# Co-evolution of strategies and update rules in the prisoner's dilemma game on complex networks

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Thursday 14 April 2011, Department of Physics of Condensed Matter, Zaragoza, Spain

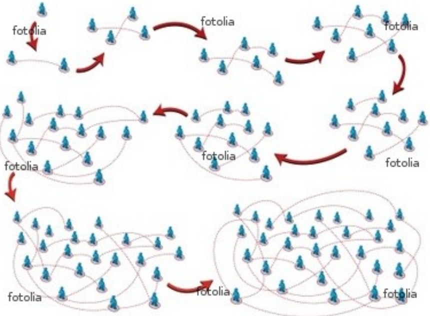
# Summary

- Introduction
- Game theory
- Update Rules
- Complex Networks
- Results
- Conclusions

# Introducing myself . . .

## Actually I am working on . . .

- Dynamical processes on networks;
- Evolutionary game theory on networks;
- Emergence of collective behaviours (i.e. cooperation);

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# What we are going to talk today . . .

New Journal of Physics
The open-access journal for physics

**Co-evolution of strategies and update rules in the prisoner's dilemma game on complex networks**

**Alessio Cardillo**[1], **Jesús Gómez-Gardeñes**[2,3,6], **Daniele Vilone**[4] **and Angel Sánchez**[3,4,5]

Published on: New Journal of Physics – **12**, 103034, 2010

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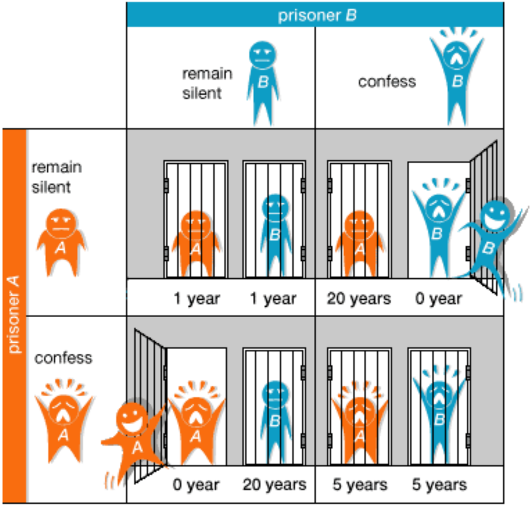
# Introduction to Prisoner's Dilemma (PD) game

**Situation: Two bank robbers have been arrested . . .**



# Introduction to Prisoner's Dilemma (PD) game

From a mathematical point of view we can *describe* the game through the payoff matrix as:

$$\begin{array}{cc} & \begin{array}{cc} \mathrm{C} & \mathrm{D} \end{array} \\ \begin{array}{c} \mathrm{C} \\ \mathrm{D} \end{array} & \begin{pmatrix} \mathcal{R} & \mathcal{S} \\ \mathcal{T} & \mathcal{P} \end{pmatrix} \end{array} \qquad \text{such that:} \qquad \mathcal{T} > \mathcal{R} > \mathcal{P} > \mathcal{S}\,.$$

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## Nash equilibrium

The Nash equilibrium of PD game is the defection state. The problem with Nash equilibrium is that:

- Players are not "smart" (they are not able to calculate the Nash equilibrium);
- Playert do not have "full knowledge" (they do not know the structure of payoff matrix);

# Introduction to Prisoner's Dilemma (PD) game

## Game theory is not enough

- In general, people do not play only once but many times;
- They learn after each round and try to choose a strategy which ensure them the best success in the next one (payoff driven selection);
- Humans are not always “fully rational” because sometimes they make counterituitive choices;

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## Evolutionary game theory

These objections could find a solution under the evolutionary game theory postulated by Maynard Smith and Price in 1973.

# Update Rules

**Game alone is not enough**

Once player's strategies are defined, one has to define also how players update their strategies during the game, i.e. their Update Rules.

# Update Rules

## Definitions

Replicator Dynamics (REP): Each agent $i$ chooses one of his neighbors at random, say $j$, and compares their payoffs. If $f_j > f_i$ agent $i$ will copy strategy and update rule of $j$ with probability:

$$\Pi \propto f_j - f_i \, .$$

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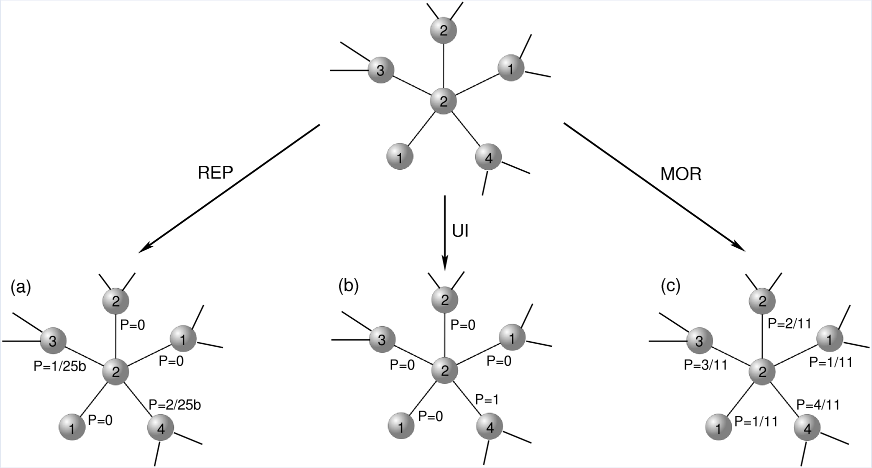
Moran Rule (MOR): Each agent $i$ chooses one of his neighbors proportionally to his payoff and changes his state to the one of the chosen one.

# Update Rules

**In particular:**

- REP $\longrightarrow$ is stochastic with partial information;
- UI $\longrightarrow$ is deterministic with full information;
- MOR $\longrightarrow$ is stochastic with full information;

## An example:



2
3
1
2
1
4
REP
UI
MOR
(a)
2
P=0
3
1
P=1/25b
2
P=0
P=2/25b
1
P=0
4
(b)
2
P=0
3
1
P=0
2
P=0
P=1
1
P=0
4
(c)
2
P=2/11
3
1
P=3/11
2
P=1/11
P=4/11
1
P=1/11
4

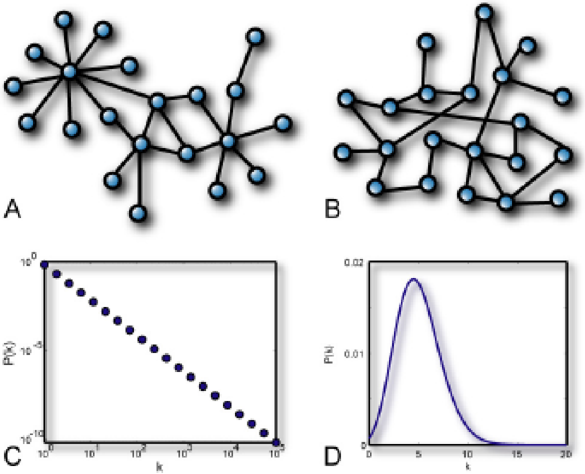
# Complex Networks

### Interaction patterns

Once the game is fully set-up, we have to decide how the players interact between them. Mean-field and regular lattices are two paradigmatic examples but complex topologies are best suited to represent a real-scenario.

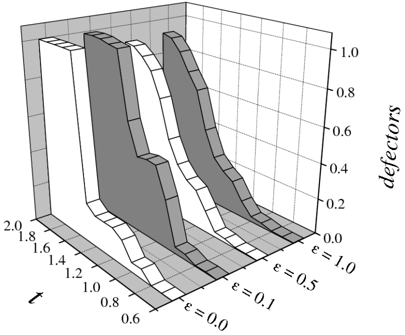
# Complex Networks

Two of the most common topologies used are: Scale Free (SF) and Erdős-Rényi (ER) graphs.



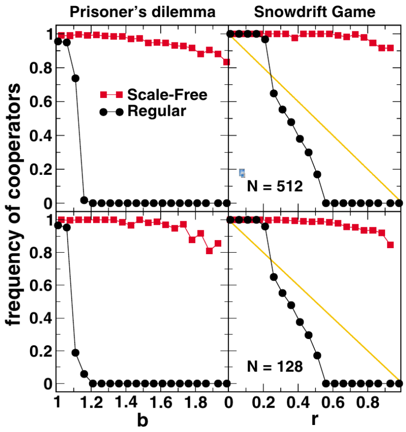
# Complex Networks

## Erdős-Rényi & Lattice




G. Abramson and M. Kuperman, PRE, **63**, 030901(R) (2001)

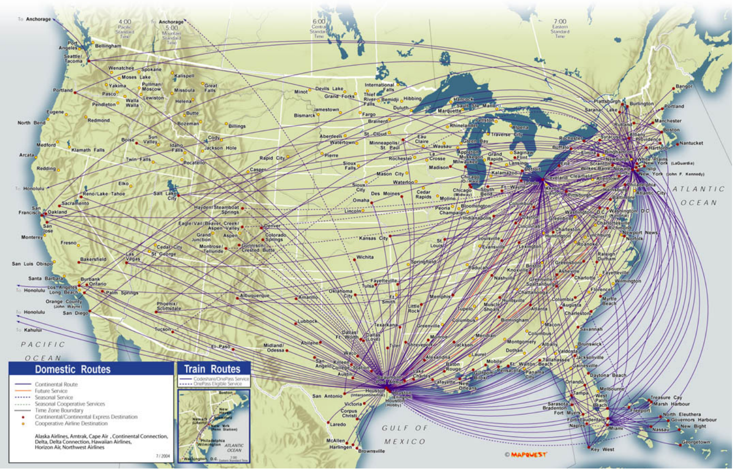
## Scale-free




F.C. Santos and J.M. Pacheco, PRL, **95**, 098104 (2005)

# Complex Networks

Scale-free networks are very important because many real systems display such kind of structure.



# Complex Networks

## Interaction patterns

In order to consider topologies spanning from ER networks to SF ones, we used the Interpolation Model of Gardeñes *et al.*

J. Gómez-Gardeñes and Y. Moreno, Phys. Rev. E – **73** 056124, 2006

# The question is: Why Co-evolution?

- Papers in literature show that the outcome of evolutionary game on complex networks, in terms of cooperative behavior, strongly depends on topology but also on the update rule used.

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- Papers in literature show that the outcome of evolutionary game on complex networks, in terms of cooperative behavior, strongly depends on topology but also on the update rule used.
- Since there are not any a-priori reasons to fix the update rule or to impose one over the others, we treat update rule in the same manner as strategy: we let it evolve, allowing the system itself choose what he “likes” most.
- We want to see if the coexistence of different update rules, in association with different underlying topologies, changes the overall cooperative behavior present in literature.

# Experimental setup

**Game:** Weak Prisoner's Dilemma with payoff-matrix given by:

$$\begin{array}{c c} & \begin{array}{cc} \mathrm{C} & \mathrm{D} \end{array} \\ \begin{array}{c} \mathrm{C} \\ \mathrm{D} \end{array} & \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \end{array} \qquad \text{with } b \in [1, 2] \,.$$

**Update Rule:** REP, UI, MOR;

**Topologies:** SF ($\alpha = 0$), Intermediate ($\alpha = 0.5$), ER ($\alpha = 1$)
$N = 5000$ and $\langle k \rangle = 6$;

Other information:

- Pairwise game with two different update rules per "realization";
- Initial fraction of players with a certain rule $x_{\text{rule}}(0) \in [0, 1]$ ;
- Initial fraction of cooperators and defectors $f_C(0) = f_D(0) = 0.5$;
- Payoff does not accumulate through gaming and the update of the strategies is synchronous.
- Dynamic evolution of the system = 4000 game rounds;
- All simulations averaged over 100 different realizations for each set of parameters;

# Results

Results are analyzed looking at the behavior of two quantities (with respect to the temptation parameter *b*):

- Average cooperation level in the asymptotic regime $\langle C \rangle = \frac{n_C}{N}$;

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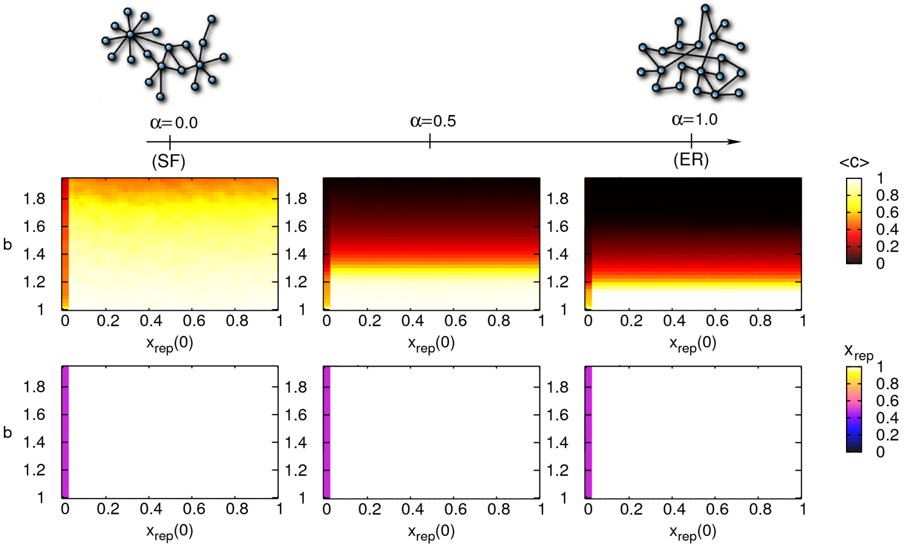
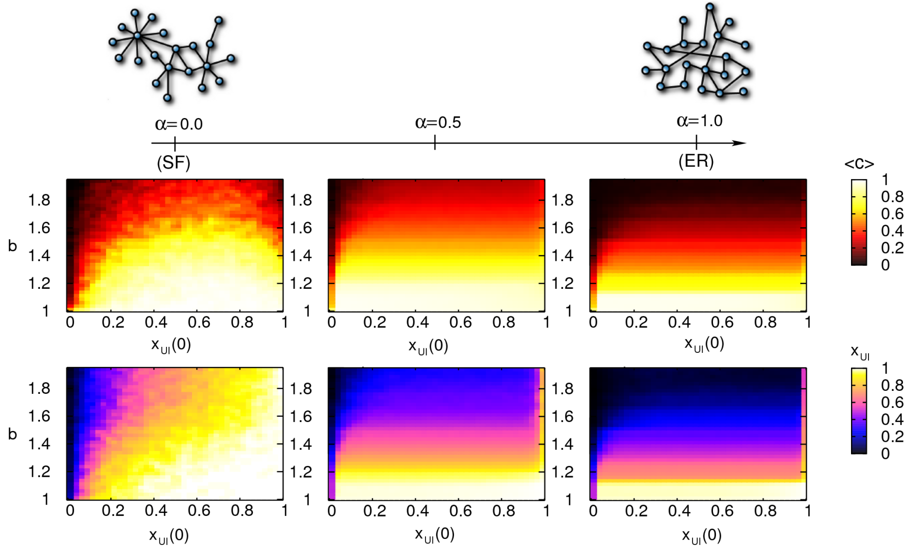
- Average cooperation level in the asymptotic regime $\langle C \rangle = \frac{n_C}{N}$;
- Average final fraction of players with a certain rule $\langle x_{\text{rule}} \rangle = \frac{n_{\text{rule}}}{N}$.

# MOR vs REP

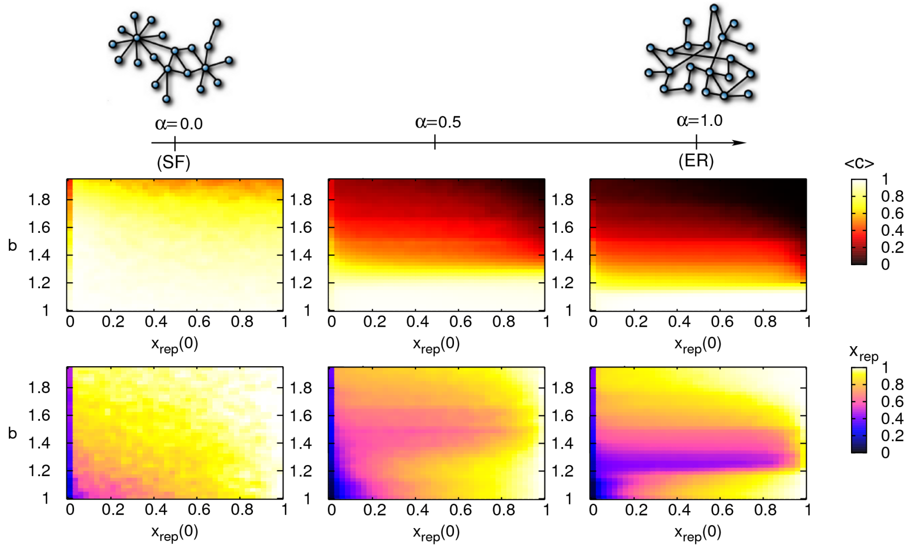


α=0.0
α=0.5
α=1.0
(SF)
(ER)
<c>
1
0.8
0.6
0.4
0.2
0
b
1.8
1.6
1.4
1.2
1
0
0.2
0.4
0.6
0.8
1
x_rep(0)
x_rep

# MOR vs UI



# REP vs UI



α=0.0
(SF)
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<c>
1
0.8
0.6
0.4
0.2
0
b
1.8
1.6
1.4
1.2
1
0
0.2
0.4
0.6
0.8
1
x_rep(0)
x_rep

# Conclusions

## Conclusions

- Evolutionary dynamics on networks is very different from the mean-field problem;
- Evolution on Scale-Free networks allows survival of cooperation even when the temptation to defect is relatively high;
- Co-evolution shows that is possible to obtain relatively large cooperation values when two update rule coexist in contrast with the single rule case. In particular we observe this in:
  - REP vs UI in ER networks;
  - MOR vs UI in SF networks;

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  - + REP vs UI in ER networks;
  - + MOR vs UI in SF networks;

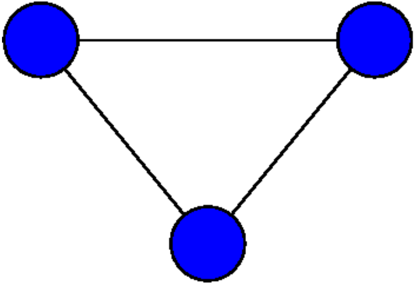
## Further Developments

- Consider different time scales for the game and change of the update rule processes;
- Study the scenario in which three rules are used simultaneously;
- Consider the use of other update rules (eg. Fermi rule) and/or other games (eg. Stag Hunt);

# Details on the interpolation model

## Interpolation Model

1. Start with a *seed network* with $m_0$ nodes fully connected between them;



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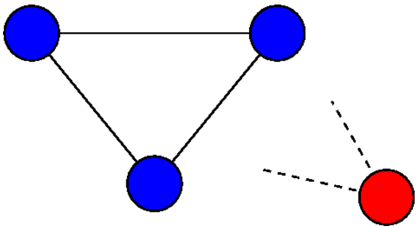
2. Add $U = N - m_0$ nodes, each with $m \leq m_0$ links;



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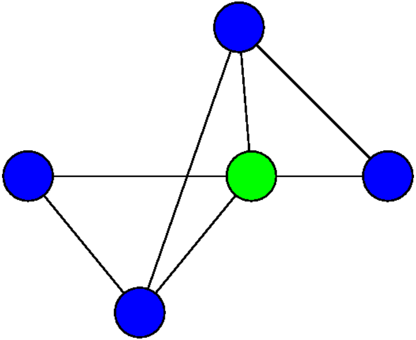
2. Add $U = N - m_0$ nodes, each with $m \leq m_0$ links;
3. Each link has a probability $\alpha$ to be attached *randomly* to one of the $N - 1$ other nodes, and a probability $1 - \alpha$ to be attached using the preferential attachment model of Barabasi and Albert;



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