ij}(w).$$

- ③ Associate for each link a p -value, γ , such that $P(w_{ij} > w_{ij}^*)$.
- ④ Select a threshold $\tilde{\gamma}$ and remove all the links with $\gamma_{ij} > \tilde{\gamma}$.

IAEWm

ECM-filter

Global filter

- ① Find the subgraph Σ with L' links that minimizes the **likelihood** of being generated by chance.

IAEWm

ECM-filter

Global filter

- ① Find the subgraph Σ with L' links that minimizes the **likelihood** of being generated by chance.
- ② Minimize the function:

$$P(\Sigma) = \prod_{i < j} [q_{ij}(w_{ij})]^{a_{ij}} .$$

IAEWm

ECM-filter

Global filter

- ① Find the subgraph Σ with L' links that minimizes the **likelihood** of being generated by chance.
- ② Minimize the function:

$$P(\Sigma) = \prod_{i < j} [q_{ij}(w_{ij})]^{a_{ij}} .$$

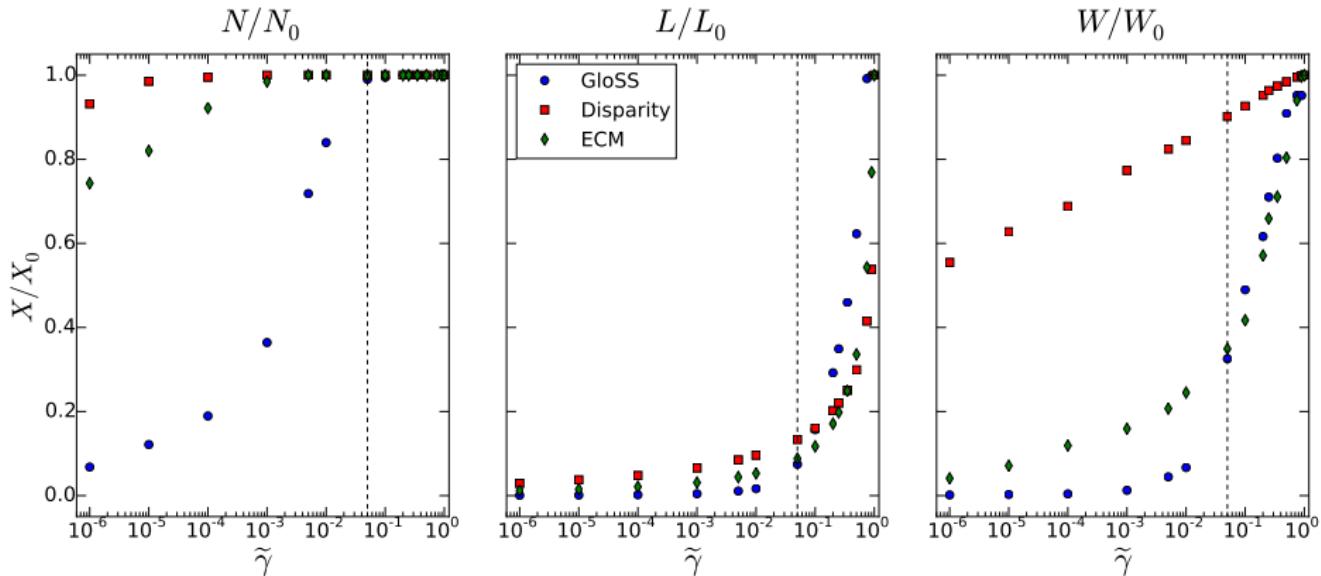
- ③ Rank edges upon their $q_{ij}(w)$ and add the L' smallest ones.

Section 3

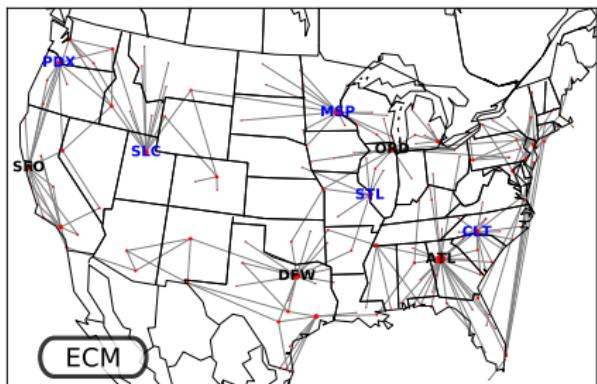
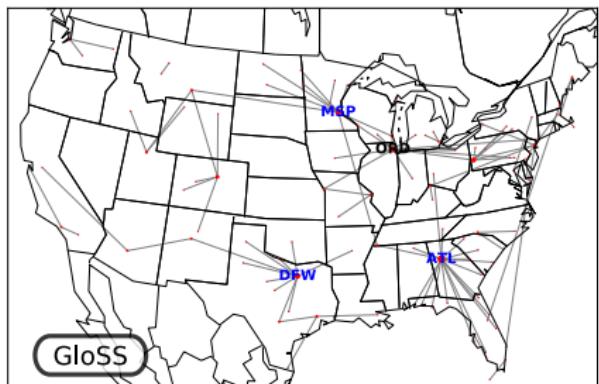
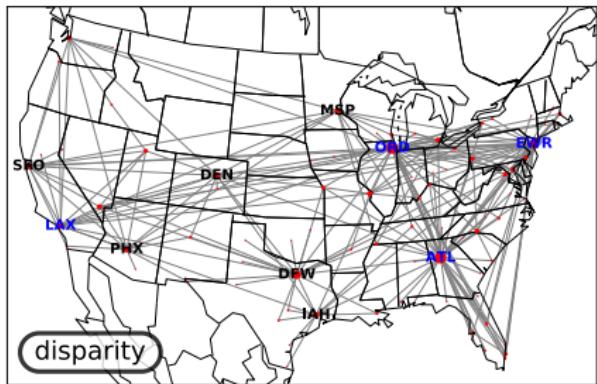
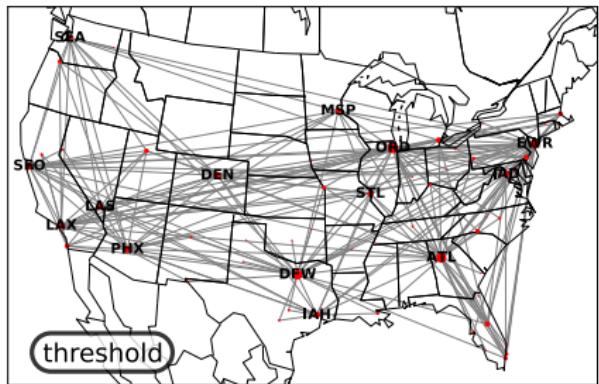
Results

KUKUAEWm

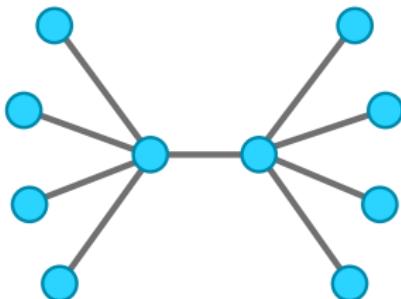
Comparison among methods



Comparison among methods

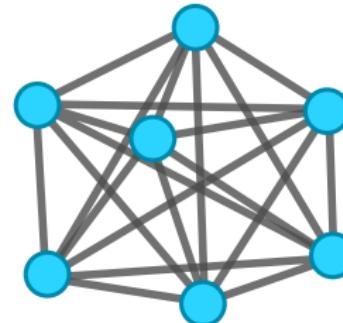


Comparison among methods



Hub and Spoke

Point to Point



KUKUAEWm

A “serious” example . . .

KUKUAEWm

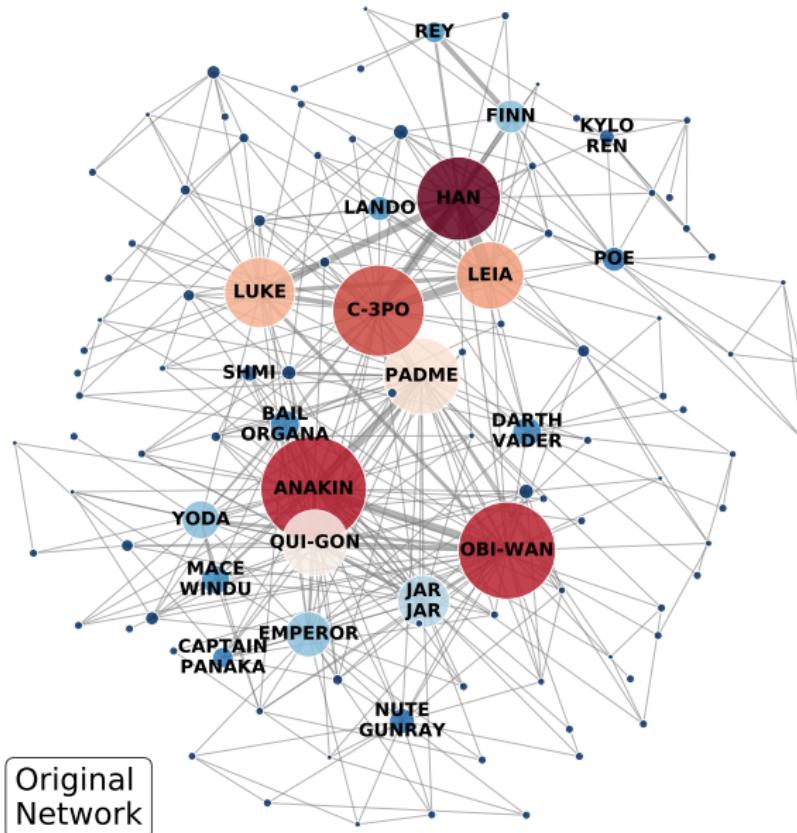
A “serious” example ...

A long time ago in a galaxy far,
far away....

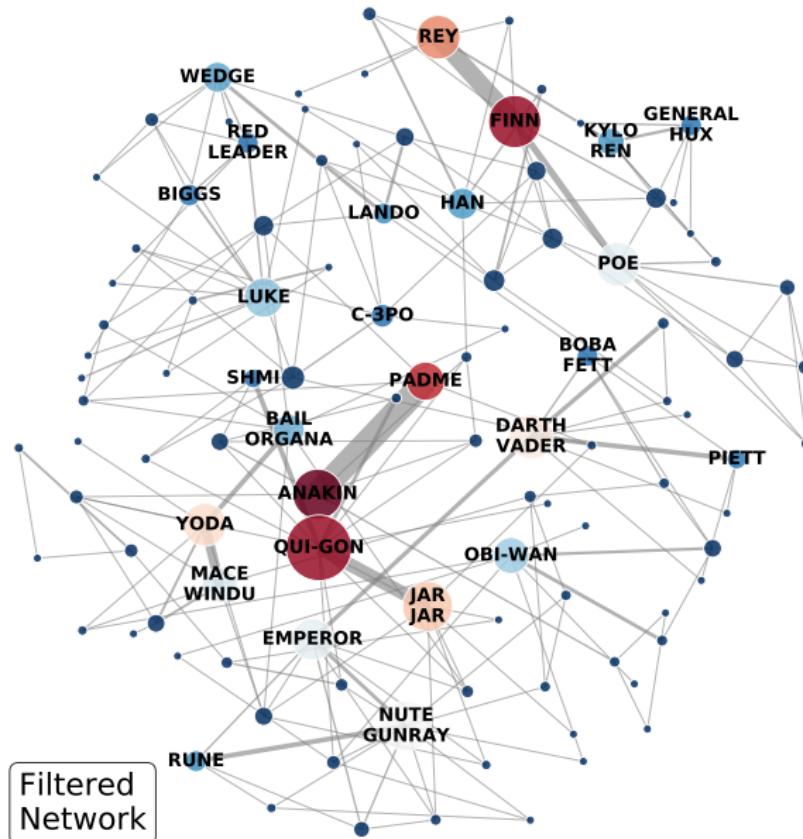
EWm

- <http://evelinag.com/blog/2015/12-15-star-wars-social-network/>

A “serious” example . . .



A “serious” example ...



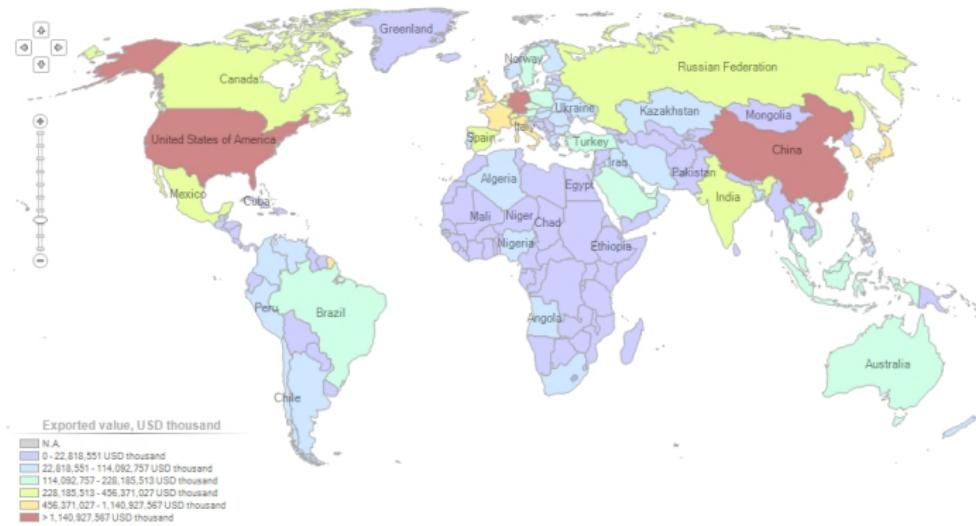
KUKUAEWm

... and a less “serious” one

International Trade Network 1998 – 2011

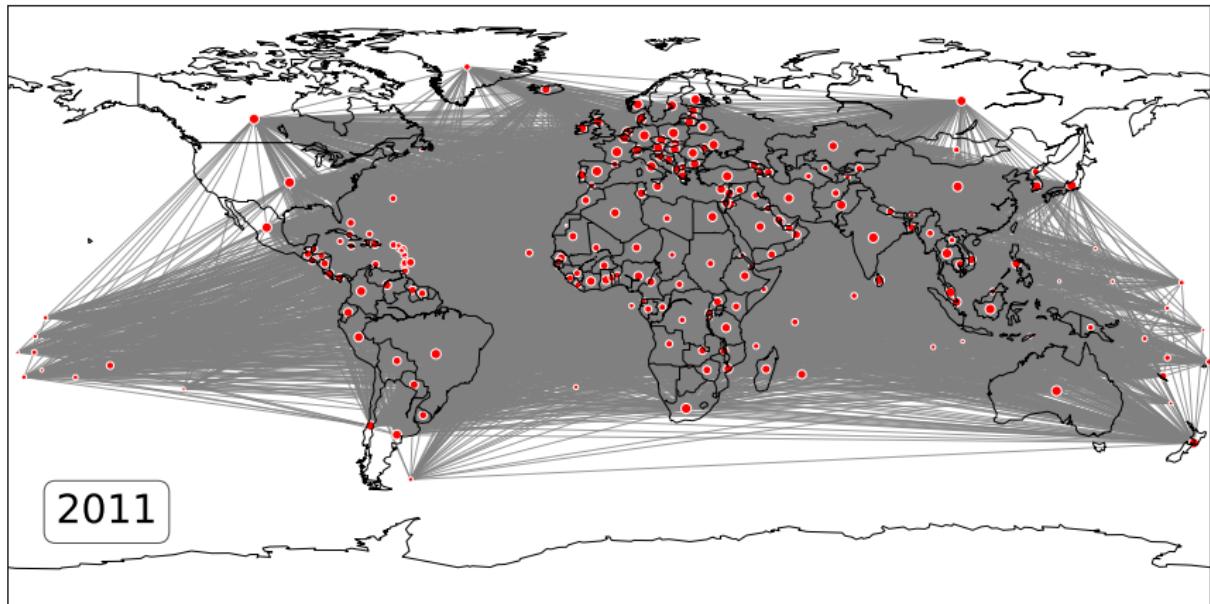


exporting countries for the selected product in 2015
Product : TOTAL All products

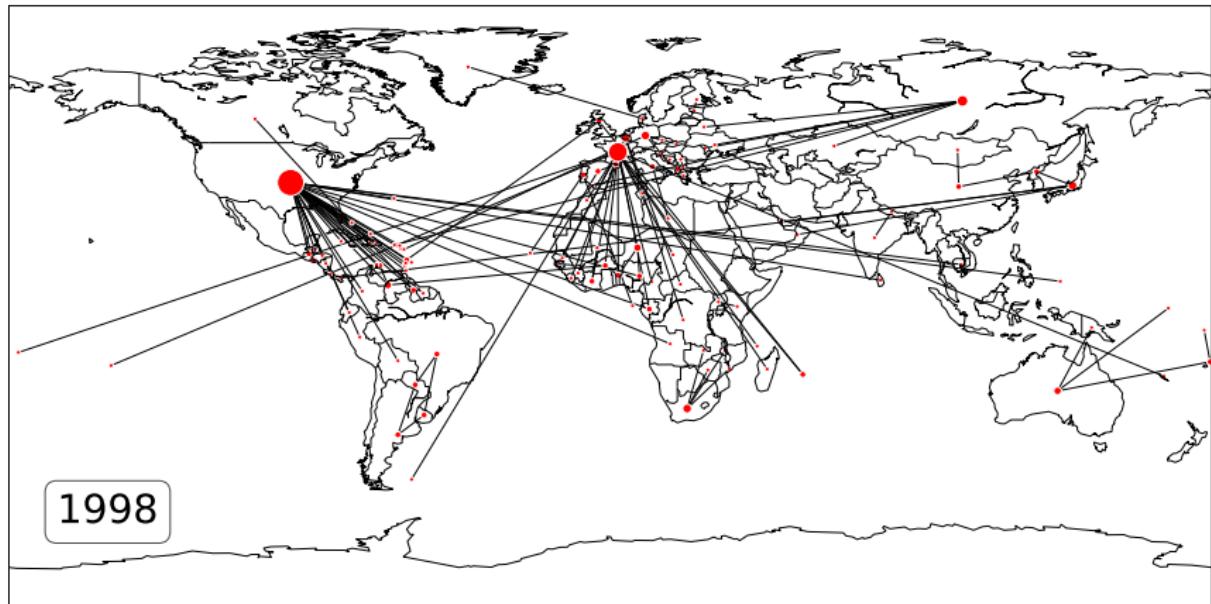


KUKUAEWm

... and a less “serious” one

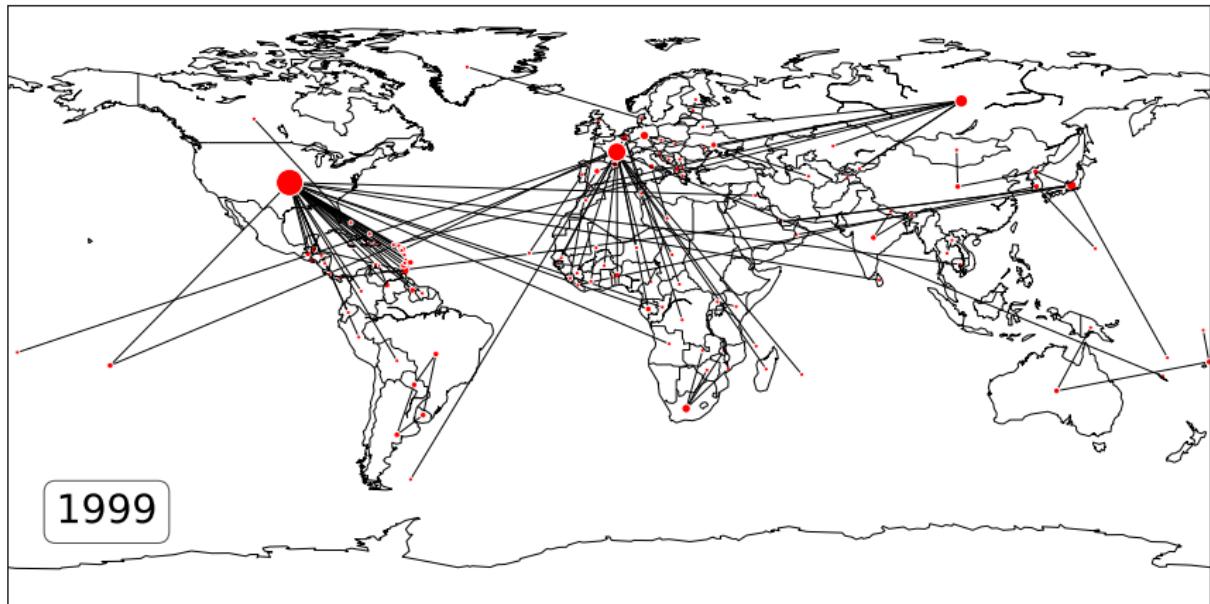


... and a less “serious” one



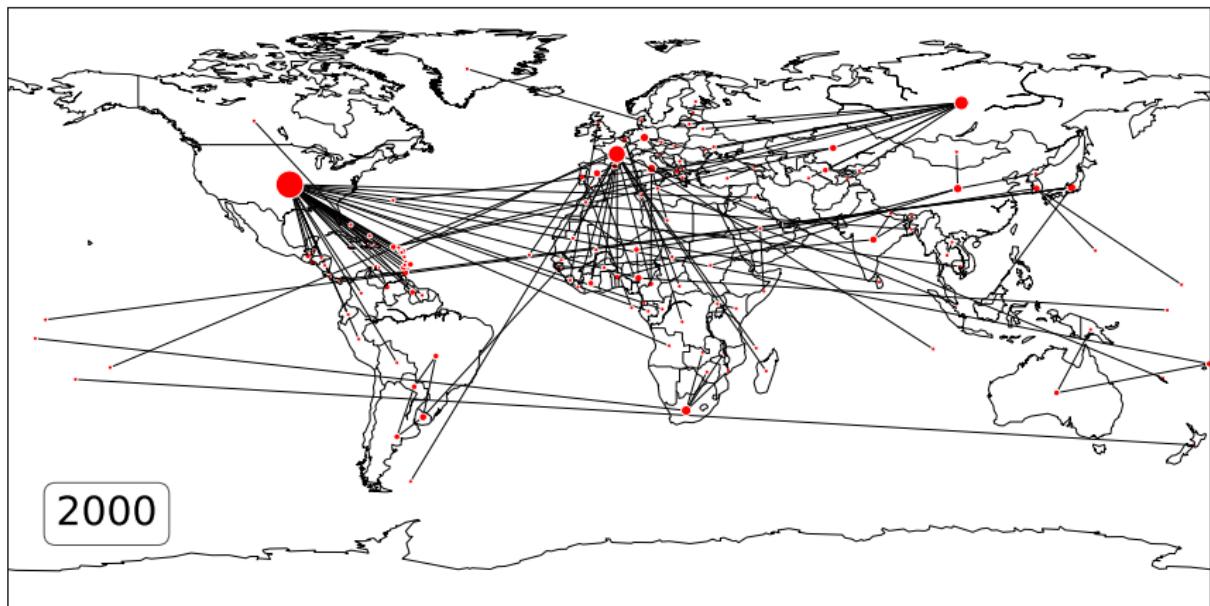
AEWm

... and a less “serious” one



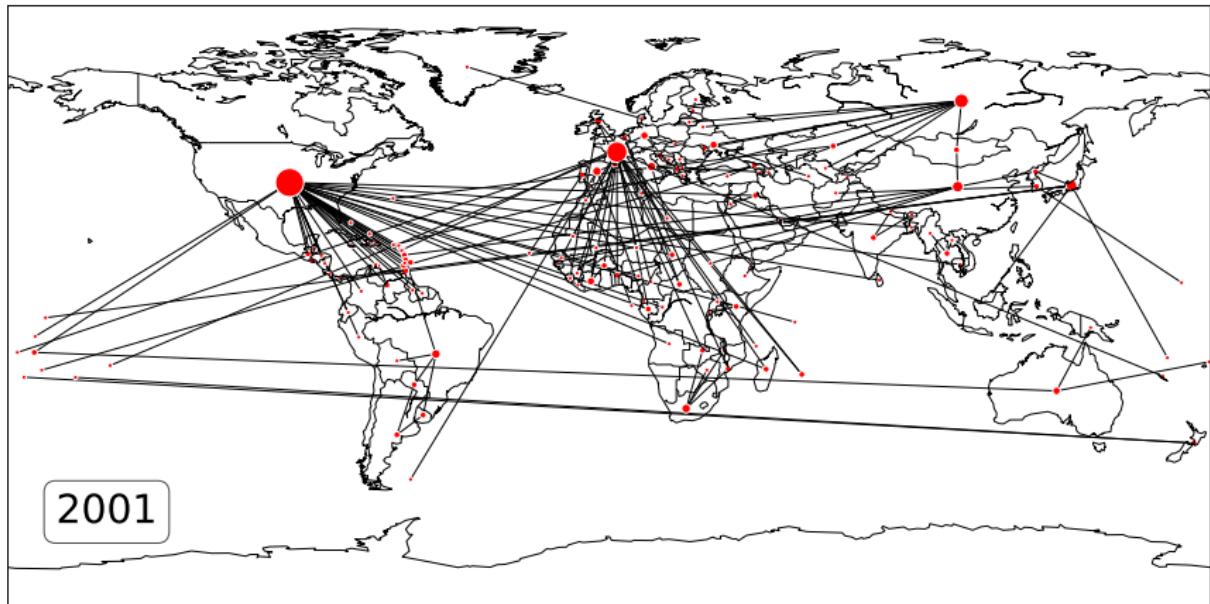
AEWm

... and a less “serious” one

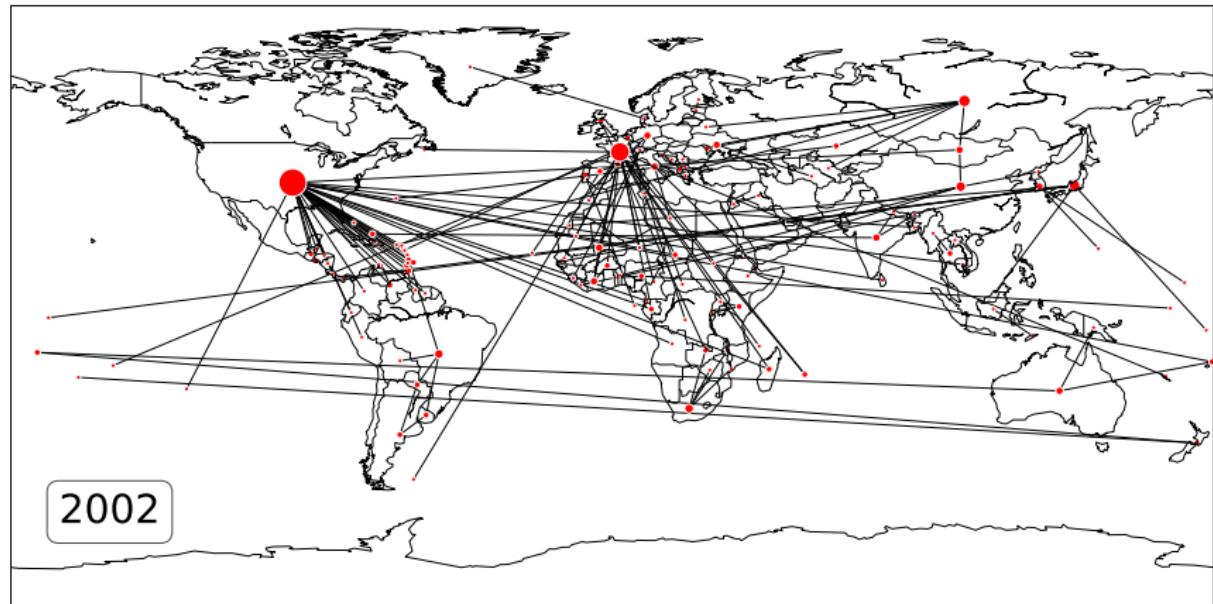


AEWm

... and a less “serious” one

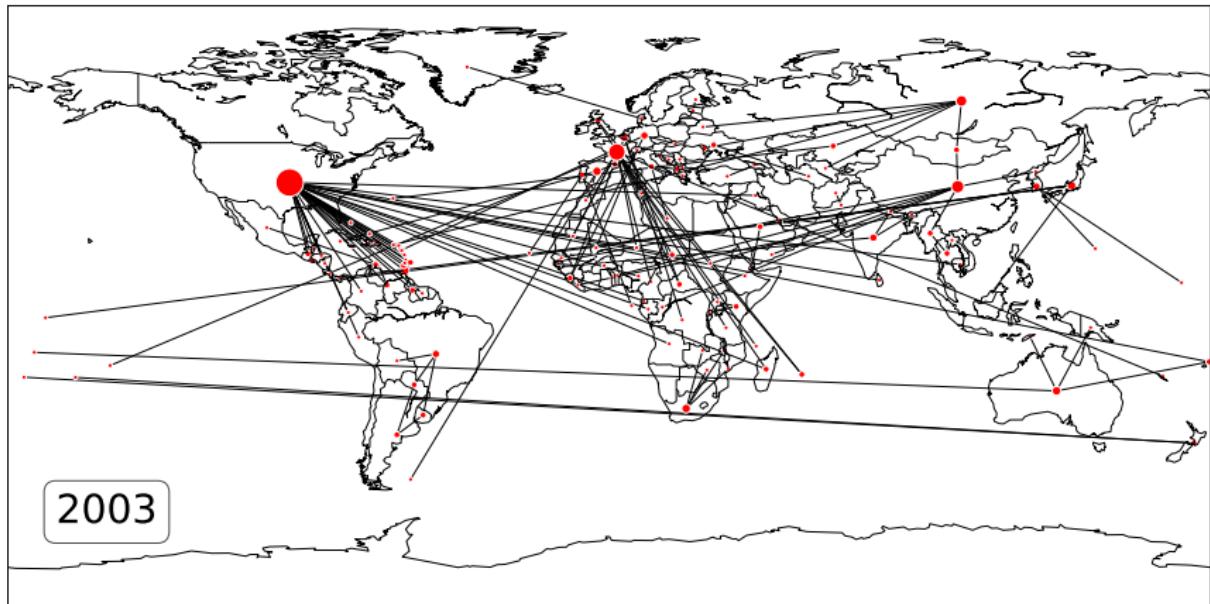


... and a less “serious” one



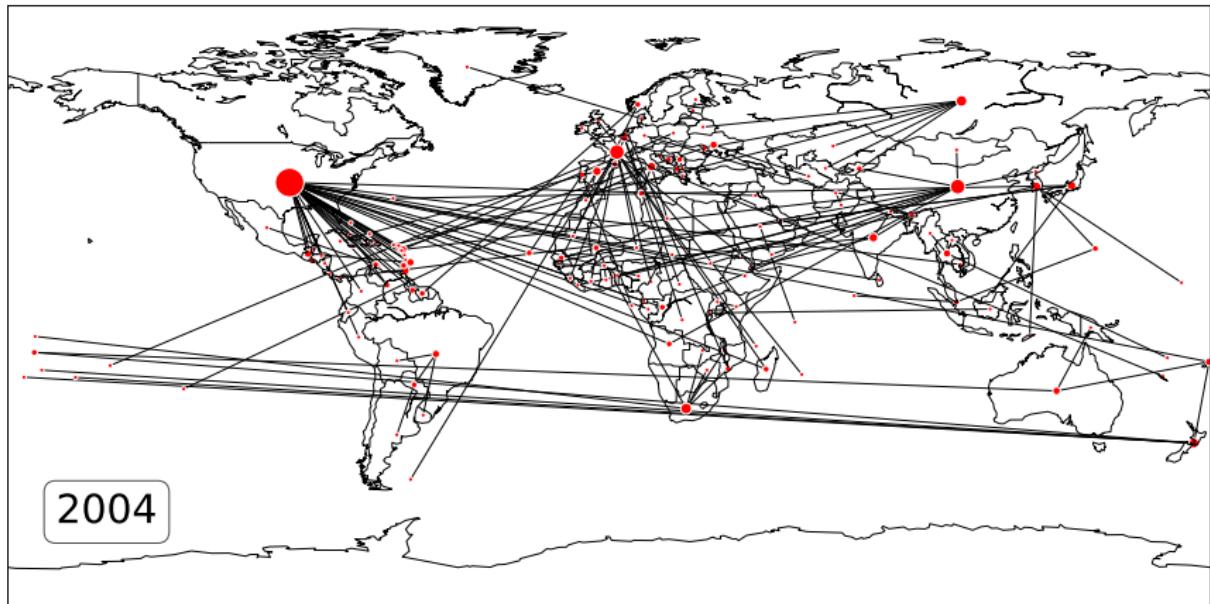
AEWm

... and a less “serious” one



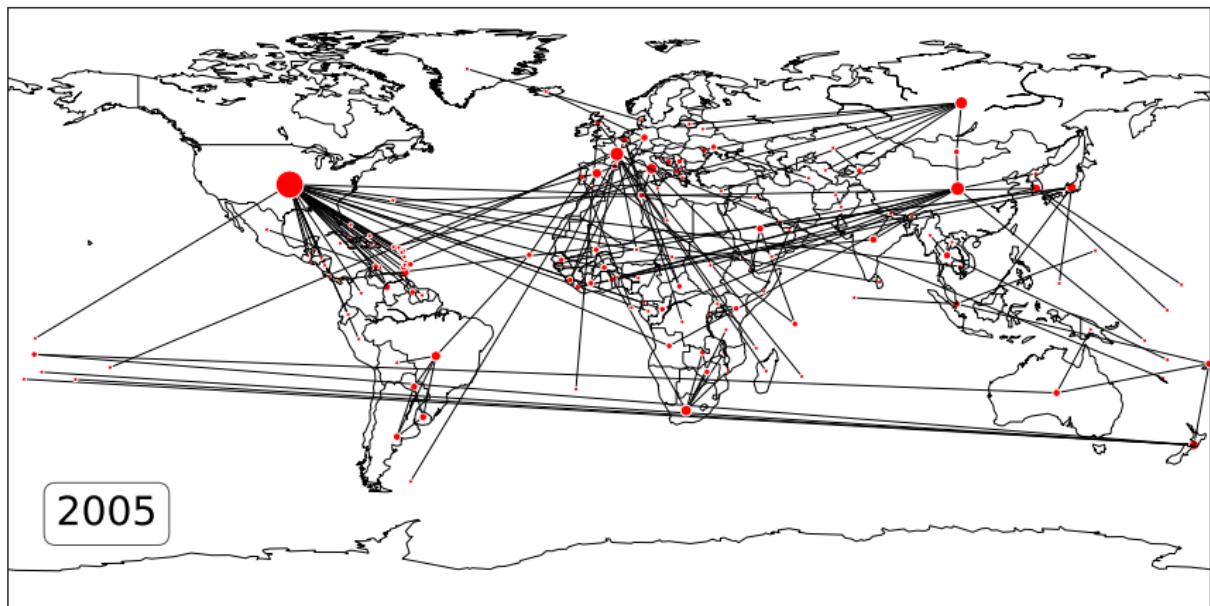
AEWm

... and a less “serious” one



AEWm

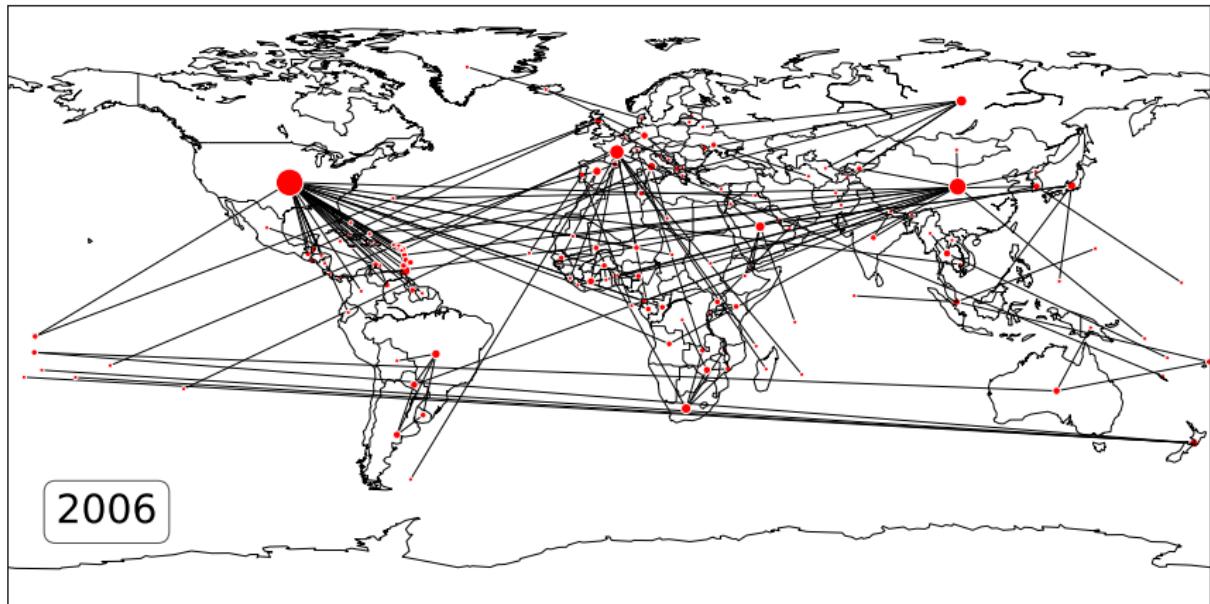
... and a less “serious” one



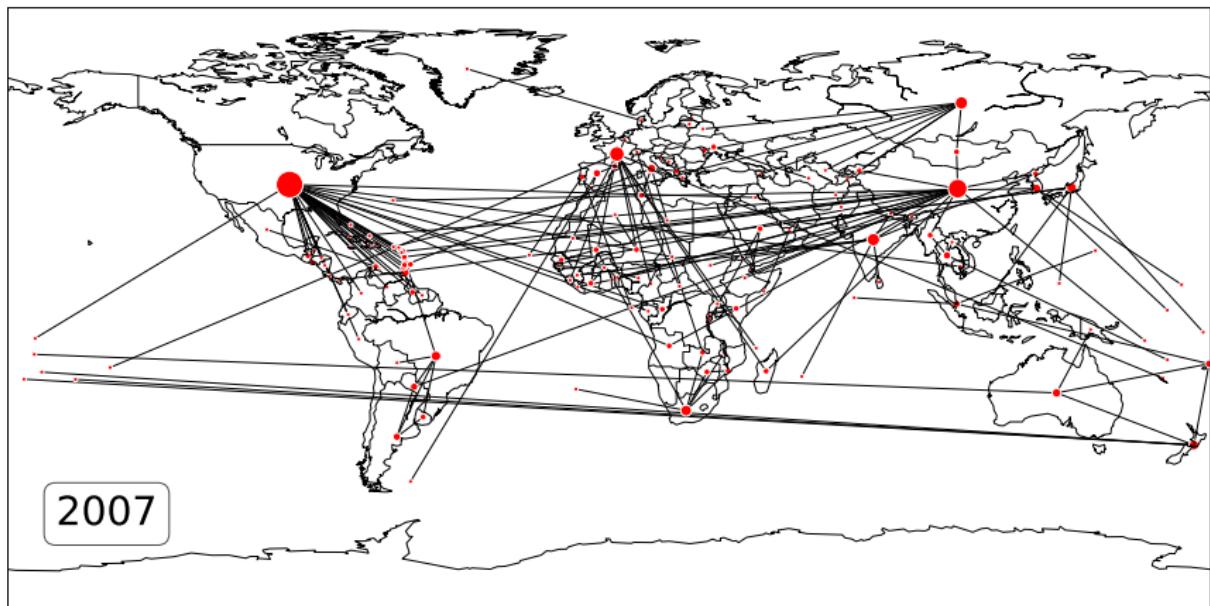
AEWm

2005

... and a less “serious” one

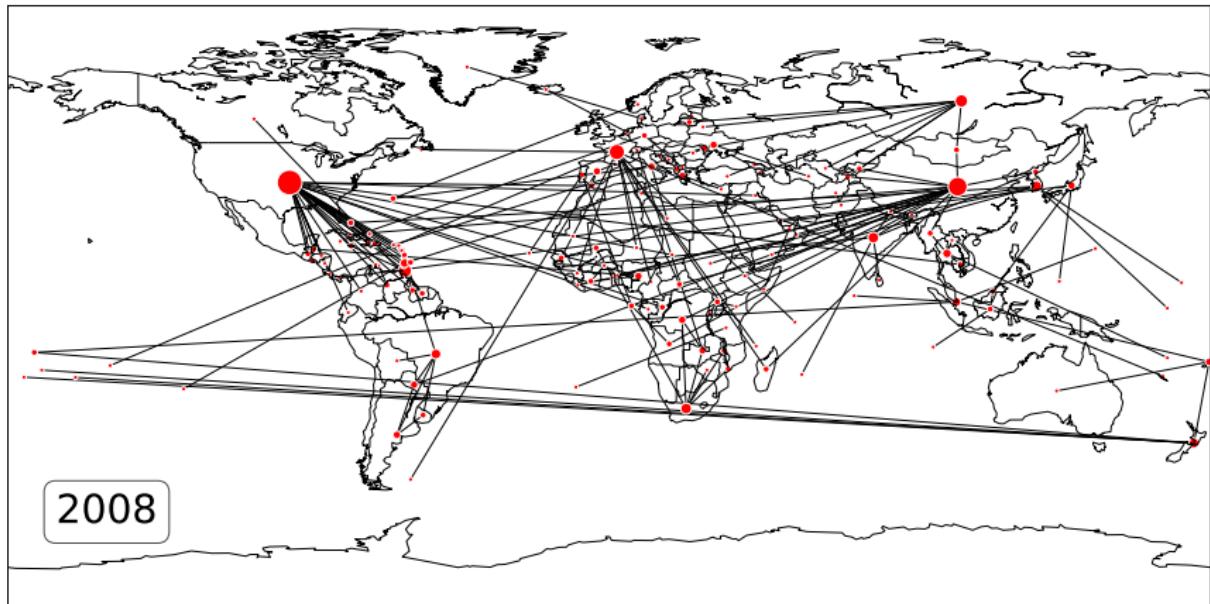


... and a less “serious” one



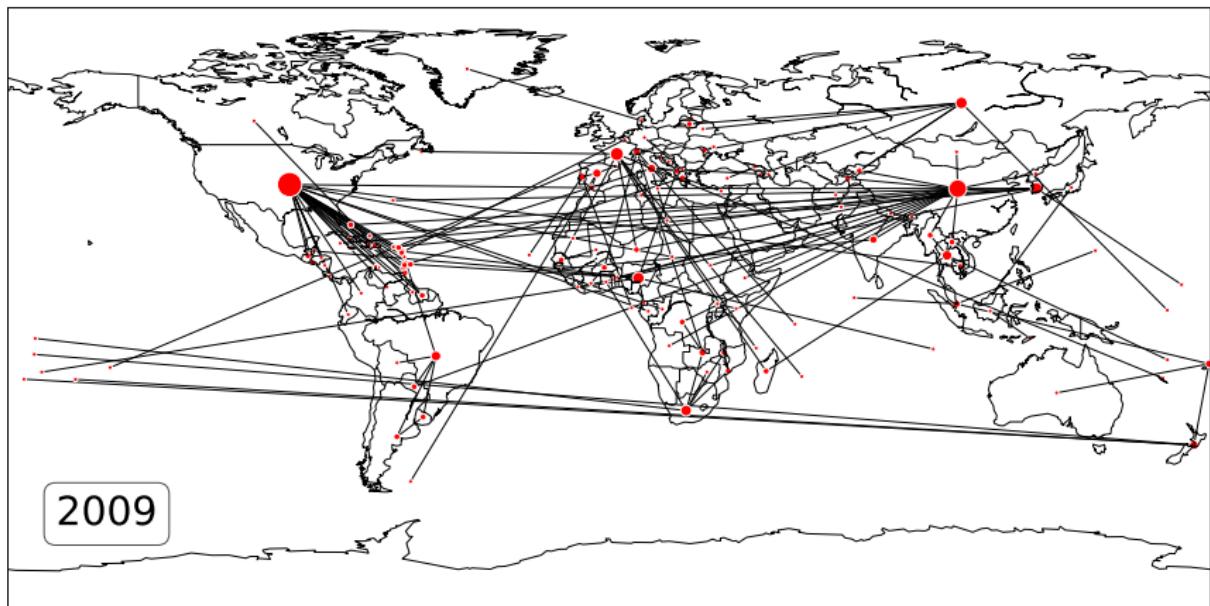
AEWm

... and a less “serious” one



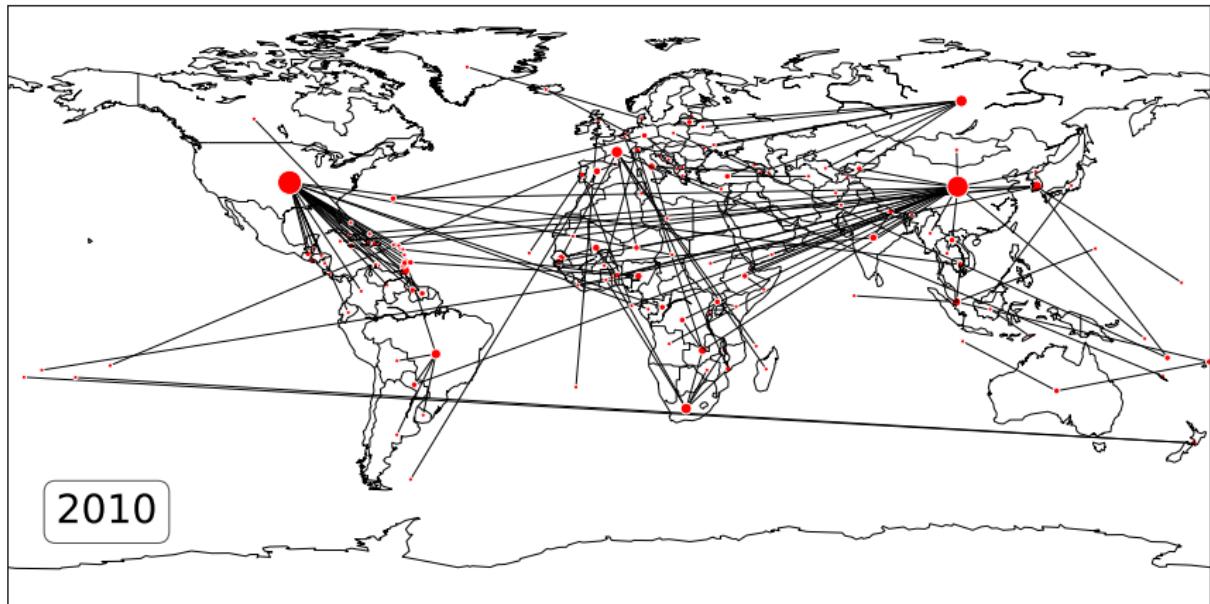
AEWm

... and a less “serious” one



AEWm

... and a less “serious” one



AEWm

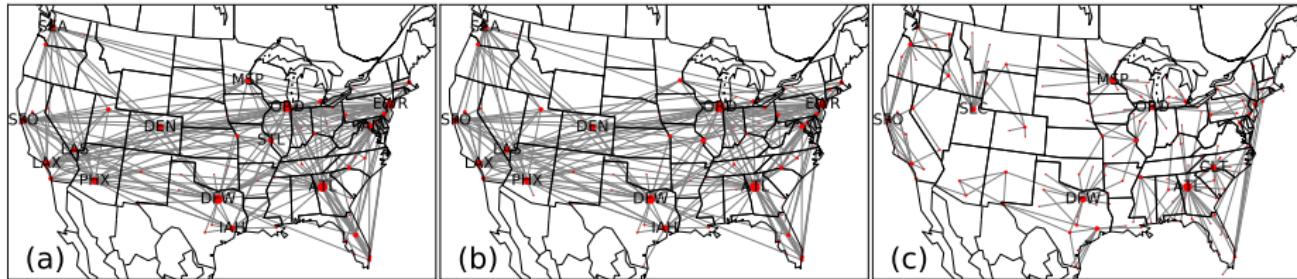
... and a less “serious” one



AEWm

2011

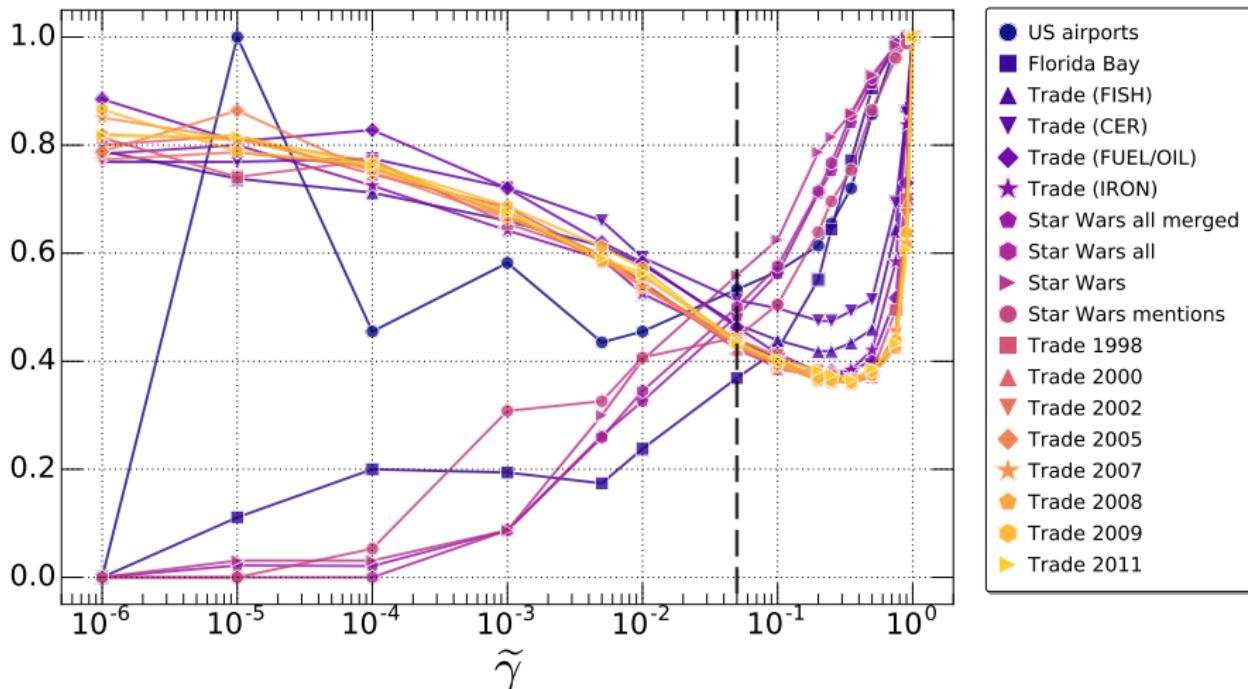
Local vs Global



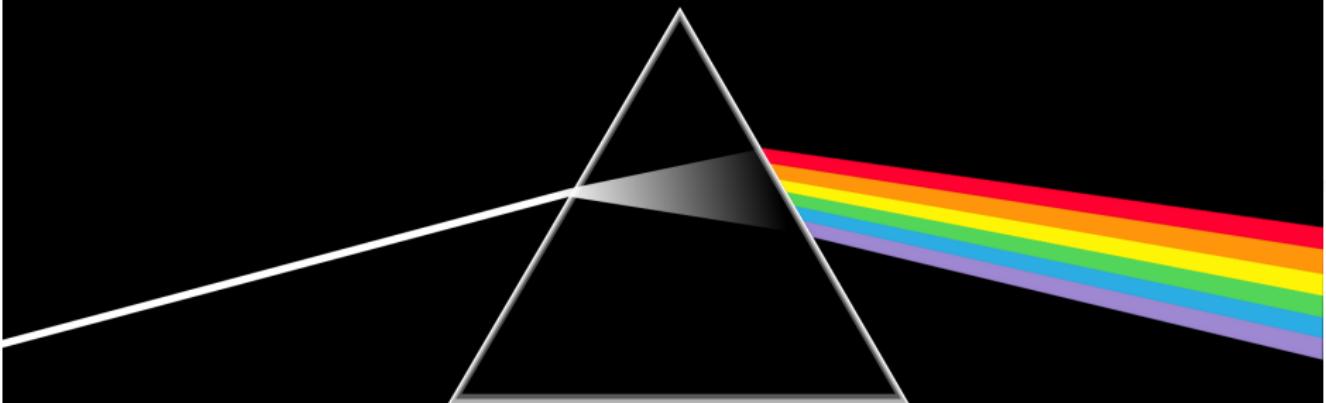
Jaccard Score

$$J = \frac{|A \cap B|}{|A \cup B|}.$$

Local vs Global



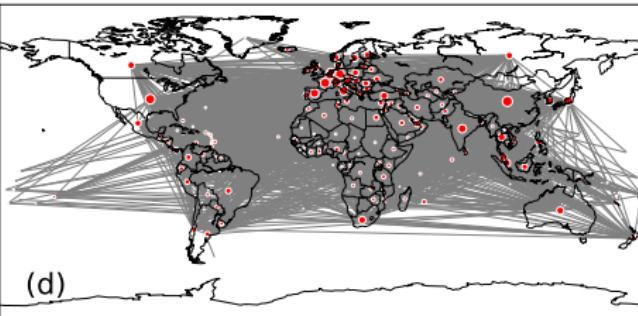
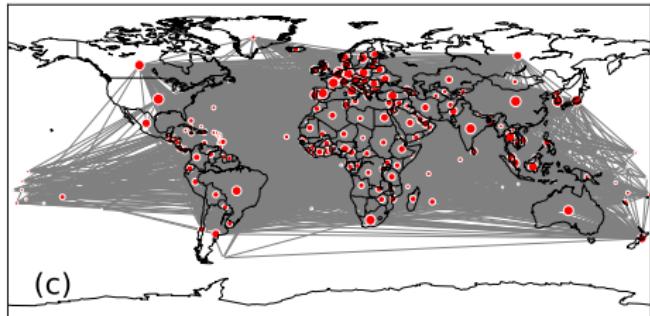
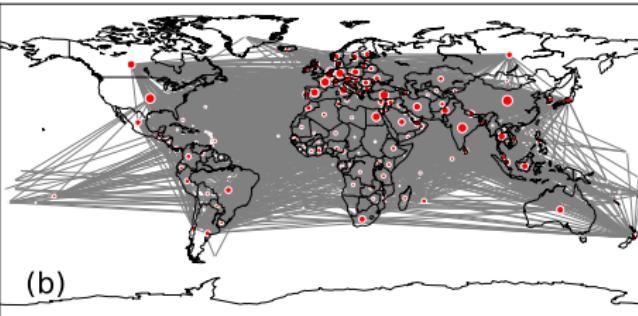
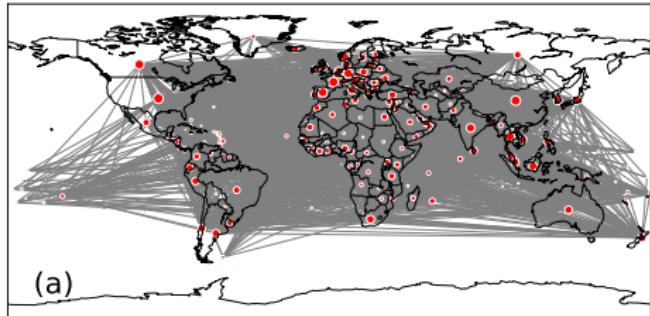
ECM and Multiplex



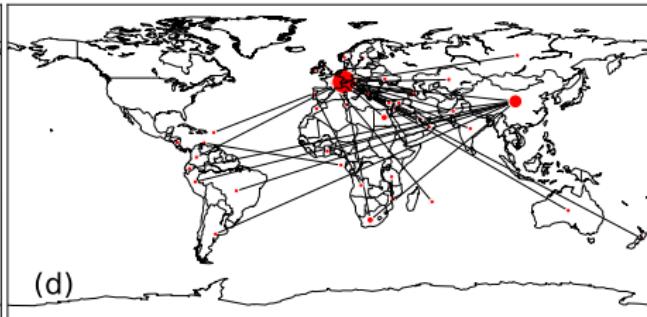
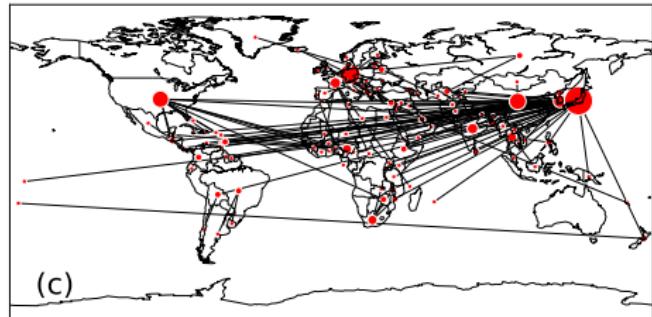
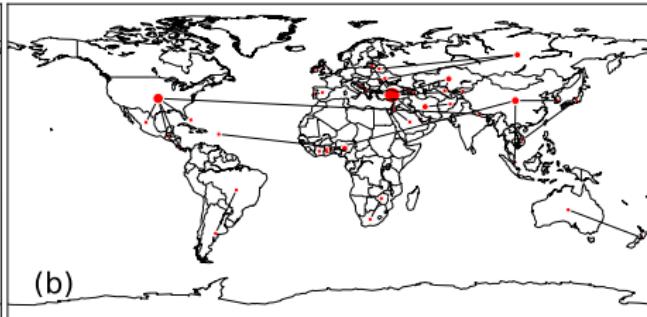
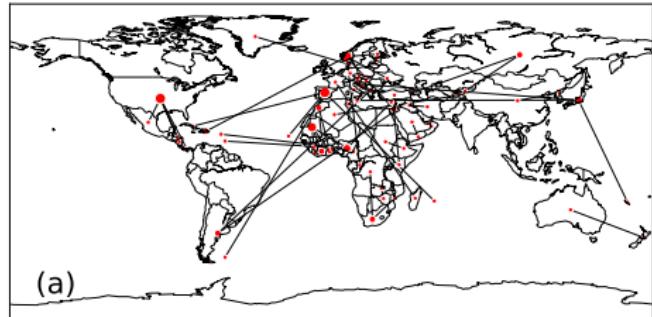
ECM and Multiplex



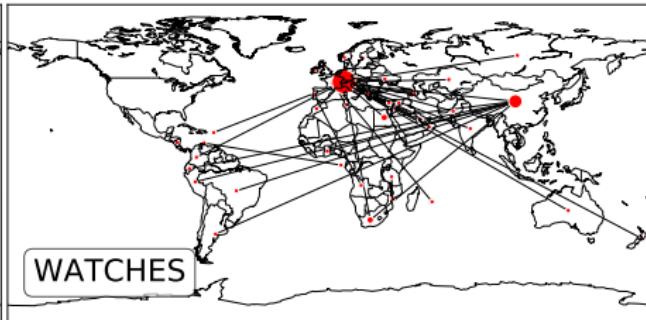
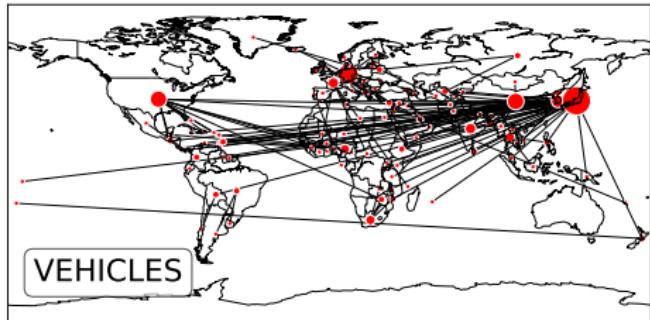
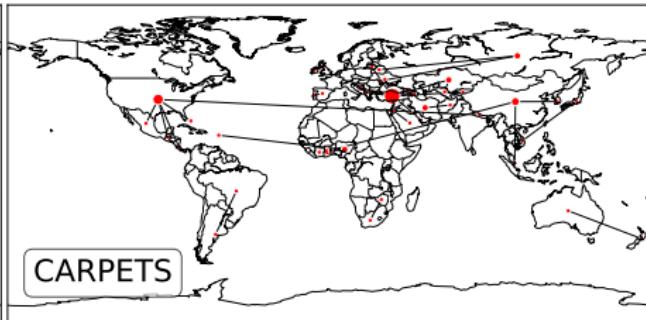
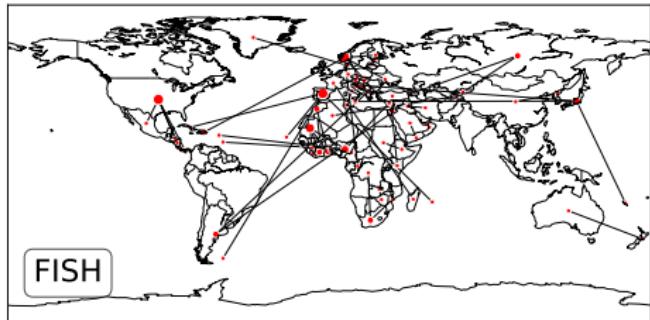
ECM and Multiplex



ECM and Multiplex



ECM and Multiplex



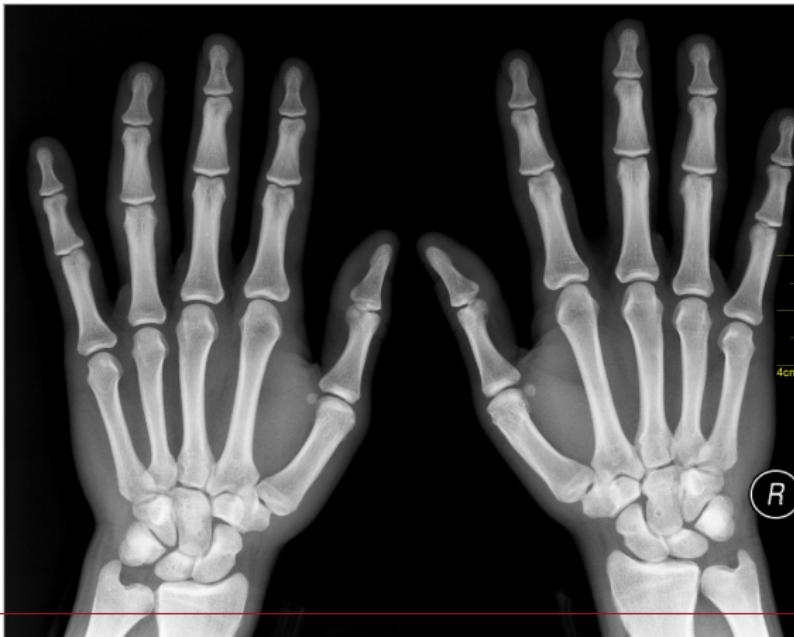
Section 4

Conclusions

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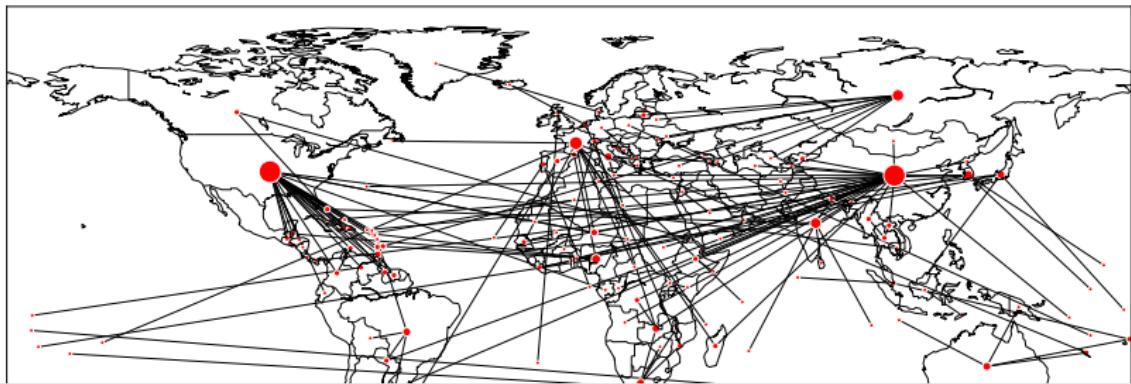
Take home messages

Filtering is becoming a more and more **required** step to continue using networks in complex systems.



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Take home messages



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ECM filter overcomes limitations of previous approaches and retrieves non trivial features.

Acknowledgements



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Valerio Gemmetto

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Wanna know more?

Stay tuned on the arXiv ...

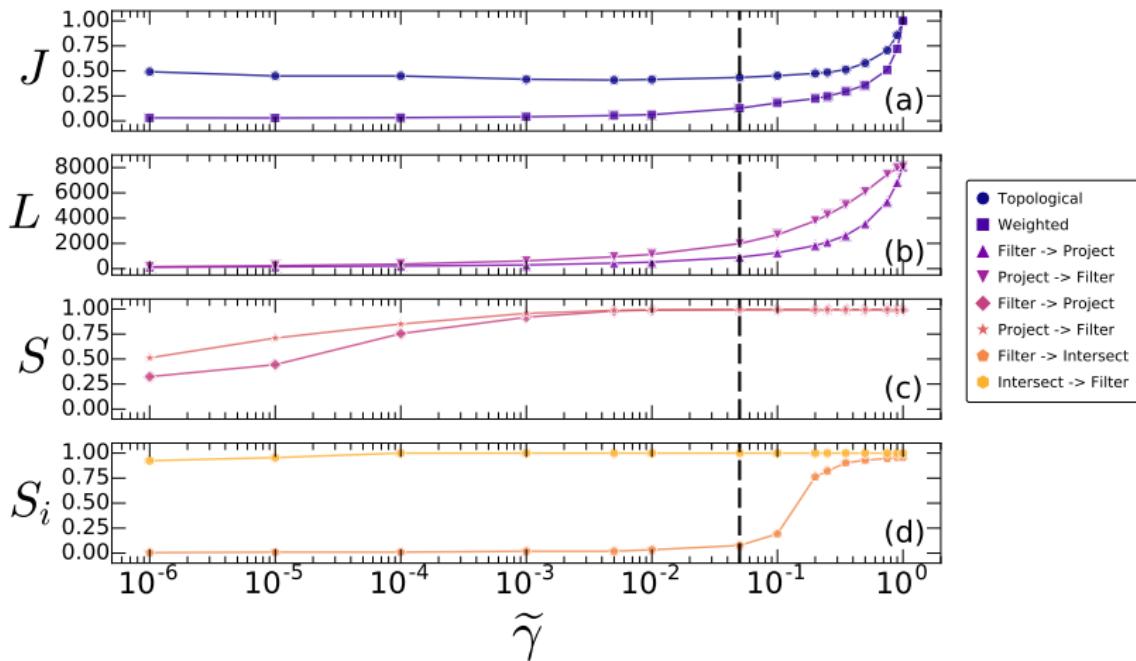
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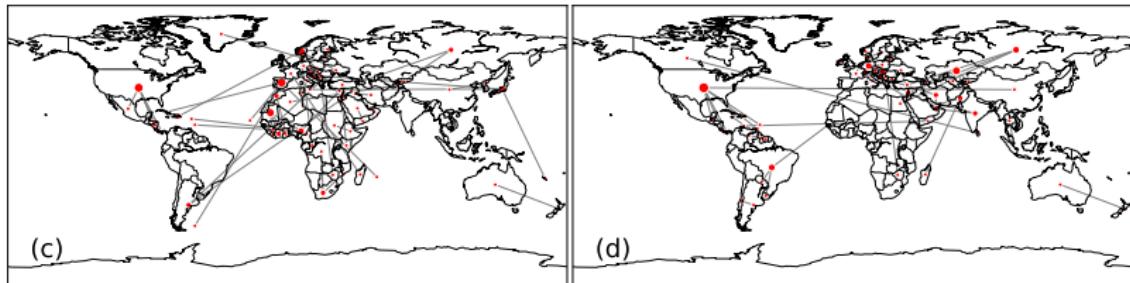
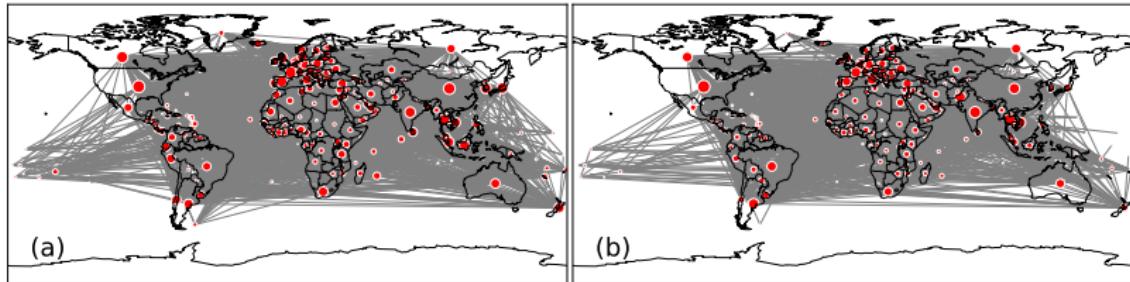
Filtering & Multiplexity



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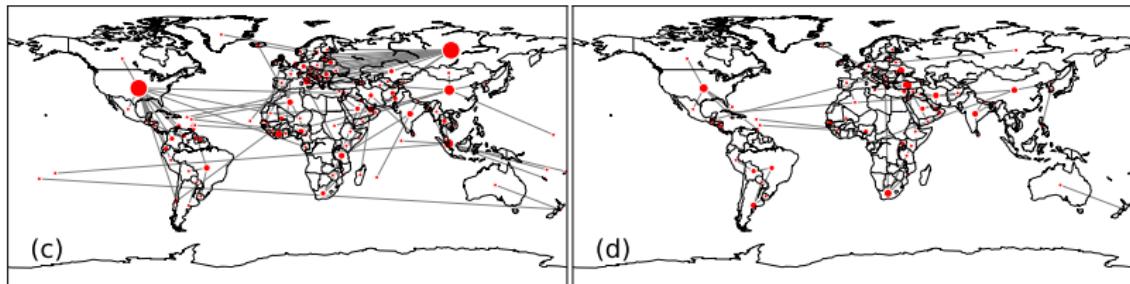
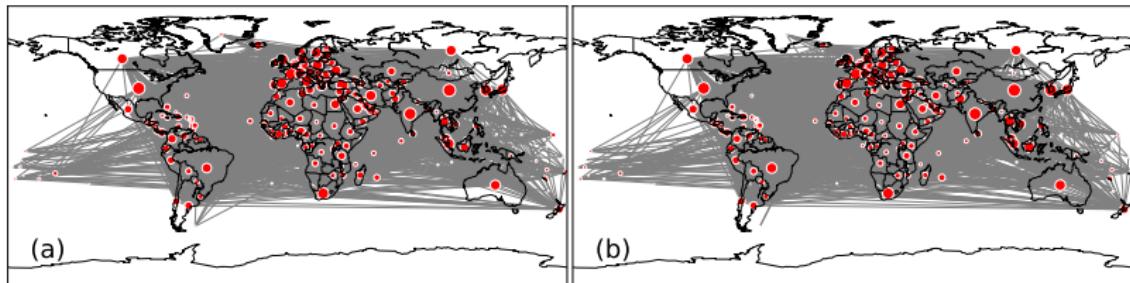
Trade: other commodities

FISH – CEREALS in 2011



Trade: other commodities

FUEL – IRON in 2011



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